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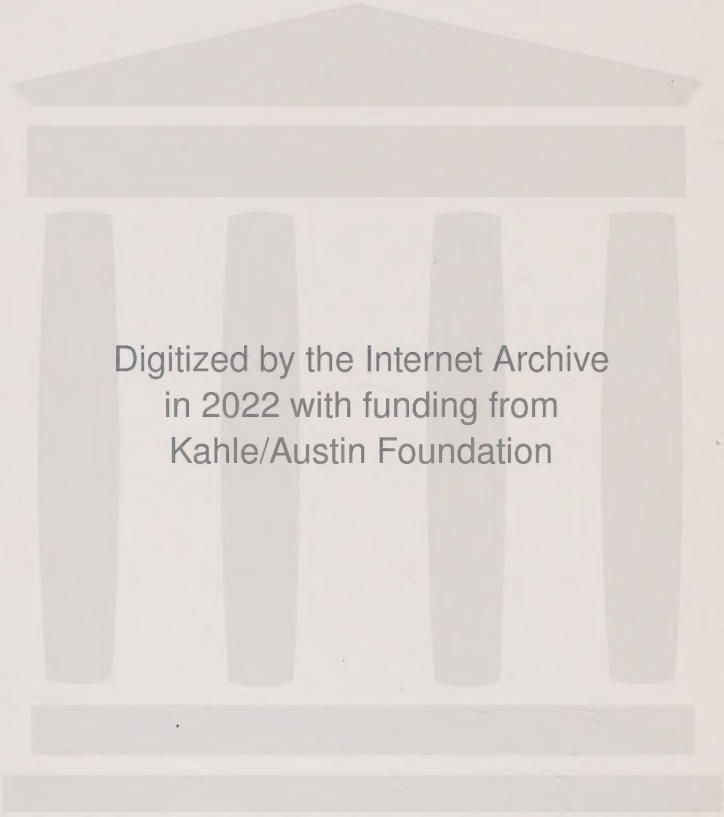
PRINCIPLES UNDERLYING THE DESIGN OF ELECTRICAL MACHINERY



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PRINCIPLES UNDERLYING THE DESIGN OF ELECTRICAL MACHINERY

BY

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PREFACE

THIS book is developed from a course of lectures, given at Columbia University and from the author's own experience as a designing engineer. It is to some extent an amplification of the series of articles on the Design of Electrical Machinery, written by the same author for Pender's "Handbook for Electrical Engineers."

An important feature of the present work is the derivation of formulæ from fundamental principles, an explanation of each formula and the reasons for the various standards of practice, all of which are explained in such a manner as to convey a mental picture of the fundamental physical phenomena leading to them.

The object is to give a practical method of design, with explanations of the physical meaning of the arbitrary constants used by the professional designer.

It is hoped that the book will be found useful, not only as a text for a course on design in technical schools, but also as an aid to the young engineer by explaining the "why" of certain conventional practices.

A systematic method of procedure and a complete sample calculation are given for the design of each type of machine treated. The tables also give complete specifications and the results of calculations on several actual modern machines of each type.

The calculations of the mechanical features have been purposely omitted as they are considered to be outside the scope of the book, which is strictly electrical.

The methods and the data are the result of an experience of seven years' service in the Design Department of one of the large manufacturing companies and sixteen years' experience in teaching the subject.

The author has found the works of E. Arnold to be most helpful in enabling him to correlate theory and practice, and wishes to

acknowledge here the influence these books have had upon his treatment of the subject. He also wishes to express his thanks to the General Electric Company, to the Westinghouse Company, and to the members of their staffs, for data and for illustrations for the book.

W. I. SLICHTER.

CONTENTS

CHAPTER I

General Principles and Fundamental Relations 1

CHAPTER II

The Continuous-current Generator 27

CHAPTER III

The Salient-pole Alternating-current Generator 124

CHAPTER IV

The Turbine-driven Alternating-current Generator 196

CHAPTER V

The Transformer 226

CHAPTER VI

The Induction Motor 268

Index 309

THE DESIGN OF ELECTRICAL MACHINERY

CHAPTER I

GENERAL PRINCIPLES AND FUNDAMENTAL RELATIONS

Current (I) expresses the quantity (coulombs) of electricity flowing past a given point in one second. The practical unit is the (international) **ampere** or the unvarying electric current which, when passed through a solution of nitrate of silver in water, prepared in accordance with authorized specifications, deposits silver at the rate of 0.001118 gram per second.

A direct current (d.c.) is, by convention, one that is always in the same direction although it may pulsate in value. It is, however, very common practice to use the term direct current to designate what is defined by convention as a continuous current.

A **continuous current** (c.c.) is, by convention, a direct current of constant value as distinguished from one of pulsating value.

Potential (E or V), or difference of potential between two points, is theoretically the work or energy required to move a unit quantity or charge of electricity from one point to another. It is analogous to difference in pressure, or head, of a current of water. The practical unit is the (international) **volt,** or that electrical pressure which, when steadily applied to a conductor whose resistance is one ohm, will produce a current of one ampere. For comparison and calibration the Weston Normal Cell is used, the electromotive force of which is approximately 1.0183 volt at a temperature of 20° C. These cells are calibrated by the Bureau of Standards at Washington.

Resistance (R) in an electric circuit is the property of the conductor that limits the quantity of current caused by a steady

voltage. The (international) **ohm** is the practical unit of resistance and is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, having a mass of 14.4521 grams, a constant cross-section, and a length of 106.3 centimeters.

The unit of **power** is the **watt** or the **kilowatt.** A watt is the product of one volt by one ampere, or is the rate of expenditure of energy by one ampere in a circuit having a resistance of one ohm. A kilowatt (kw.) is 1000 watts. A horse power (h. p.) in the United States is 746 watts. The metric horse power is 735 watts.

The theoretical unit of **energy** or **work** is the **joule,** which is the energy expended by 1 watt in 1 second. In terms of physical units, 1 joule is 10^7 ergs. The commercial and engineering unit is the **kilowatt-hour,** which is equal to 3,600,000 joules or watt-seconds. One kilowatt-hour (kw.-hr.) equals 2,655,000 ft.-lbs.

The **maxwell** (Φ) is the unit quantity of magnetic flux and is equal to 1 line of magnetic force.

The **gauss** (B) is the unit of magnetic flux density and is equal to 1 line per square centimeter, or 1 maxwell per square centimeter. B is also used to represent flux density in lines or maxwells per square inch and is so used in this book.

The **gilbert** (F) is the unit of magnetomotive force (m.m.f.) and is that m.m.f. which will produce 1 line or 1 maxwell in a path in air having a cross-section of 1 sq. cm. and a length of 1 cm. Ampere-turns are also used as a unit of m.m.f. One ampere-turn = 1.257 gilberts.

The **oersted** (S) is the unit of magnetic **reluctance** and is the reluctance of a path that requires 1 gilbert to set up a flux of 1 maxwell. Reluctance limits the quantity of flux which is caused by an m.m.f. Reluctance is proportional to the length of the circuit and inversely proportional to the cross-section and permeability (see below) of the circuit. Reluctance, S, in oersteds equals $l/\mu a$ where l = length in centimeters. μ equals permeability, and a equals cross-section in square centimeters.

Permeability (μ) is the ratio of the flux density that would be set up in a magnetic path in any material compared to the flux density that would be set up in a path in air of the same dimensions, the same m.m.f. being used in both cases. It is a characteristic of the material of the path. Iron, steel, and nickel have a

permeability greater than unity. Air and most other materials have a permeability of unity.

Permeance (P) of a magnetic path is the quantity by which the m.m.f. must be multiplied to obtain the flux. It is directly proportional to the permeability of the material and the cross-section of the path, and inversely proportional to the length of the path. It is the reciprocal of the reluctance $P = \mu a/l$.

The **henry** (L) is the unit of **inductance**. It is the inductance of an electric circuit in which a variation in current of 1 ampere in 1 second will induce an electromotive force of 1 volt. The inductance in henries of an electric circuit of s turns, interlinked with a magnetic circuit of permeance, P, is $L = 0.4\pi s^2 P/10^8$.

The **effective resistance** of an electric circuit carrying an alternating current is found by dividing the total power loss in watts in the circuit, due to the combination of true resistance, eddy currents, and hysteresis, by the square of the effective value of the current.

The action of all electrical machinery is based upon a very few fundamental principles, and it is a great help to the thorough understanding of the formulae used to have a good, clear mental conception of their physical meaning. The following review includes the fundamental relations that are most frequently met in design work.

A Unit Magnet Pole.—Any magnet exerts a force on an adjacent piece of iron or another magnet. If two magnet poles are brought into proximity to each other, the force between them is proportional to the strength of the magnets and inversely proportional to the square of the distance between them; thus, if m_1 and m_2 are the strengths of the two magnet poles, r is the distance between them, and f is the force, then: $f = \frac{m_1 m_2}{r^2}$.

Two magnet poles of equal strength, such that when r is 1 cm. the force is 1 dyne,[1] are called unit poles.

In the neighborhood of any magnet there is said to be a field of force. This means that in that neighborhood any other magnetic material would be subject to a force, just as near the Earth every object is subject to the force of gravity.

[1] 445,000 dynes = one pound.

Intensity of Field.—The intensity of a magnetic field (of force), H, is measured by the amount of force (number of dynes) which that field exerts on a unit pole placed in it. The intensity of a magnetic field is represented by imaginary lines drawn in the direction of action of the force, the number of lines per square centimeter being made equal to the units of force on the unit pole.

Thus, if a field is said to have an intensity of $H = 10$ units, this means that a unit pole in that field would be acted upon with a force of 10 dynes, and that the field would be represented by 10 lines in every square centimeter.

At 1 cm. distance from a unit magnet pole, the force on another unit pole is everywhere 1 dyne. Thus the field intensity is unity and is represented by 1 line per square centimeter. The surface of a sphere with a radius of 1 cm., surrounding a unit pole, is 4π sq. cm. Hence there are 4π lines traversing its surface, and therefore there are 4π lines diverging from a unit magnet pole.

As the quantity 4π is constantly met in all calculations of the magnetic circuit, it is important to get a physical conception of the hypothetical unit pole with the 4π or 12.57 hypothetical lines radiating from it, since this is the reason for the use of the quantity 4π in all our magnetic calculations. (Fig. 1.)

FIG. 1.—Visualization of a Unit-magnet pole.

FIG. 2.—Work done by a conductor carrying a current and moving in a magnetic field.

Current, Magnetism and Work.—Experiment shows that if a conductor having a length of L cms., carrying a current of I absolute units,[2] is situated in a magnetic field of H lines per square centimeter, the conductor is acted upon by a force which has a value, $f = H \times I \times L$, dynes. If we move the conductor through a small distance, ds (see Fig. 2), against the force, work or energy is exerted or expended.

$$\text{Work} = f\,ds = HIL\,ds \text{ (ergs)}.^{3}$$

[2] One absolute unit of current equals 10 amperes.

[3] 10^7 ergs equal 1 watt-second equals .738 ft.-lb.

$L\,ds$ is an area, a, and in it there are $H \times a$ lines of magnetic force which may be designated by $\phi = Ha$. Thus the work is $I\phi$.

Transforming this into amperes and practical units of work, we say that if a conductor carrying I amperes is moved in a magnetic field so that it cuts ϕ magnetic lines, then an amount of work has been done equal to $I\phi \times 10^{-8}$ watt-seconds or $I\phi \times .738 \times 10^{-8}$ ft.-lbs.

Magnetomotive Force (m.m.f.).—This is the name given to a quantity which is not a force at all but is the measure of the ability of a current to produce magnetism and is expressed in units of work or energy. M.m.f. is a difference in magnetic potential and is measured by the amount of work, in ergs, required to move a unit magnet pole from one place to another in a magnetic field. The work done in moving a unit magnet pole through a loop of wire having S turns, and carrying a current of I absolute units, so that all the flux cuts all the turns of wire, is $\phi \times SI$. See Fig. 3. But the flux from the unit pole is 4π lines, thus $\phi = 4\pi$ and the work is $4\pi SI$ ergs. This amount of work is the measure of the difference in magnetic potential passed through in moving the pole. Hence the m.m.f. of a coil of wire of S turns is $4\pi SI$ or, if I is to be in amperes, then the m.m.f. $= .4\pi SI$ (ergs).

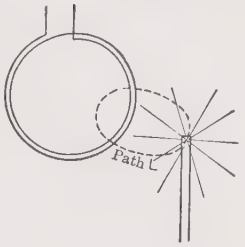


FIG. 3.—Measurement of the magnetomotive-force of a coil of wire.

Flux Produced by a Turn of Wire.—In performing the preceding experiment (Fig. 3), let L be the length in centimeters of the complete path through which the unit pole has been moved. Since the total work done was $4\pi SI$ when I was in absolute units then the average or mean force against which the pole was moved was

$$f = \frac{\text{total work}}{\text{total distance}} = \frac{4\pi SI}{L} \text{ dynes.}$$

But the force on a unit pole is a measure of the field intensity, H; thus the average intensity, $H = \dfrac{4\pi SI}{L}$ or if I is in amperes, $H = \dfrac{.4\pi SI}{L}$ = lines per square centimeter. If the density in any

particular area, of a square centimeter, is H, then the total flux passing through a is $Ha = \frac{.4\pi SIa}{L} = \phi$. $\frac{a}{L}$ is the permeance (P) of the magnetic path in air. Thus $\phi = .4\pi SIP$. If the material is iron or steel of which the permeability is μ, then $\phi = \frac{.4\pi SI\mu a}{L}$. Sometimes it is more convenient to use the Reluctance, which is the reciprocal of the permeance, $R = \frac{1}{P}$ and $\phi = \frac{.4\pi SI}{R}$. In figuring the main magnetic circuit of most machines, the reluctance of the circuit is most convenient to use because the different parts are in series; but in figuring the leakage flux of all machines, both in field and armature and in figuring the distribution of flux in the main poles of turbo-generators, it is more convenient to use the permeance because the different paths are in parallel with each other. If L and a are in inches and square inches, then $\phi = \frac{3.2SI\mu a}{L}$.

Magnetism and E.m.f. The Generation of an E.m.f.—It may be demonstrated experimentally that, if a conductor be moved in a magnetic field so that it cuts magnetic lines of force, an electromotive force is set up in the conductor, and if a circuit is provided to connect the two ends of the conductor, a current will flow which will develop heat energy or do work (see Fig. 2). The heat energy is known to be $EIdt$ where E is the difference of potential set up, I is the current, and dt is the length of time the current flows. If work is done on the conductor by moving it through a small distance, ds, then the flux cut is $HLds = d\phi$, the work done is $Id\phi$ and may be called the input, while the heat energy, $EIdt$, is the output. From the law of conservation of energy we have, ignoring losses, input = output; therefore $Id\phi = EIdt$, whence $E = \frac{d\phi}{dt}$ in absolute units of potential. If instead of one length of conductor we have S turns in series, the effect is multiplied S-fold, as this is the same as multiplying the length by S, hence $E = \frac{Sd\phi}{dt}$. To express this in the practical units, volts, we note that 10^8 absolute units of potential equal 1 volt; thus we have the very useful formula $E = \frac{Sd\phi}{10^8 dt}$ where E

is volts, S is number of turns, ϕ is number of lines, and dt is time in seconds.

Self-inductance.—If a coil of wire of S turns surrounds a magnetic path of reluctance, R, and has a current of I amperes sent through it, the flux that is set up in the coil changes from O to $\phi = \frac{.4\pi SI}{R}$ when the current changes from O to I amperes. $R = \frac{l}{a}$, where l is the length and a the cross-section of the magnetic path in centimeters and square centimeters. If all this increasing flux cuts the S turns of wire it induces an e.m.f. of $E = \frac{Sd\phi}{10^8 dt}$ volts, where $d\phi/dt$ is the rate at which the flux increases, expressed in lines per second. Substituting for ϕ its value above, we have $E = \frac{S}{10^8}\frac{d}{dt}\left(\frac{.4\pi SI}{R}\right)$. In this equation I is the only quantity that varies with time, provided the coil is in air, where the permeability, and hence the reluctance, is constant. Therefore,

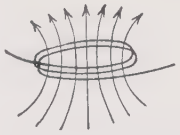


FIG. 4.—Coil being cut by flux produced by its own current.

$$E = \frac{.4\pi S^2}{10^8 R}\frac{dI}{dt} = L\frac{dI}{dt},$$

where L is called the coefficient of self-inductance, is expressed in henries, and is equal to $\frac{.4\pi S^2}{10^8 R}$.

This is the physical value and meaning of the coefficient of self-inductance. It is proportional to the square of the number of turns in the electric circuit and inversely proportional to the reluctance of the magnetic circuit enclosed by the coil.

Another useful relation and convenient formula for determining the self-inductance of a circuit, enclosing magnetic material at a particular value of flux or flux density, is deduced as follows:

Since $E = \frac{S}{10^8}\frac{d\phi}{dt}$ and $E = L\frac{dI}{dt}$ then $L = \frac{S}{10^8}\frac{d\phi}{dI}$ or $= \frac{S}{10^8}\frac{\phi}{I}$.

If the permeability μ is constant (air), then the rate ϕ/I is constant and L is constant. If the permeability changes with ϕ, and I as in steel, then ϕ/I and L are not constant and the calculation can be made only for a particular value of ϕ.

Examples

1. How many magnetic lines (maxwells) diverge from a magnetic pole having a strength of 90 units?

If a unit pole has 12.56 lines, then a pole of 90 units has $90 \times 12.57 = 1130$ lines.

2. A magnet pole has a strength of 90 units. What is the intensity of field at 3 cm. from the pole?

A unit pole placed at 3 cm. from the given magnet would be acted upon by a force of: $f = \frac{mm}{r^2} = \frac{1 \times 90}{9} = 10$ dynes.

Hence the intensity of field, H, would be 10 gausses.

3. A direct-current motor of four poles has 400,000 lines of flux emanating from each pole; the armature has 784 conductors on it, each carrying 12 amperes. What is the work or energy per revolution?

The total flux cut by each conductor in one revolution is $4 \times 400{,}000 = 1{,}600{,}000$ lines. The total product of current $\times$ conductors is $12 \times 784 = 9408$ ampere-conductors; and

$$\text{work} = \phi I \times \frac{.738}{10^8} = 1{,}600{,}000 \times 9408 \times .738 \times 10^{-8}$$
$$= 110 \text{ ft.-lbs. per revolution.}$$

If the armature is running at 1200 r.p.m. the power will be

$$110 \times 1200 = 132{,}000 \text{ ft.-lbs. per minute}$$

or

$$132{,}000 \div 33{,}000 = 4 \text{ horse power.}$$

4. What is the m.m.f. of a coil of 100 turns carrying 9 amperes?

M.m.f. $= .4\pi sI = .4\pi \times 100 \times 9 = 1130$ gilberts.

5. The field poles of a certain machine have 1160 turns of wire each carrying 1.6 amperes. One of the leakage paths in the air between the poles has a cross-section of 20 sq. ins. (129 sq. cm.) and a length of 5 ins. (12.7 cm.). How many lines of flux pass across this path?

NOTE.—The field coils of the two adjacent poles combine to force flux across this path.

$$\phi = \frac{.4\pi \times 1160 \times 1.6 \times 129 \times 2}{12.7} = 58{,}000 \text{ lines or maxwells.}$$

6. If a conductor is moved across a magnetic field so that it cuts 1 million lines of flux in 0.01 second, what is the voltage generated in it?

The rate of cutting is $1{,}000{,}000 \times 100$ lines per second, turns $= 1$.

$$\text{Volts} = \frac{1{,}000{,}000 \times 100}{10^8} = 1.$$

7. (*a*) A coil of 100 turns encloses a magnetic path in air of a cross-section of 20 sq. ins. (129 sq. cm.) and a length of 10 ins. (25.4 cm.). What is the self-inductance in henries? Reluctance, $R = \frac{25.4}{129} = .197$.

$$\text{Inductance, } L = \frac{.4\pi s^2}{10^8 R} = \frac{.4\pi \times 100^2}{10^8 \times .197} = .0064 \text{ henry.}$$

(*b*) When a current of 1.5 amperes flows through a coil of 1160 turns wound on an iron core, it sets up a flux of 500,000 lines. What is the self-inductance of the coil under this particular condition?

$$L = \frac{s \times \phi}{10^8 I} = \frac{1160 \times 500{,}000}{10^8 \times 1.5} = 3.86 \text{ henries.}$$

MATERIALS OF DESIGN

The materials used in a dynamo-electric machine may be grouped into four divisions according to their functions. These functions and the specific properties of the materials, which concern the designing engineer, differ according as the materials form a portion of one or another of the following parts of a machine:

Magnetic Circuit
Electric Circuit
Dielectric Circuit
Mechanical Structure.

Some parts, as the field frame, perform dual functions, magnetic and mechanical. In this case it is necessary to investigate two properties of the materials, magnetic permeability and mechanical strength.

The Magnetic Circuit

The magnetic circuit includes the field yoke, field poles, pole shoes, air gap, and armature laminations. For each of these parts (except the air gap), it is necessary to choose materials that have suitable magnetic properties. The various materials and their characteristics are as follows:

Cast iron was formerly much used for the field frame or yoke because it is the easiest and cheapest material to cast, but lately it has been almost entirely superseded by cast steel. The permeability of cast iron is so much less than that of cast steel that to provide a path of a given permeance the greater quantity of cast iron required would probably cost more than would the cast steel for the same magnetic permeance, in spite of the lower cost per pound of the cast iron. At present cast iron is seldom used except in small machines where bulk of material is not objectionable. The magnetic saturation curve of a typical sample of cast iron is given in Fig. 5. As cast iron is never used for an alternating flux, no data on hysteresis or eddy-current loss are given. For mechanical purposes cast iron is usually operated at a tensile stress of from 3000 to 4000 lbs. per square inch. Cast iron weighs 0.26 lb. per cubic inch.

Cast steel is used for the field frame of most d.c. machines and for the pole pieces of many. It is used for the field yoke, or spider, of all slow-speed and moderate-speed alternators and for the pole pieces of a very few alternators and synchronous motors. The saturation curve for a representative grade of cast steel is given in Fig. 5. For mechanical purposes it may be operated at a tensile stress up to 16,000 lbs. per square inch. Its weight is figured on a basis of 0.28 lb. per cubic inch. In the yoke, or spider, of revolving field alternators, the metal has to withstand the centrifugal force, and therefore the mechanical stresses must be as carefully investigated as are the magnetic conditions.

Forged steel is used for the entire revolving-field structure of many high-speed turbo-generators It has excellent mechanical properties and very good magnetic permeability. The magnetic properties of a typical grade of forged steel are shown in Fig. 5. The maximum working tensile stress is 16,000 lbs. per square inch. This is very important, as the proportioning of the revolving structure to withstand the great centrifugal forces due to the necessary high peripheral velocities is the controlling feature in the design of turbine-driven a.c. generators. The weight of forged steel is 0.28 lb. per cubic inch.

Laminated-steel or sheet-steel punchings are used for the armature core and teeth of all types of machines, for the pole pieces of most alternators and some d.c. machines, and for the revolving field of some turbo-alternators (small sizes). Laminated steel is also used for the cores of all transformers. Sheet steel is available in various thicknesses and qualities. The most common thicknesses are 0.014 in. (3.5 mm.) for armatures and transformers, and 0.025 in. (6.5 m.m.) for field structures. Of the various grades of sheet steel, the most common for electrical machinery are:

(*A*) *Special sheet steel*, having a very high permeability to be used in special cases where high densities must be used or a low excitation is desired.

(*B*) *Commercial sheet steel*, which is a standard product having good permeability and reasonable coreloss. It is in most general use, particularly for field structures, pole pieces and for the armatures of most d.c. machines and in a.c. machines of moderate size and speed where the frequency or magnetic density is not excessive. It is also desirable where good mechanical strength is required.

(*C*) *Silicon sheet steel*, which has a lower permeability than

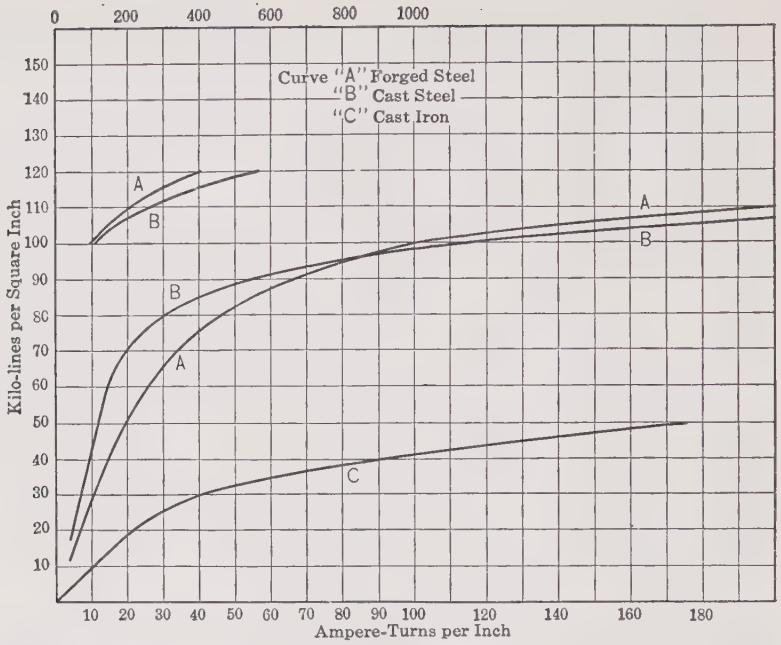


FIG. 5.—Magnetization Curves of solid samples of forged steel, cast steel and cast iron.

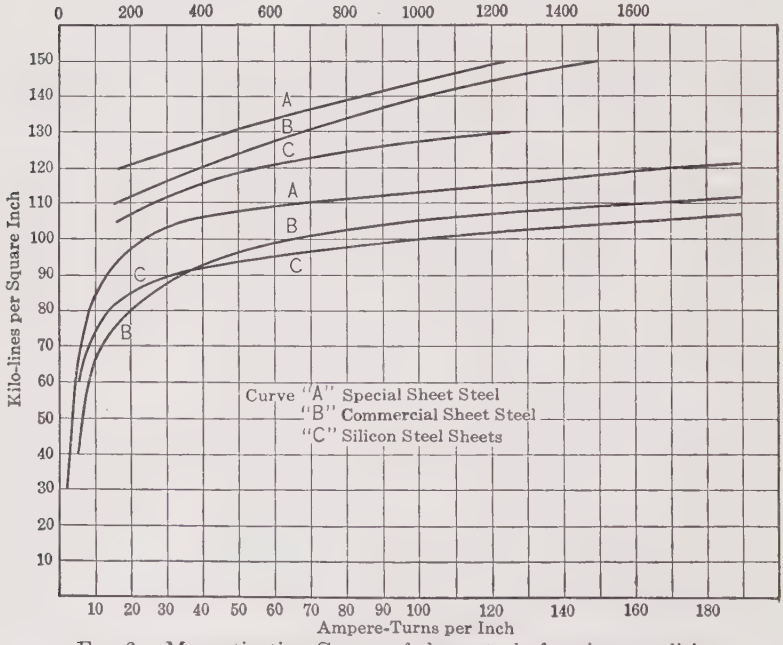


FIG. 6.—Magnetization Curves of sheet steel of various qualities

the commercial but also a much lower hysteresis and eddy loss. It is not as strong mechanically as the others. It is used in transformers, in stationary armatures, and in particular cases where the magnetic density or frequency is high and it is desired to avoid excessive coreloss.

Representative magnetization curves for these three materials are given in Fig. 6.

In air the number of ampere-turns per inch length of path is $\frac{B}{3.2}$ where B represents density in maxwells per square inch.

Representative curves for the hysteresis and eddy loss in standard or commercial sheet steel are given in Fig. 7. These are based on a value of η of .003 and of ϵ of .0002.

The hysteresis loss is expressed in watts per cubic inch for 1 cycle per second at various densities and any thickness of sheet. The total loss in any part is found by taking the value of watts per cubic inch for the proper density from the curve and multiplying it by the volume in cubic inches and the frequency in cycles per second.

The eddy loss is expressed in watts per cubic inch for 1 cycle per second at various densities and for a thickness of 0.014 in. (3.5 m.m.). The total loss in any part is found by taking the value from the curve corresponding to the proper density and multiplying it by the volume in cubic inches and by the square of the frequency.

For any other values of η or ϵ the losses will be directly proportional to the value of these constants.

The curves of Fig. 7 are useful when standard grades of steel are employed, but if a grade of steel is to be used whose quality, characteristics or constants differ from the conventional then it is much more convenient and practical to use the general formulae and to substitute in them the constants obtained by tests of samples or supplied by the steel manufacturer. For this purpose the following relations are used:

Hysteresis Loss in watts $= K_1 \eta V f B^{1.6}$,

where K_1 = a constant, 0.00525;

η = a constant, see table below;

V = volume of steel in cubic inches;

f = frequency in cycles per second;

B = magnetic density in kilolines per square inches.

Eddy-current Loss in watts $= K_2 \epsilon V(tfB)^2$,
where $K_2 = 0.254$ a constant;
ϵ = a constant, see table below;
V, f, B as before;
t = thickness of sheet in inches.

TABLE I

Coreless Constants in Sheet Steel

Material	A	B	C	D
	η	Watts per Pound, Hysteresis	ϵ	Watts per Pound, Eddy
Silicon steel	.0006–.0015	.53–1.32	.00004–.0001	.104–.26
Standard steel	.001 – .004	.88–3.52	.00012–.00025	.312–.66
Cast steel	.003 – .012	2.64–10.5		
Forged steel	.015 – .025	13.–22.		

Columns B and D give the loss in watts in 1 lb. of material at a frequency of 60 cycles and at a magnetic density of 10,000 gausses (64,500 lines per square inch) in sheets having a thickness of 0.014 inch (3.57 mm.). This is the generally accepted specification for comparative tests on sheet steel (Am. Soc. for Testing Materials).

The Electric Circuit

The **electric circuit** consists of the armature winding and the field winding in the majority of dynamo-electric machines, and the primary and secondary windings of transformers, induction motors and induction regulators. These windings consist of single wires, stranded wires, or solid bars, of copper. The wires or bars are usually individually insulated (see "Dielectric Circuit") and wound in coils of various shapes on special models or forms and pressed to an exact shape before being placed on the machine.

The use of these **form-wound coils** is almost universal at present and about the only exceptions are the small fractional horse-power motors where the coils are sometimes wound directly

on the armatures. Some of the most usual types of form-wound coils are shown in Figs. 8 to 11.

Rectangular or flattened wires are sometimes used in order

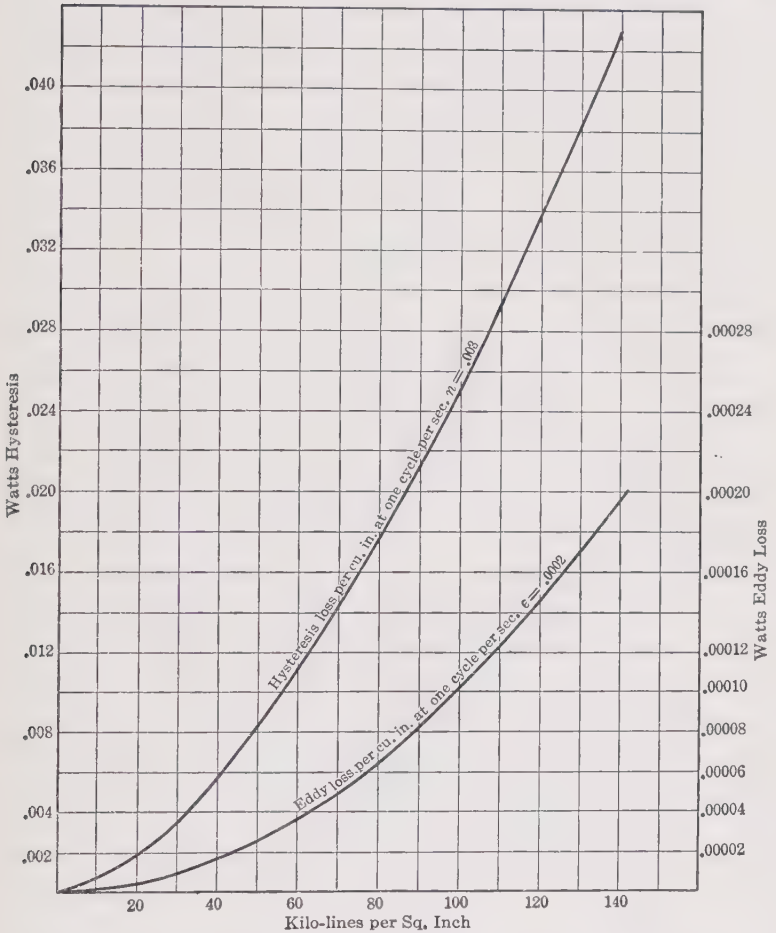


FIG. 7.—Hysteresis and Eddy Current Losses in sheet steel of thickness of .014 inch for $\eta = .003$ and $\epsilon = .00002$.

to get more copper into a slot of a given size and thus reduce the I^2R loss of the machine. It is evident that the waste space between round wires is greater than between rectangular wires packed together. Stranded wires or bars made up of many strands

pressed into rectangular shape are frequently used to reduce the loss from eddy currents as this loss becomes very great in solid bars if the magnetic density is very great or the rate of change of flux density, i.e., the frequency, is high.

The conductors of electric machines are usually made of annealed soft-drawn copper, the electrical resistance charac-

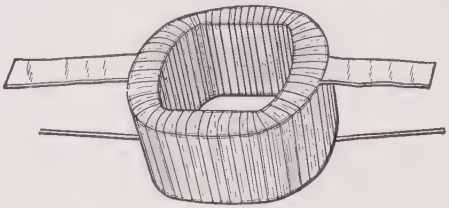


FIG. 8.—Field coil of a compound machine showing terminals of series winding and of shunt winding.

teristics of which are given in Table II. This is called copper of 100 per cent conductivity and the figures are taken from the values given by the Bureau of Standards in Circular No. 31 and adopted as standard by the American Institute of Electrical Engineers.

TABLE II

CHARACTERISTICS OF COPPER AS A CONDUCTOR, 100 PER CENT CONDUCTIVITY

Temperature, Centigrade	0	25	75	100
Resistance of 1000 ft. of 1 sq. in.	.0075	.0083	.0099	.0107
Resistance of 1 ft. of 1 cir. mil.	9.55	10.55	12.6	13.63
Relative resistance	1.00	1.108	1.32	1.427

The sizes of round wires are classified in an arbitrary convention, the standard for the United States being known as Brown and Sharpe (B. & S.) gauge or the American Wire Gauge (A. W. G.) these being two names for the same thing. Table III on page 18 gives characteristics of the various B. & S. sizes of copper wire of 100 per cent conductivity.

The variation in the resistance of a copper wire due to a change of temperature is frequently used to determine the temperature of

an electric machine. The exact law for the rate of change is fairly complicated, but for all practical purposes the formula

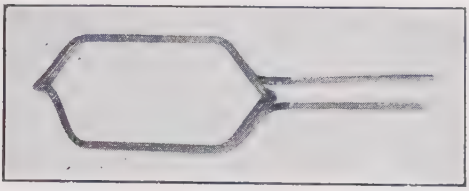


FIG. 9.—Wire wound coil for an armature winding.

$R_t = R_o\ (1 + .00427\ t)$ gives the relation with sufficient accuracy. In this, R_o is the resistance at 0° C. R_t is the resistance at any

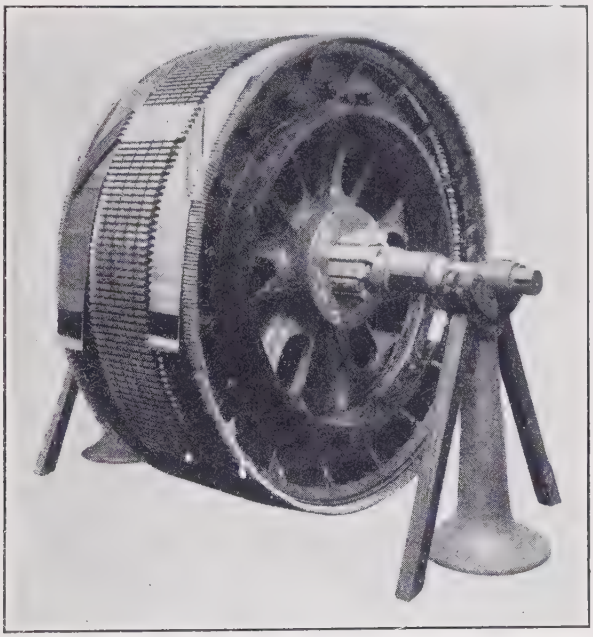


FIG. 10.—Armature with winding of rectangular bars in process of construction.

other temperature t degrees centigrade above zero. A convenient method of using this on the slide rule is expressed in the formula:

$$\frac{R_t}{R_o} = \frac{234 + t}{234}.$$

It is customary to operate the copper conductors of electric machines at densities of current from 1000 amperes per square inch to 3000. On small or very well ventilated machines or parts of machines, the higher values are found.

TABLE III

Wire Table

B. & S. or A. W. G. Size	Bare Diameter, Inches	Double Cotton-covered Diameter, Inches	Area, Circular Mils	Weight, Pounds per 1000 Ft.	Resistance Ohms per 1000 Ft.	
					25° C.	75° C
	1.152	.....	1,000,000	3050	0.0106	0.0127
	1.035	.....	800,000	2440	0.0132	0.0158
	0.891	.....	600,000	1830	0.0176	0.0210
	0.819	.....	500,000	1525	0.0211	0.0253
	0.728	.....	400,000	1220	0.0264	0.0316
	0.590	.....	250,000	762	0.0422	0.0506
0000	0.460	.....	211,600	640	0.0499	0.0596
000	0.4096	.....	167,810	507	0.0629	0.075
00	0.3648	.....	133,100	402	0.0794	0.095
0	0.3249	.....	105,500	318.8	0.1000	0.1195
1	0.2893	0.303	83,690	252.8	0.126	0.151
2	0.2576	0.272	66,370	200.5	0.159	0.190
3	0.2294	0.243	52,640	159	0.201	0.240
4	0.2043	0.216	41,740	126.1	0.253	0.303
5	0.1819	0.194	33,100	100	0.319	0.381
6	0.1620	0.174	26,250	79.3	0.402	0.48
7	0.1443	0.156	20,820	62.9	0.504	0.602
8	0.1285	0.140	16,509	49.9	0.640	0.765
9	0.1144	0.126	13,094	39.6	0.810	0.968
10	0.1019	0.112	10,381	31.4	1.017	1.215
11	0.0907	0.101	8,234	24.9	1.280	1.530
12	0.0808	0.091	6,529	19.7	1.617	1.93
14	0.0641	0.075	4,107	12.4	2.57	3.07
15	0.0571		3,257	9.9	3.26	3.89
16	0.0508	0.059	2,582	7.8	4.09	4.9
17	0.0453		2,048	6.2	5.22	6.23
18	0.0403	0.048	1,624	4.9	6.50	7.76
21	0.0285	0.0365	810	2.46	13	15.5

The loss of power in the copper due to I^2R is proportional to the volume of the copper and to the square of the current density,

but varies somewhat with the temperature. Thus at a temperature of 75° C. (a standardized reference temperature) the loss of power in watts in any copper conductor is

$$U^2V \times .825 \times 10^{-6}$$

where U = amperes per square inch;
V = cubic inches.

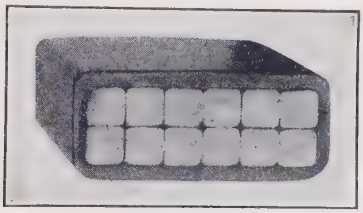


FIG. 11.—Cross-section of a coil showing insulation for high voltage.

THE DIELECTRIC CIRCUIT

The **dielectric circuit** consists of the insulating material which separates the copper conductors from each other and from the steel core and frame of the machine. The function of this circuit is to make it possible to arrange conductors lying adjacent to one another in a series circuit so that the voltages induced in them will be additive in a predetermined manner. It also confines the current to a definite useful path in the wires instead of allowing it to stray through the frame of the machine.

The properties which a good insulating material should have are five in number, namely:

1. A high resistance to the flow of current. This is known as **insulation resistance.**

2. The ability to withstand a high voltage across a small thickness without breaking down and permitting current to flow. This is known as **dielectric strength.**

3. A reasonable mechanical strength to withstand bending, vibration, abrasion, and shock.

4. The ability to retain its chemical and physical properties after being subjected to temperatures of from 80 to 125° C. for prolonged periods.

5. The ability to conduct the heat energy from the conductors

to the body of the machine without an excessive difference in temperature.

The insulation resistance of a machine is measured from the copper of the winding through the insulation to the frame of the machine. Its magnitude depends greatly upon the presence of dirt and moisture, particularly moisture. Drying or baking will usually increase the insulation resistance greatly. Usual values for the proper insulation resistance of machines vary from 50,000 to several million ohms. The higher values are for small machines and for high-voltage machines. (See "Standardization Rules" A. I. E. E.)

The dielectric strength of an insulating material is measured by applying an alternating voltage to a sample of the material of a specified thickness, and gradually increasing the voltage until the insulation breaks down, a spark passes, and an appreciable current flows. The root mean square of the voltage which caused the break-down is specified as the dielectric strength of the sample. A conventional figure for the dielectric strength of a material is often specified as the root mean square value of that alternating voltage which a sample of a thickness of 1 mil (0.001 in.) will stand for 1 minute. Values for the dielectric strength of several materials are given in Table IV, page 22.

The **mechanical strength** of an insulating material is not capable of being definitely specified. One test is to determine how many times a thin sample may be bent at right angles, first one way and then the other, without breaking.

In reference to the effect that a high temperature will have on them, insulating materials are divided into classes. The "Standardization Rules" of the A. I. E. E. state that cotton, silk, paper, and similar materials impregnated with varnish, will not stand a higher temperature than 105° C. without becoming permanently damaged, while mica, asbestos and similar materials can stand temperatures up to 125° C. without becoming permanently damaged. It is an inherent characteristic of electric machines that their various parts become heated during operation because the various losses turn into heat. Consequently the limit to the output of such a machine is the maximum temperature which its most delicate part will stand without permanent injury. The insulation is most easily injured by heat, hence the necessity of knowing what temperature it can stand and limiting the output

of the machine to that value which will not produce a temperature so high that the insulation will be ruined.

The ability to conduct heat is desirable, since the copper winding of the machine is usually the hottest part and the only way for this heat to get out is through the insulation. The better the heat-conducting qualities of the insulation the lower will be the difference in temperature between the copper and the surrounding air. It is unfortunate that all the materials that are insulators for electric currents are also very poor conductors of heat, so that the designer has to do the best he can to get around this tendency of nature. Impregnating cotton fabrics with oil or varnish and making them more solid is one way of making them better conductors of heat.

In high-voltage machines another phenomenon enters, namely the dielectric strain in the insulating material, which gradually breaks down its chemical composition, particularly at bends or opposite sharp edges of the core. This action depends upon the **specific inductive capacity** of the material and the stability of its chemical composition.

Insulating Materials.—Cotton is applied in the form of a spun braid to wires before they are wound into coils. **Single cotton-covered** wire has one layer on it, and this layer has a thickness of about 3 mils (0.003 in.) on each side. It is used only for small sizes. **Double-cotton covered** (d.c.c.) wire has a thickness of from 5 to 10 mils, the greater thickness on the larger wires. **Triple cotton covering** is sometimes specified for very large conductors or for machines in which the voltage between adjacent wires is very great. Wire tables giving the diameter over the cotton covering are available for design work (see Table III, page 18). The cotton is left dry and natural until the coil is made up, and then the whole coil is impregnated with a varnish in order to exclude moisture and to give the insulation better heat-conducting qualities. Cotton tape is also sometimes used to bind up the wound coil, usually for mechanical protection.

Enamel is a coating of a special varnish, usually black, applied to small wires, before use, by running them through a vat of the liquid and then through a drying-oven. The coating is very thin (0.001 in.) and is only used for very low voltages such as those occurring in field coils, or is reinforced by a cotton covering in addition. Enamel becomes brittle when dry and hence can

only be used on fine wires or in cases where the wire does not make sharp turns. Its most general use is in the field coils of medium-sized motors and the armatures of fractional horse-power motors.

Oiled or **varnished cambric** or **treated cloth** is used in insulating the group of conductors constituting a coil and in insulating large conductors or bars. It may be used either as tape or in sheets cut to convenient shapes. It has a high dielectric strength but deteriorates under a high temperature. The A. I. E. E. states that it is seriously deteriorated if the temperature exceeds 105° C. for an appreciable period of time. It is manufactured by impregnating sheets of cotton in a special form of linseed oil, then drying and smoothing before using.

TABLE IV

SPECIFIC DIELECTRIC STRENGTH OF COMMON INSULATING MATERIALS

Material	Average R.M.S. Volts per Mil at Break-down
Asbestos	125
Cotton, single c. c	275
Cotton, double c. c	225
Red fiber	200
Mica, pure	2000
Micarta	400
Mica-paper	300
Cloth, treated	500
Paper, treated	700
Paper, ordinary	150

GENERAL LIMITATIONS IN DESIGN AND CHOICE OF CARDINAL DIMENSION

In order to choose intelligently the approximate dimensions to be used in preliminary calculations, it is necessary to recognize the limiting factors in machine design and to know how these are related to each other.

In the design of every dynamo-electric machine, there are three fundamental limiting factors. If reasonable values are

assigned to these, a workable design is easily deduced. These factors are:

(*a*) The peripheral speed at the periphery of the armature or revolving part, usually expressed in feet per minute, and here represented by V. $V = \pi \times \text{r.p.m.} \times \frac{D}{12}$ if D is the diameter of armature in inches.

The allowable value of V depends upon the mechanical design of the machine and varies as follows:

2,000– 5,000 in small machines and slow-speed machines.
5,000–10,000 in large high-speed machines.
15,000–20,000 in turbine-driven machines.

(*b*) The magnetic density in the air gap or at the pole face, represented by B and here expressed in lines per square inch. The value of this is limited by the amount of m.m.f. we can afford to put on the poles and the amount of coreloss and consequent heating in the iron, as a high value of B in gap infers also a high density in the armature.

(*c*) The density of the current stream on the surface of the armature, expressed in ampere conductors per inch of periphery, that is, the product of the total number of conductors on the armature surface multiplied by the current in each conductor and divided by the perimeter of the armature. It can be seen that this is a measure of the total loss in the copper of the armature of the machine and of the heat produced thereby.

With these factors it is possible to deduce a simple relation controlling the main features of any design.

The force on any conductor of length l in inches, carrying a current I amperes in a field of B lines per square inch, is

$$f = BIl \times .885 \times 10^{-7} \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

On an armature the length l of conductor lying in an active field B is

$$l = ZL\rho, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where Z = total number of conductors;
L = length of armature body;
ρ = ratio of actual pole arc to the pitch of poles or ratio of armature perimeter covered by poles to total perimeter.

Pole arc is the arc subtended by one pole face measured along the periphery of the armature.

Pole pitch is the arc measured along the periphery of the armature from the center of one field pole (N pole, say) to the center of the next adjacent field pole (S pole).

Let I be the current in each conductor and D the diameter of armature in inches; then the ampere conductors per inch of periphery,

$$\sigma = \frac{ZI}{\pi D} \quad \text{or} \quad I = \frac{\pi D \sigma}{Z}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting in (1) the values for l and I from (2) and (3), we have

$$f = \pi B D \sigma L \rho \times .89 \times 10^{-7}.$$

The power of the machines in kilowatts is $\frac{fV \times .746}{33{,}000} = Q$, hence

$$Q = \frac{.89 \pi \rho B \sigma V D L \times .746}{33{,}000 \times 10^7} = \frac{\rho B \sigma V D L}{159 \times 10^9},$$

where Q = rating of machine in kilowatts;
ρ = ratio of pole arc divided by pole pitch;
B = magnetic density per square inch in gap or at pole face;
σ = ampere conductors per inch periphery of armature;
V = peripheral speed at perimeter of armature, feet per minute;
D = diameter of armature at face, inches;
L = length of armature or pole parallel to shaft, inches;
159 a constant for d.c. machines but a different value must be used for a.c. machines (see page 26).

This is the fundamental equation determining the approximate dimensions of a machine D and L and contains all the variables of design. By selecting suitable values for the arbitrary constants ρ, B, σ, and V from experience, judgment, or tables, we can at once determine reasonable values of D and L for preliminary calculations. With these preliminary values, several designs are calculated with slight variations in the constants, and by trial and comparison a final design is reached.

Discussion of the Arbitrary Constants

ρ, the per cent of pole arc, is governed by two antagonistic phenomena. For the sake of small magnetic leakage and good form factor of e.m.f. wave or good commutation, a low value of ρ is desired. For the sake of low reluctance to the main flux and economy of material, a high value is desired. The range in value is from .5 to .7 and the most usual value is .6 to .67.

B, the pole-face density, is limited by the exciting ampere-turns on the field required to produce the flux, the length of air gap, and sometimes by the heating of the iron from coreloss. The range is from 30,000 to 60,000. B depends to a great extent on the size and type of machine, and specific values recommended for each size and type are given in their proper place further on.

σ, the number of ampere conductors per inch of periphery at the gap, or the density of armature current stream, is limited by the copper loss it produces and the distortion produced by the armature m.m.f. If ventilation is good and the insulation thin, or if the slots are very deep, the values of σ may be high. The range of σ is from 250 to 1200 and it is very closely connected with the size and type of machine. It is more fully discussed in connection with each type.

V, the peripheral velocity, is altogether a function of the mechanical design and varies from 2500 to 25,000 ft. per minute. With the higher values very careful design of the mechanical strength is necessary. The usual values are fron 2000 to 6000.

D, the diameter of the armature (in inches), is usually set when V is chosen, since ordinarily the speed in revolutions per minute at which the machine is to run is one of the conditions given in the specifications of the problem. Thus $D = \frac{12V}{\pi N}$ where N is revolutions per minute.

If, however, the speed is not given, we can substitute in the equation $\frac{\pi DN}{12}$ for V and solve for D^2L, in which case we have a choice of interdependent values of D and L.

L, the length of armature between heads (inches), or the length of pole face parallel to shaft, is usually the only unknown quantity and is obtained from the equation by substitution of the chosen values for the other quantities.

For an alternating-current machine, the value of the constant 159 is changed to allow for the fact that the effective or virtual value of the alternating current is 1.11 times the average value and hence will produce 11 per cent more torque, consequently the machine may be that much smaller, or the constant should be $159 \div 1.11 = 144$.

CHAPTER II

THE CONTINUOUS-CURRENT GENERATOR

CHARACTERISTICS

The fundamental principle involved in the construction and operation of any type of dynamo-electric machine is the production of an electromotive force in one or more conductors by the relative motion of these conductors and a magnetic field. Such a machine may, as a general thing, be used either as a generator or as a motor.

In all dynamo-electric machines of the more usual forms (excepting the homopolar, or acyclic), the electromotive force induced in any one conductor or turn varies in value and alternates in direction. If the terminals of any coil or group of coils are brought out to an external circuit by means of revolving contacts, such as slip rings, an alternating current will flow in the circuit. In order to obtain a direct, or continuous, current it is necessary to add a rectifier or commutator. Thus the most simple and elementary form of electric generator is an alternating-current generator, and the direct-current generator is a special form, adapted by the addition of a commutator to give a continuous, or unidirectional, current.

Direct-current, or as it is more properly called, continuous-current, apparatus was in very general use before the alternating-current type was developed. It is simpler in its calculation, and the phenomena of its action are more easily understood.

Wherever the distance to which energy is to be transmitted is not a factor, there is no doubt that the continuous-current system is to be preferred. However, if energy is to be transmitted over long distances, the alternating-current system with its transformer has unequivocal advantages. The result of these two conditions is that a compromise is adopted. The majority of the generating stations provide alternating current and many of the

motors use continuous current supplied by synchronous converters or motor-generator sets. Thus the continuous-current apparatus and the alternating-current apparatus supplement each other rather than compete with or displace each other, since each has its own appropriate field.

The principal parts of a continuous-current machine are the field frame, field poles, field coils, armature. armature winding, commutator, and bearings.

The field frame is usually a large ring of cast steel or cast iron having sufficient cross-section to carry the main magnetic flux from pole to pole and sufficient mechanical strength to act as a support

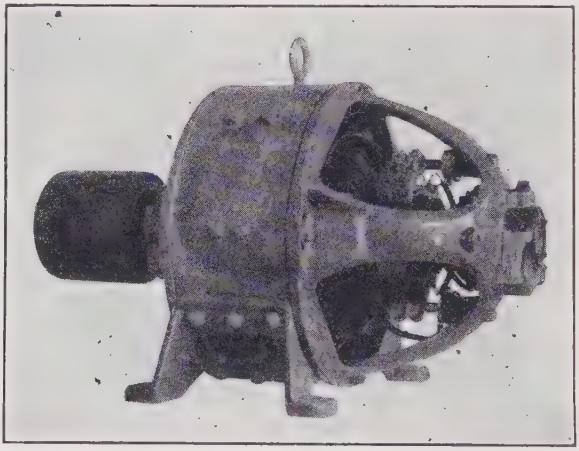


FIG. 12.—Small direct-current machine showing exterior of field frame.

for the field poles. It usually has cast integral with it the feet upon which it stands or the lugs which support it upon the bedplate.

The field poles, or **pole pieces,** are usually of cast steel or built up of laminated steel sheets, and are bolted to the field frame. At one end the pole pieces flare out to increase the surface facing the armature and to hold the field coils in place. This enlarged portion is called the **pole shoe.**

The field coils (Fig. 8) consist of spools of insulated wire wound concentric with the pole pieces and carrying the exciting current which is the magnetizing force that produces the flux in the poles.

The armature is a cylinder or drum, built up of laminated or sheet steel in the form of circular punchings. In the periphery of the cylinder are longitudinal slots into which coils of insulated wire are placed. These coils are connected up in a definite manner to form the **armature winding.** The armature is sometimes mounted directly on the shaft (in small machines) and sometimes upon a wheel-like structure known as the spider (in large machines).

The commutator is a cylinder built up of segments of copper insulated from each other and from the support by sheets of mica. Each segment is electrically connected to the connection between

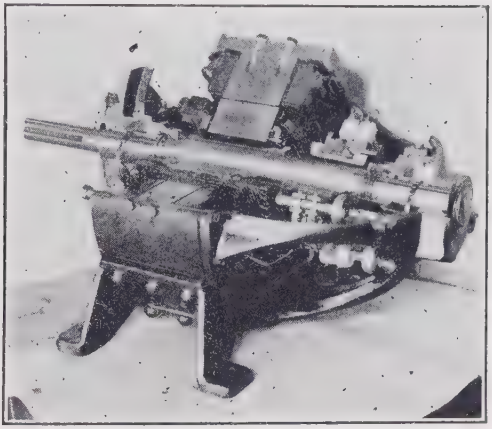


FIG. 13.—Direct-current machine cut through showing sections of frame, pole piece, armature and commutator.

two coils of the armature winding. Carbon brushes bear upon the commutator at definite positions to collect the current from the winding and lead it to the external circuit.

The bearings are either held in a frame-head attached to the field (small machines) or supported by pedestals upon the bed-plate (large machines).

In all continuous-current machines the armature revolves inside the field structure, and as the conductors of the armature winding move past the pole pieces they cut the magnetic flux which issues from these pole pieces, with the result that there is generated in them an electromotive force in accordance with the law discussed on page 35. The various conductors of the winding

may be connected in groups in series in order to obtain a high value of e.m.f. or may be connected in multiple groups in order to carry a large current.

VARIOUS TYPES OF CONTINUOUS-CURRENT MACHINES

Separately Excited Machine.—In this form the current for the field coils is supplied from some other machine. Although this was, rather naturally, the first type of machine built, it is no longer in general use. It is sometimes used, however, in large

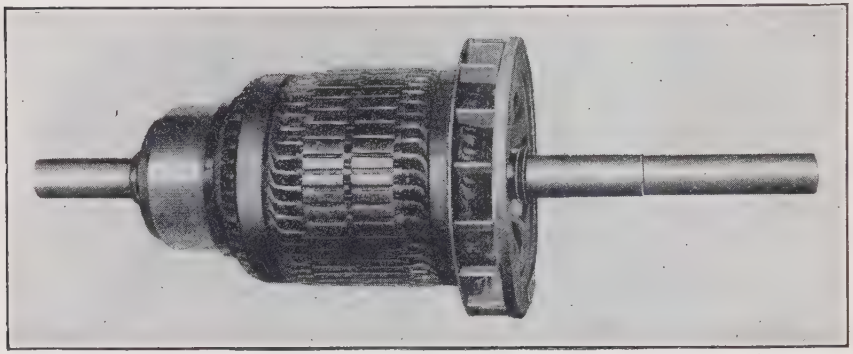


FIG. 14.—Armature of a d. c. machine showing commutator and ventilating fan.

stations where the exciting current is readily obtained from special bus bars and a voltage regulator is used.

Shunt Machine.—A shunt-wound machine, either generator or motor, is one in which the entire field excitation is derived from a circuit of many turns and high resistance, connected in **shunt** or in multiple with the armature circuit. The characteristic of a shunt machine is poor regulation, that is, the voltage of a shunt generator decreases as the load increases. This is so marked that some shunt generators may be short-circuited, their terminal voltage dropping to zero, without resultant harm. (See Figs. 16 and 17.)

Series Machine.—A series machine, generator or motor, is one in which the entire armature current flows through the field winding, the conductors being made large enough to stand it. The characteristic of a series generator is a terminal voltage of

zero at no load and increasing very rapidly as the current increases. Series generators are very seldom used, but series motors are very general, particularly for railway use.

Compound-wound Machine.—This type of machine has on each field pole, in addition to its shunt winding, a few turns of thick wire which carry the load current and are known as the **series turns,** or series winding. This winding tends to increase the excitation as the load increases and therefore tends to keep the terminal voltage of the machine constant or even to increase it. If the series-field windings are proportioned so as to cause

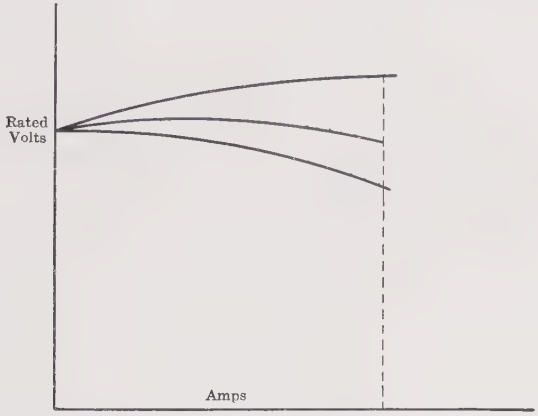


FIG. 15.—Compounding curves of a d. c. generator showing relation between terminal voltage and output in amperes. Top, over-compounded; middle, flat-compounded and lowest, under-compounded.

the terminal voltage at full load to be higher than the voltage at no load, the machine is said to be **over-compounded. A flat-compounded** machine has the same voltage at full load as at no load, and an **under-compounded** machine has a lower voltage at full than at no load.

A compound-wound machine may be connected **short shunt** with field connected directly to the terminals of the armature as in Fig. 16, or **long shunt** with the shunt field spanning both the armature and series field as in Fig. 17. The choice is merely a matter of convenience of station wiring.

Commutating-pole or Interpole Machines.—These machines have small auxiliary poles alternately placed with respect to the

main poles and excited by a few turns in series with the load. The effect of these poles is to improve the operation of the machines in the matter of commutation (see page 93).

Bipolar Machines.—Small continuous-current machines are usually of bipolar or two-pole type, the two poles projecting inward from a cylindrical field frame more or less enclosing the armature.

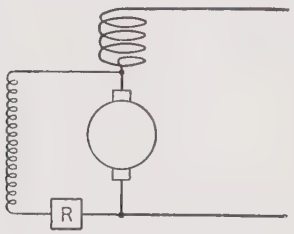


FIG. 16.—Circuit diagram of short-shunt compound-wound generator.

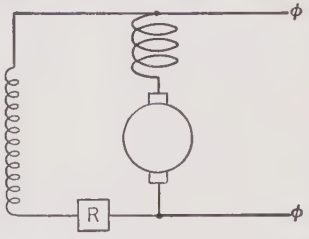


FIG. 17.—Circuit diagram of long-shunt compound-wound generator.

(See Fig. 18.) The Edison bipolar type and the bipolar **overtype** are two forms which were formerly much used but have not been very generally manufactured in recent years.

Multipolar Machines.—(Fig. 19.) Large continuous-current machines are of the multipolar type, that is, they have a large number of radial pole pieces as shown in Fig. 19.

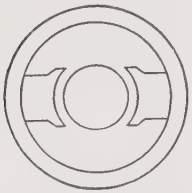


FIG. 18.—Magnetic circuit of a cylindrical bipolar machine.

Belt-driven and Direct-connected Types.—Generators may be designed to be driven by means of a belt or to be direct-connected on the same shaft as the prime mover. Machines for direct connection to reciprocating engines are usually **slow-speed** machines, revolving at from 100 to 300 turns per minute, while machines for direct connection to steam turbines turn at from 1500 to 3000 r.p.m. and are known as high-speed machines. For belt-driven machines or machines driven from a steam turbine by means of a reduction gear, the speed is from 400 to 1000 r.p.m., and these are known as **moderate-speed** machines.

The rating of an electric machine is the output, usually expressed in kilowatts, which it will give without injury from high temperature or commutation, and without excessive strain of its mechanical or dielectric strength.

The continuous rating is the output which the machine can give continuously without exceeding these limitations. For the continuous rating the rise in temperature is usually the limiting factor.

The short-time or intermittent rating is the output which the machine can give during a limited period, to be specified in each

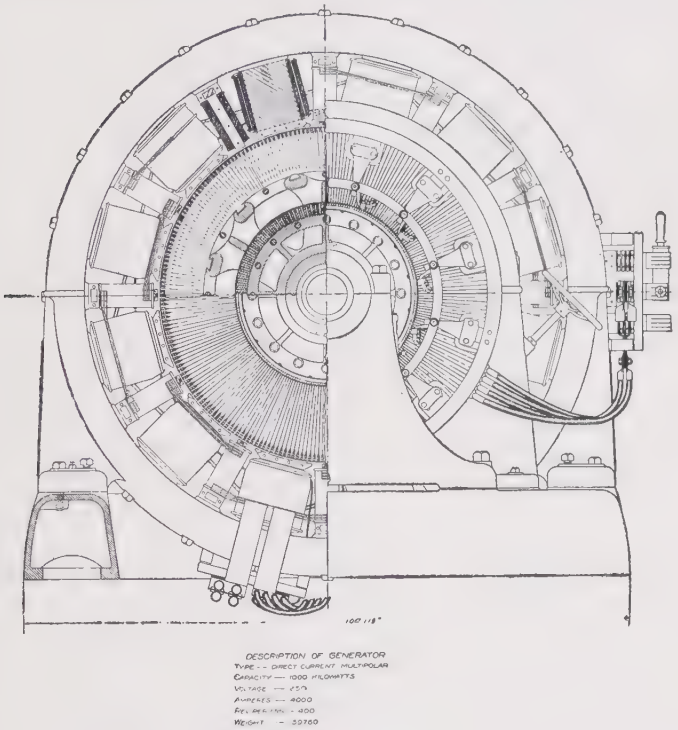


FIG. 19.—Elevation and part section of a large multipolar d. c. machine with commutating poles.

case, without exceeding those limitations. The rated output of a machine is always greater for a shorter period.

The maximum allowable temperature of the hottest part of any machine is set by the "Standardization Rules" of the A. I. E. E.

If the insulation is made up of cotton, silk, paper, or other similar material with no special treatment to strengthen its heat-resisting qualities, the limiting temperature is 95° C. If these

materials are treated or impregnated in such a manner as to increase their heat-resisting qualities, the limiting temperature is 105° C. If the insulation consists of mica, asbestos, or similar materials capable of withstanding high temperatures, the limiting temperature is 125° C.

In every case the limiting temperature is that of the **hottest spot,** and as this spot is not usually accessible to measuring devices the allowable observable temperature is less. Thus if the temperature is measured by thermometer it is assumed that this instrument cannot record within 15° of the hottest spot inside, and therefore 80, 90 and 110° respectively, according to the type of insulation, are considered the observable limits allowable.

If the temperature is measured by determining the increase in the resistance of the windings caused by the rise in temperature, the figures 85, 95, and 115 are considered to indicate that the hottest spots are at the limits of 95, 105 and 125, respectively.

Since the losses in the machine cause a **rise in temperature** proportional to those losses, the temperature of the surrounding air has an important effect upon the load the machine can carry without exceeding a definite final temperature. Thus, if the surrounding air is at a temperature of 25° there is a possible allowable rise of 70°, bringing the temperature to 95°, for ordinary insulations; while if the surrounding air has a temperature of 40° there is only a possibility of an observable rise of 40°, or an actual rise of 55° at the hottest spot. Thus, with a lower temperature of the surrounding air a machine may be rated slightly higher without injury to the insulation. (For details see the "Standardization Rules" of the A. I. E. E.)

Standard Voltages

The voltages which are usually employed in continuous-current circuits and for which c.c. generators and motors are designed are:

110– 125 volts for lighting and incidental small motors, household appliances, etc.

220– 250 volts for power in factories and for lighting and power combined by means of the three-wire system of distribution.

500– 600 volts for power alone, particularly for railway service.

1200–1500 volts for special interurban railway service.

2000–3000 volts for heavy main-line electrification of railways.

ELECTROMOTIVE FORCE INDUCED IN THE ARMATURE

If an armature having a number of conductors, or **inductors,** on its surface is revolved in a magnetic field produced by any given number of poles p, each inductor cuts all the lines of flux emanating from each pole every time it makes a revolution. If we call the number of magnetic lines, or **maxwells,** which pass from one pole into the armature ϕ, then each inductor cuts $p\phi$ maxwells per revolution. If the armature is revolving at N revolutions per minute or $\frac{N}{60}$ per second, the flux cut per second by 1 inductor is $\frac{p\phi N}{60}$.

Cutting flux at the rate of 100,000,000 lines per second induces 1 volt; therefore the average voltage induced in each inductor is $\frac{p\phi N}{60 \times 10^8}$. Let there be Z inductors on the armature arranged in m groups so that there are $\frac{Z}{m}$ inductors in series in each group. The average voltage generated in each group will therefore be $E = \frac{p\phi ZN}{m60\ 10^8}$ volts. If the m groups are connected in multiple, the voltage of all will be the same and this will be the voltage given by the armature.

It should be emphasized here that this is the average voltage of all the inductors in series and that the voltage induced in each individual inductor varies from zero to a maximum and back to zero again. The commutator groups these inductors so that all the inductors in one group are generating a voltage in the same direction.

The electromotive force in each inductor alternates (i.e., passes through a complete cycle of positive and negative values) once every time a pair of poles is passed, or with a frequency f. $f = \frac{pN}{120}$ cycles per second. Hence the formula for the average e.m.f. between brushes is $E = \frac{2f\phi Z}{10^8 m}$ volts.

The number of turns, S, in series between a positive and a negative brush is $\frac{Z}{2m}$, since two inductors form 1 turn. Thus E may also be expressed as

$$\text{Average } E = 4fS\phi \times 10^{-8} \text{ volts.}$$

This is the most general form of this relation and is directly comparable with the formula for the generation of an alternating e.m.f., differing from it only by the **form factor** of the alternating wave, which with a sine wave is 1.11.

For the effective value of an alternating e.m.f., we have $E = 4.44 fS\phi \times 10^{-8}$.

Commutator.—The preceding formulae give the average value of the e.m.f. This can only be obtained by means of a commutator or rectifier which reverses the connection of each coil in turn, to the external circuit, at the instant that the e.m.f. in that coil is zero and is reversing. Taps leading from the front connections or coils of the armature winding are connected to respective segments of the commutator, so that, as the segments come successively in contact with the brushes, the coil passes from one group to another and the current in it reverses, while the coil is short-circuited by the brush. By this means the coils are arranged in two or more groups or circuits so that all those coils in one circuit are generating an e.m.f. in the same direction and helping one another.

ARMATURE WINDINGS

Though there is a large variety of types of armature windings, the practical man, and even the designing engineer, seldom meets types other than (1) the multiple-drum or lap winding, and (2) the series-drum or wave winding. The other types, multiplex windings, may be used in a few special and exceptional machines but the designer may work for many years without finding any necessity for their application.

Most d.c. machines are constructed with **double-layer** windings, having formed coils placed in slots on the periphery of the armature. This means that coils of from one to many turns are wound upon a form and insulated before being placed in a machine. These coils are designed to fit into each other with all the crossings or lappings of coils at the ends of the armature. To accomplish this the coils are so shaped that one side of every coil will lie in the top layer of one slot and the other side in the bottom layer of another slot. The beginning and end of each coil are brought out, and the beginning of one coil is connected to the same commutator segment as the end of the preceding coil in a multiple-drum winding. The entire winding thus forms a complete **closed-**

circuit ring and it is only by means of the brushes that it is arranged into definite paths.

Multiple-drum or Lap Winding.—This type of winding is very common in direct-current machines, rotary converters, and induction motors, and is used to some extent in alternating-current generators. Its chief advantage is that it affords a very free choice in the number of slots and coils and is very simple to lay out and connect up. The disadvantages are that it is not easily adapted to high voltages in small machines and it gives trouble if the armature is not in true magnetic balance, that is, if one pole

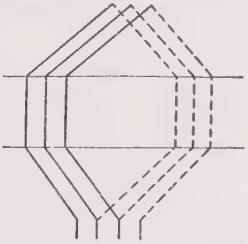


FIG. 20.—Connection of successive coils in a multiple-drum or "lap" winding.

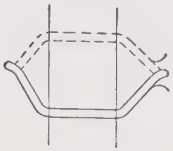


FIG. 21.—Typical coil and coil leads for a multiple-drum winding.

is nearer the armature than the others. Its chief characteristic is that there are as many circuits or paths through the armature, and as many brush studs, as there are poles. The distinguishing feature is that the two terminals of each coil go to adjacent commutator segments, as shown in Fig. 20. The characteristic connection of the coils is shown in Fig. 21.

Conditions for Lap or Multiple-drum Winding.—Generally, though not necessarily, the number of slots, and hence the number of coils, is a multiple of the number of poles, and each coil spans a number of slots equal to the slots per pole. This is expressed in the formula:

$$Z = pz = sb,$$

where Z = total number of coil-sides or bars, which is twice as great as the number of coils;

p = number of poles;

z = average pitch of connection = half the sum of the front pitch plus the back pitch;

s = total number of slots on armature;

b = number of coil-sides or bars per slot.

The total number of coil-sides (Z) is double the number of coils as each coil has two sides. Z must be a multiple of the number of slots and is here a multiple of the number of poles.

Let the coil-sides be numbered consecutively from slot to slot as shown in Fig. 22, the top coil-sides odd and the bottom coil-sides even, and let a point travel through the coil-sides in the order in which they are to be connected. This point, starting at coil-side or bar No. 1, passes through the back connection to coil-side No. 14 and then through the front connection to No. 3. Thus the front pitch is $14 - 3 = 11$ and the back pitch $14 - 1 = 13$, the average pitch being 12. (Fig. 22.)

The average pitch must be an even number so that the front and back pitches may be different and odd $(z + 1)$ and $(z - 1)$, respectively. There are always two layers of coil-sides, top and bottom. One side of each coil lies in the top layer in one slot (odd number) and the other side in the bottom layer in another slot (even number). The actual front and back pitches of a coil must be odd because they span from a top layer (odd number) to a bottom layer (even number) as in Figs. 22 and 23.

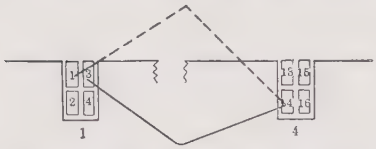


FIG. 22.—Front and back pitch connections in a multiple-drum winding.

In order that the winding may progress and include successive coils, the front pitch, or pitch of commutator connections, must differ by 2 from the back pitch or actual pitch of coils. The actual spread of a formed coil is the same front and back and is equal to the back pitch, while the front pitch discussed is that of the ends or connecting leads.

In order that all the formed coils may be alike, have the same spread, and be grouped into polycoils when there are more than two coil-sides per slot, the average pitch should be a multiple of the coil-sides per slot.

The total number of coils, $Q = \frac{Z}{2}$ is equal to the number of commutator segments; it must be a multiple of the number of slots and is usually a multiple of the number of poles, so that the number of turns in series in each of the multiple paths is the same.

A multiple-drum winding can be, however, and sometimes is,

made with a number of slots and coils not a multiple of the number of poles. This is done to help commutation by preventing two coil-sides in the same slot from being commutated simultaneously as explained under "Commutation." For the same purpose the

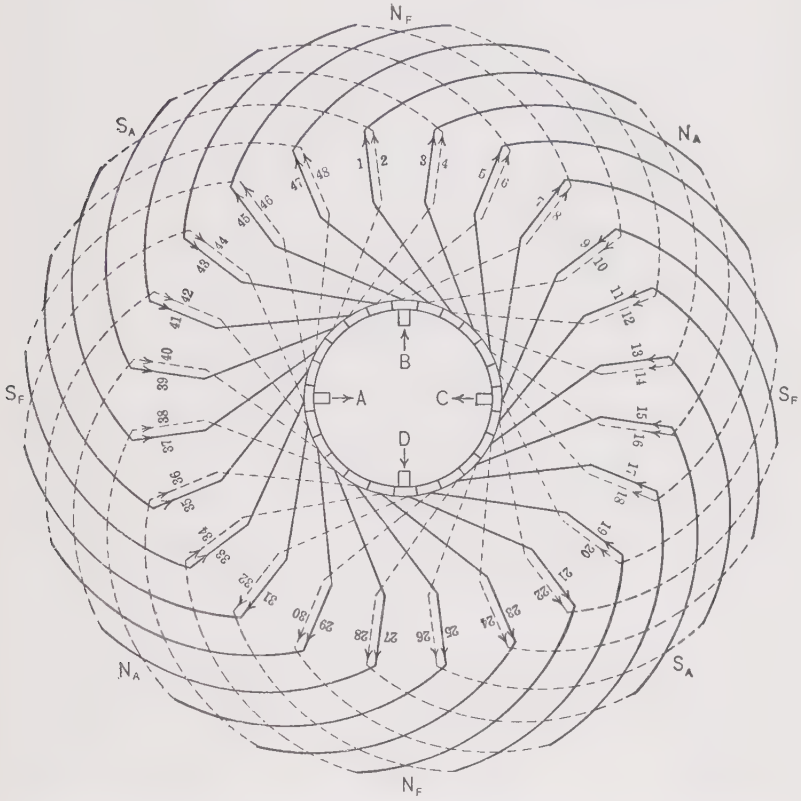


FIG. 23.—Winding diagram for a four-pole multiple-wound armature with 24 slots, 24 coils and 48 coil-sides.

average pitch is sometimes made less than $\frac{Z}{p}$. But these are unusual expedients.

A typical diagram of connections of a multiple-drum winding for a four-pole armature with 24 slots and 24 coils is shown in Fig. 23. Here $Z = 48 = 4 \times 12 = 2 \times 24$.

There are two coil-sides in each slot. Those in the upper layer

are represented by full lines and odd numbers; those in the bottom layer, by broken lines and even numbers. The front pitch is $14 - 3 = 11$ and the back pitch is $14 - 1 = 13$; thus the average pitch is 12.

Starting with coil-side No. 1 and following the back connection (outside) we come to No. 14, then the front connection to commutator and to No. 3. The order of connecting up the coil-sides would thus be: 1–14–3–16–5–18–7, etc. If there were several turns in each coil they would span from 1 to 14 both front and back. Thus the front ends of 1 and 14 are the two terminals of one coil and go to commutator segments Nos. 1 and 2, respectively.

When the winding is completed we place four brushes at four equidistant places on the commutator as shown in the figure. Two brushes (A and C) will be positive (where the current leaves the armature) and two (B and D) will be negative (where the current enters the armature). Considering negative brush B, the current passes from the brush to the segment and there finds two paths in multiple, one going through coil-sides Nos. 45–10–47–12, etc., to the positive brush C and the other going through Nos. 8–43–6–41, etc., to brush A. At brush D there are also two paths for the current to enter and there are two paths coming out at each of brushes C and A. There are thus four paths in multiple for the current through the armature.

Following the current through each of these four paths (Fig. 23) and noting with an arrow the direction of the current through each slot, we find that six slots in succession carry currents in the same (outward) direction and the next six slots carry currents all in the opposite (inward) direction, thus showing that the armature has four poles. The resultant poles of the armature would be, between coil-sides 8 and 9, 20 and 21, 32 and 33, and 44 and 45 respectively while the centers of the field poles would be in line with the brushes as shown at N, F, S, F, etc.

When the armature moves so that each brush straddles two commutator segments, then there are four coils removed from the active circuits and short-circuited by the four brushes, 43 and 8, 7 and 20, 19 and 32, 11 and 44; and it will be noted that the eight sides of the four short-circuited coils lie in only four different slots, 7 and 8, 19 and 20, 31 and 32, 43 and 44. This is the act of commutation and shows how, with this full-pitch winding, the mutual

inductance of the coils having their sides in the same slots aggravates the problem of commutation.

On the other hand, when a commutating pole is used the condition is favorable for a very narrow pole to affect 2 coils simultaneously.

With a large number of slots and many coils, there would still be only one coil short-circuited by each brush; and as these are short-circuited only at the time they are least active in producing e.m.f., the loss in voltage is negligible.

Series-drum or Wave Winding.—This type of winding is used in direct-current armatures where the multiple drum would give too low a voltage or would require too many turns of fine wire for a given voltage. Its advantages are that it gives an armature that is better balanced magnetically and that only two studs of brushes are necessary. A greater number of brush studs may be used, however. There are always two circuits or paths in multiple, regardless of the number of poles. The two ends of 1 coil go to commutator segments far apart.

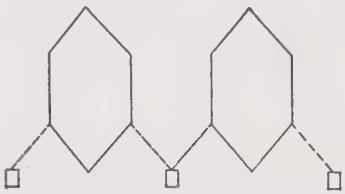


FIG. 24.—Coils of a series wound armature showing leads connecting to commutator.

Conditions for Wave Winding.—The conditions to be fulfilled in laying out a wave winding are quite complex and definite and are expressed by the formula

$$Z = pz \pm 2 = sb,$$

where Z = total number of coil-sides or bars;
p = number of poles;
z = average pitch of end connections;
s = total number of slots;
b = number of coil-sides or bars per slot.

The total number of coil-sides (Z) is double the number of coils and must also be a multiple of the number of slots unless a dead coil is used (see later).

The pitch of end connections must always be in one direction and not alternately forward and back as in the lap winding. The average pitch (z) may be odd or even, but since the actual pitch of connection must be odd, then if z is even the front and back

pitches are $(z + 1)$ and $(z - 1)$. For coils having more than 1 turn of wire, the back pitch (span of coil) must be one greater than a multiple of b, the coil-sides per slot, in order that all coils may be alike and fit into the slots.

The total number of slots (s) must be an odd number for a four-pole armature and must not be divisible by 3 for a six-pole armature.

The bars or coil-sides per slot (b) must be an even number, cannot be 4 or 8 for a four-pole armature, and cannot be 6 or 12 for a six-pole armature, unless the expedient of leaving a dead coil is used.

Usual Arrangements.—For a four-pole machine, $b = 6$ and $s = 29, 33, 37, 41$, etc. For definite values of s and b there may be two values of z corresponding to the positive and negative signs in the formula. With the positive sign, the end connections are slightly shorter; and with z an odd number, the front and back pitches are the same. These two conditions are desirable in bar-wound machines or in machines manufactured in large quantities.

For a six-pole machine, b may be 2, 4, 8 or 10, and s any number not divisible by 3.

Fig. 25 shows a diagram of a typical series-drum winding for a four-pole armature having 23 slots, 23 coils, 46 coil-sides and 23 commutator segments in accordance with the formula;

$$Z = 46 = 4 \times 11 + 2 = 2 \times 23.$$

There are 2 coil-sides per slot, the upper represented by full lines and odd numbers, the lower by broken lines and even numbers. The front and back pitches are both 11 and are always forward.

Starting with coil-side No. 1 and following the front connection to commutator, then to No. 12, and thence by back connection to No. 23, the order of connection is 1–12–23–34–45–10–21, etc., thus taking in 4 coil-sides spaced almost equally around the periphery and returning to the next slot to the starting point. By continuing this system of connection we shall finally end up by joining on the back at No. 1 where we started. If there were several turns of wire in each coil they would span from 2 to 13, 4 to 15, etc. the same as the back connections.

Thus the front connections again represent the terminals of

the coils, and the two terminals of one coil go to commutator segments almost diametrically opposite or 11 segments apart.

Two brushes only are necessary for this winding; the current entering at the negative brush, *A*, divides into two paths, one through coil-sides 30–41–6, etc., and the other through coil-sides 19–8–43, etc., both ending at positive brush *B*. But since brush

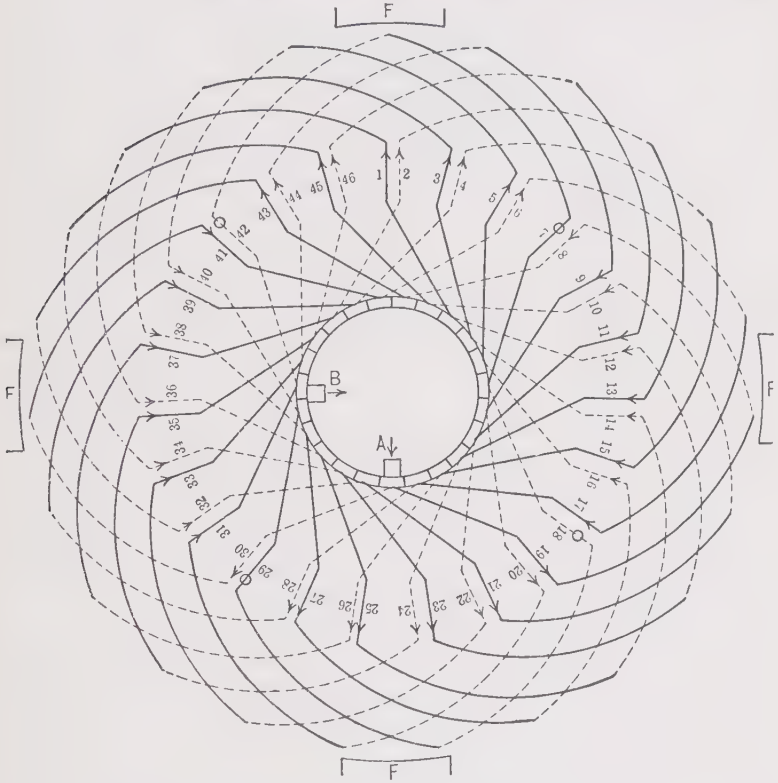


FIG. 25.—Winding diagram for a four-pole series-wound armature with 23 slots, 23 coils and 46 coil-sides.

B straddles 2 commutator segments we find a local path through coil-sides 42–7–18–29 short-circuited by this brush. Thus of the 46 coil-sides only 42 are active as indicated by the arrows showing the direction of the generated voltage in them, and four are short-circuited as indicated by the small circles. There are thus two paths in multiple for the current, each having nominally 23 coil-

sides in series, but in reality one has 20 and the other 22. As the segments move under the brushes the number in series will vary, from 23 when there is no straddling, to 20 under the condition shown in the diagram. Since the short-circuited coils are in the inactive position (concentric with the field poles), their absence from the circuit has no appreciable effect upon the magnitude of the voltage generated. To calculate the average voltage generated, we may use the total number $Z = 46$.

Two more brushes could be added, symmetrically spaced with respect to brushes A and B, without in any way affecting the value of the voltage generated, as diametrical points of the commutator are equi-potential points. The current carried by each brush would then be reduced to one-half. This is sometimes done when it is desired to restrict the size of the individual brushes or keep the commutator short.

The field poles would be placed as shown while the commutating poles would be placed midway between field poles or opposite the slots containing conductors marked with circles.

Use of a Dead Coil.—By leaving one coil dead, or out of circuit, a greater choice of numbers for slots and coils is available. Four coil-sides per slot for a four-pole winding and six coil-sides per slot for a six-pole winding may then be employed with any number of slots.

This is stated by the formula $Z = pz \pm 2 = sb - 2$.

Where Z = number of useful coil-sides.

Fig. 26 shows a series-drum winding with a dead coil. This armature has 4 poles, 24 slots, 24 coils, 48 coil-sides and 23 commutator segments. The coil comprising coil-sides Nos. 2 and 13 has its ends taped up or insulated. This leaves 46 active coil-sides and these are connected up as in the example shown in Fig. 26, the only difference being the uneven spanning of the front connections in order to reach past the dead ends. The back connections are all uniform. This diagram corresponds to the formula:

$$Z = 46 = 4 \times 11 + 2 = 2 \times 24 - 2.$$

As this type of winding is quite frequently used in small-size motors and as the actual connecting up is rather puzzling, the following instructions are given for this example:

Four poles, 24 slots, 2 coil-sides per slot, 23 commutator segments. Make all back connections (coil span) a pitch of 11 coil-

sides, 2–13, 4–15, etc. Assume coil containing sides Nos. 2 and 13 is to be dead, and tape up its two terminals. Begin at the front terminal of coil-side No. 1, go to commutator and then to No. 14 (ignoring No. 13 as if it did not exist), then proceed in accordance with the following table, reading across the table.

Front	Back	Front	Back
1	14	25	36
47	12	23	34
45	10	21	32
43	8	19	30
41	6	17	28
39	4	15	26
37	48	11	24
35	46	9	22
33	44	7	20
31	42	5	18
29	40	3	16
27	38	(49=1)	14

Nos. 2 and 13 are dead

Multiplex Wave Windings.—These windings are infrequently employed when it is desired to provide an armature winding which has more than two paths in multiple but not as many paths as the number of poles. Thus, if it is desired to build a six-pole machine with four paths in multiple, which with a given total number of inductors would give a greater voltage than a multiple-drum but not as much as a series-drum, a duplex winding could be used. Or if a still lower voltage is desired than that given by a multiple drum, then a quadruplex winding would be indicated. This would have eight paths in multiple.

In these windings the closed circuit passes completely around the armature many times. If, in connecting up, there are two entirely independent electric circuits, the winding is said to be a doubly reentrant duplex winding and is represented by the symbol 00. If, on the other hand, the winding closes upon itself after passing twice around the armature, it is said to be a singly reentrant duplex winding and is denoted by the symbol $\bigcirc_2$, which quite graphically indicates its characteristic. The multiplicity and complexity may be carried to a great extreme, such as **quadru-**

plex quadruply reentrant, but this is seldom done in commercial practice.

Conditions for Multiplex Windings.—The conditions are imposed by the formula:

$$Z = pz \pm 2m = sb,$$

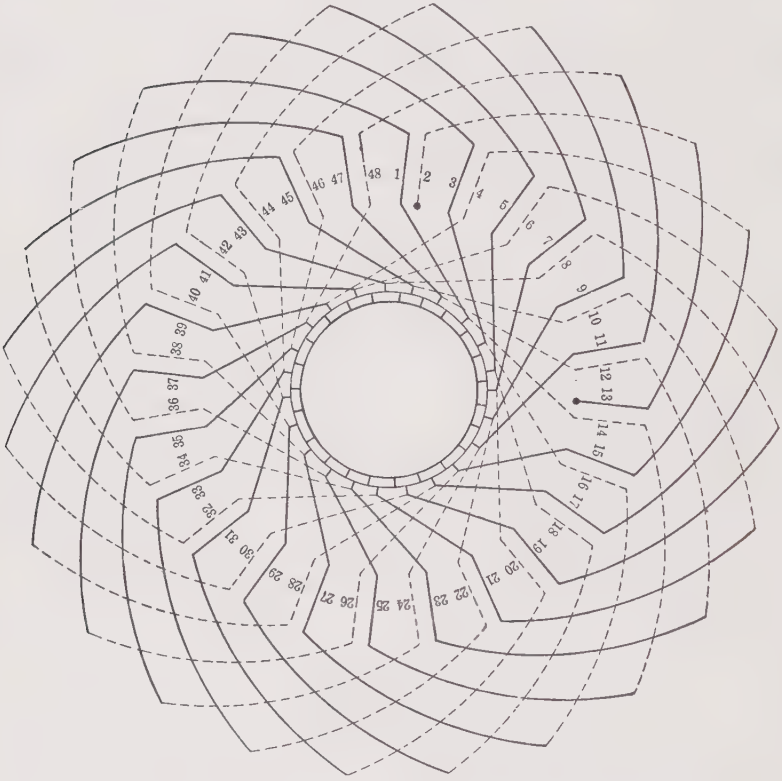


Fig. 26.—Winding diagram for a four-pole series wound armature with one dead coil.

where the symbols have the same meaning as before, m is the number of multiple windings, and $2m$ the number of paths in multiple for the line current. If m and z are prime to each other the winding is singly reentrant. The greatest common factor of m and z gives the number of times the winding is reentrant or in other words the number of independent windings.

Example of Multiplex Winding.—Assume it is desired to construct an armature for a six-pole machine and have four circuits in multiple; then two of several possible combinations would be:

$$Z = 610 = 6 \times 101 + 2 \times 2 = 61 \times 10, \quad . \quad . \quad (1)$$

or

$$Z = 640 = 6 \times 106 + 2 \times 2 = 64 \times 10. \quad . \quad . \quad (2)$$

In case (1) there are 61 slots, each with 10 bars or coil-sides winding connection pitch 101, $m = 2$ giving a duplex singly reentrant winding. Starting with bar No. 1, we should connect up by going first to commutator, then to bar 102, back connection to bar 203, to commutator, to bar 304, back connection to bar 405, then to 506, 607, to (708 = 98), etc. Each time we go around we come to a bar four places short of the preceding, instead of two, as in the common wave winding, but after going around a definite number of times this winding will close upon itself at bar No. 1.

In case (2), however, $z = 106$ and $m = 2$ have the common factor 2 and consequently the winding will close upon itself when only one-half the bars have been connected up. By proceeding to connect up the remaining bars in the same manner, we get a second winding exactly like the first but entirely independent of it, thus giving a doubly reentrant winding. Since the pitch of this winding is even, the front and back pitches must be different and odd (105 and 107). Both windings have four paths in multiple but the brushes must span at least 2 bars in order to lead the current into both windings.

ARMATURE REACTION OR ARMATURE MAGNETOMOTIVE FORCE

The armature of a dynamo-electric machine consists of turns of wire wound upon an iron core and is therefore a strong electromagnet whenever current flows through the wire. This magnetomotive force and the magnetic flux which it produces are the causes of many undesirable but unavoidable minor actions in the operation of such a machine. The armature winding is divided by the brushes into as many sections as there are poles on the field structure, and each of these sections constitutes an independent m.m.f. and magnetic circuit which in normal operation passes through the armature core, comes out between the poles and returns outside the armature across the pole face (see Fig. 28).

Armature reaction is the name given by the designing engineer to the magnetomotive force of one of these sections, commonly called the **armature reaction per pole,** and is always expressed in ampere-turns per pole rather than in gilberts. The quantitative value of armature reaction is expressed by the formula $AR = \frac{ZI}{2p}$, where Z = the total number of conductors on the armature;
I = the current in each conductor;
and p = the number of poles.

The armature reaction of a d.c. machine has a very important influence upon the commutation and regulation of both generators and motors. It is the effect of the magnetomotive force of the current in the armature conductors in distorting and weakening the magnetic flux set up by the field coils.

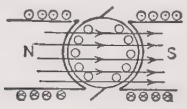


FIG. 27.

FIG. 28.

FIG. 29.

FIG. 27.—Flux set up by the field coils when there is no current in the armature.

FIG. 28.—Flux set up by the armature coils when there is no current in the field.

FIG. 29.—Resultant distorted flux when there is current in both the amature and the field.

Effect of Armature Reaction.—When current flows in the field coils and no current flows in the armature, a flux is set up following a path directly across the armature from pole to pole as in Fig. 27. On the other hand, when current flows in the armature and there is no current in the field, a flux is set up in the armature across the axis of the pole piece as in Fig. 28.

The result of these two actions is that when current flows in both the field and armature as under operating conditions, the resulting flux is shifted around so that one tip of each pole has its density increased and one tip has its density decreased, as in Fig. 29. If the brushes are at the geometrical neutral between poles they are no longer on an axis at right angles to the flux, that is, they are no longer at the neutral point with respect to the resultant flux. When the brushes are in this position the coil that is

being short-circuited by a brush is cutting flux and generating a voltage which sends current across the brush, and sparking results. The brushes must, therefore, be moved a small angle in the direction of rotation in a generator (against rotation in a motor) until they are in the real neutral axis. This shift of the brushes aggravates the flux distortion, but nevertheless, unless the armature reaction is too great, a position can be found in which a brush short-circuits a coil that is in the real neutral position and is inactive.

Relations Shown Vectorially.—In Fig. 30, F_1 and A_1 are the m.m.f.'s of the field and armature respectively when the brushes are on the geometrical or apparent neutral axis. R_1 is the resultant of these two. The brushes, which are in line with A_1, are not at right angles to R_1. If the brushes are moved to an axis at right angles to R_1 as at A_2, then the resultant becomes R_2 and the desired

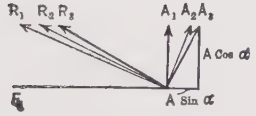


FIG. 30.—Vector combination of the field and the armature m.m.f.'s.

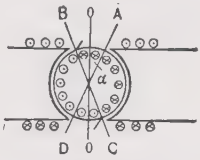


FIG. 31.—Cross and back ampere-turns in the armature.

conditions have not been attained. By moving the brushes still further, to A_3, giving the resultant R_3, it is possible to get A_3 and R_3 at right angles to each other. The stronger the field m.m.f. as compared to the armature m.m.f. the smaller will be the angle through which the brushes must be moved.

When the brushes have been moved through an angle α to the position A_3 there is one component of the armature m.m.f., $A \sin \alpha$, directly opposed to the field m.m.f. This is called the **back m.m.f.** Another component, $A \cos \alpha$, is at right angles to the field m.m.f. and is called the **cross m.m.f.**

Demagnetizing and Cross-magnetizing Action.—Let the movement of the brushes from A to A_3 be represented in Fig. 31 by the movement form O to B through the angle α. Then the conductors in the angle 2α, between A and B and 2α between C and D carry currents whose m.m.f. directly opposes the effect of the field coils. These are known as the **back,** or **demagnetizing,** conductors. The remainder of the armature conductors in A–C and B–D carry

currents and are placed so that they constitute an m.m.f. at right angles to the axis of the field and cause a distortion of the flux. These are known as the **cross-magnetizing** conductors.

It should be noted that while all the conductors in the sections A–B and C–D, subtending an angle 4α, constitute back ampere-turns, these are opposed to the strength of two field poles. In the quantitative treatment of this phenomenon, in which the unit of the designer is the strength of 1 pole, it is customary to consider the turns of the angle 2α as the back ampere-turns per pole.

Armature Inductance.—Each coil of the armature carries a current flowing in one direction as it travels from a positive brush to a negative brush, and in the opposite direction as the coil travels from a negative to a positive brush. Thus the direction of the current in each coil reverses during the time in which the commutator bars to which the coil is connected pass under the brush. To reverse the current in any circuit, it is necessary to take out the energy stored up in the form of magnetic flux linked with the circuit and put back an equal amount of energy represented by an equal flux in the opposite direction. In doing this there is induced in the circuit a voltage which is proportional to the time rate of change of the flux and which opposes any change in the value of the current. This voltage is the e.m.f. of self-inductance of the coil $\left(L\frac{di}{dt}\right)$.

Methods of Improving Commutation.—To minimize the voltage of self-inductance it is necessary to keep the number of ampere-turns in each coil as low as possible and to cause the reversal to take place as slowly as possible. It is also possible to neutralize the voltage of self-inductance by introducing an opposing voltage, which is accomplished in practical machines by moving the brushes slightly or by using commutating poles or interpoles.

Increasing the resistance in the path of the short-circuit current also aids commutation. This changes the time constant of this circuit, that is, some of the stored energy of the magnetism is dissipated in I^2R loss in this resistance instead of in sparking at the brush. The usual method of accomplishing this is to use carbon brushes, which have a higher resistance of contact than metal brushes. Another method is to introduce a relatively high resistance into the connection between the winding and the commutator segment.

Forward Lead of Brushes.—The brushes are moved in the direction of rotation in a generator (opposite direction in a motor) away from the true neutral until the inductors of the coil undergoing reversal are moving in a flux coming from the adjacent pole tip, of such a density that it induces, by rotation, in that particular coil, a voltage that opposes and neutralizes the voltage of self-induction.

In Fig. 32 let F represent the distribution of the field flux in the air gap when there is no current in the armature, and let A represent the distribution of a strong armature flux when there

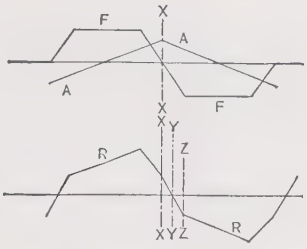


Fig. 32.

is no field current. Then R shows the distribution of the resultant flux when both field and armature are excited. It will be noted that the neutral point has been shifted from XX at no load in a direction with rotation in a generator, to YY at the load considered. The brushes would have to be shifted from XX at no load to a point ZZ beyond YY at this load, in order to commutate a coil lying in a flux which is producing a voltage helpful to commutation.

THE DESIGN OF CONTINUOUS-CURRENT GENERATORS AND MOTORS

The form in which the problem usually comes to the designing engineer is that it is required to lay out the design of a machine of specified power in kilowatts, of a specified terminal voltage at full load. The speed is frequently specified in advance, but not always, and sometimes the period of operation at rated output is

stated as 1 hr., 2 hrs., etc. When not otherwise specifically stated, continuous operation at rated load is to be understood.

The steps in the systematic procedure in the design of a d.c. generator or motor of a given voltage and power rating are given below:

1. Statement of problem.
2. Suitable speed.
3. Number of poles.
4. Diameter and length of armature.
5. Length of air gap, preliminary.
6. Number and size of slots, preliminary.
7. Total useful flux, preliminary.
8. Number of turns on armature, preliminary.
9. Armature reaction ampere-turns.
10. Number of conductors per slot.
11. Form of armature winding.
12. Size of armature conductors.
13. Exact size of slots.
14. Armature resistance and heating.
15. Design of commutator and brushes.
16. Reactance voltage.
17. Exact value of flux.
18. Cross-section of magnetic path; flux densities.
19. Calculation of magnetic circuit, excitation.
20. Stability factor.
21. Final length of air gap.
22. Armature reaction or interference.
23. Field windings, shunt, series.
24. Commutating poles and compensating winding.

The performance of the machine should then be calculated from this preliminary design to determine whether the design meets the imposed conditions, and such modifications as are necessary should then be made. The steps in calculating the performance are the calculations of:

25. Coreloss.
26. Friction.
27. Excitation loss.
28. Armature-circuit loss.

29. Load loss.
30. Efficiency.
31. Regulation.
32. Heating.

Symbols.—The following notation is employed uniformly throughout; other symbols and the symbols with primes or subscripts are defined in the paragraphs in which they are used.

B = flux density in the air gap, lines or maxwells per square inch.
C = total field ampere-turns per pole.
c = number of conductors per slot.
D = outside diameter of armature in inches.
E = terminal voltage of machine.
e = reactance voltage of short-circuited coil.
F = ampere-turns per shunt-field coil.
$f = \frac{pN}{120}$ = frequency of voltage induced in armature.
I = full-load line current, amperes.
L = length of armature parallel to shaft, inches.
m = number of parallel paths in armature.
N = revolutions per minute.
n = armature reaction in ampere-turns per pole.
P = power output in kilowatts in generator, input in case of motor.
p = number of poles.
P = permeance of local magnetic path of short-circuited coil.
q = cross-section of 1 armature conductor, square inches.
R_a = effective resistance of armature between brushes, ohms.
$S = \frac{Z}{2m}$ = number of turns in series between brushes.
s = total number of armature slots.
T = number of teeth under a pole carrying flux.
V = peripheral velocity of armature, feet per minute.
V_c = peripheral velocity of commutator, feet per minute.
Z = total number of inductors or active conductors on the armature.
ν = leakage coefficient.
σ = ampere conductors per inch of armature periphery.

ρ = ratio of pole arc divided by the pole pitch; pole arc is the arc subtended by the pole face; pole pitch is the arc from the center of one pole to the center of the adjacent pole measured at the air gap.

ϕ = total useful flux per pole.

1. Statement of the Problem.—The rating, in kilowatts or horse power, and the voltage are always given. The current and speed may or may not be given. The line current at full load is found as follows:

For a generator

$$I = \frac{1000P}{E}.$$

For a motor

$$I = \frac{746 \times (\text{Horse power})}{E \times (\text{Efficiency})}.$$

The proper efficiency to be assumed may be estimated from the table of usual efficiencies given on page 107.

2. Suitable Speed.—Machines are classified in commercial practice as **high-speed, moderate-speed,** and **slow-speed.** The most suitable speed for a machine of any given size for each class is given in the following table:

TABLE V

Suitable Speeds

Rating, Kw.	High Speed Poles, r.p.m.		Moderate Speed Poles, r.p.m.		Low Speed Poles, r.p.m.	
0–10	2	1500	4	1000		
10–50	4	1300	4	800		
50–100	4	1100	4–6	700	6	300
100–300	6	850	6	500	8	200
300–500	6	750	8	400	10	150
500 up	6	650	8	300	12 and up	100

3. Number of poles (p) depends upon the size, speed, and character of the machine. There is, however, much variation depending upon specific conditions. It is sometimes customary

to make alternative designs to determine the most advantageous number of poles. Reasonable arrangements are indicated in the accompanying table.

4. Diameter and length of armature are related to the rating and speed as explained in Chapter I, and preliminary values are obtained by the following formula:

$$D^2L = \frac{610P \times 10^9}{\rho\sigma BN}.$$

Values for the four design constants, ρ, σ, B, and N, may be assigned by judgment and experience as indicated in the following paragraphs and thus the numerical product of D^2L determined.

ρ, the ratio of pole arc to pole pitch, varies from 0.6 in machines with two or four poles and high speed, to .85 in machines with many poles. A value of about .7 is very common in machines without commutating poles and a smaller value if commutating poles are to be used. This is necessary to allow space for these auxiliary poles.

σ, *Ampere conductors per inch periphery, or the specific loading factor,* is the product of the total conductors around the armature, times the current in each, divided by the periphery of the armature. The limiting values depend upon the depth of the slots, the heating or ventilation, and upon the allowable value of armature reaction. In non-commutating-pole machines the last is usually the most important. Usual values for σ are given in Table VI.

B, *Magnetic density in the air gap,* has values from 25,000, or 25 kilolines, in small machines to 70 kilolines in large machines and usual values for each size are given in Table VI. The density in the air gap for general purposes is that obtained by using the actual surface of the pole face along the gap.

5. Determination of D and L.—A value for D^2L having been found, the two factors may be separated by two methods. The designing engineer generally chooses a peripheral speed suitable to the mechanical construction which it is intended to use (Table VI) and if the angular speed N is already specified or chosen this settles D by the relation $D = \frac{12V}{\pi N}$. If V and N are not known the values of D and L may be determined by assuming a square pole face (which is a desirable proportion for economy of material,

particularly of field copper), then $L = \frac{\pi\rho D}{p}$. Substituting this in D^2L, the equation may be solved for the value of D.

TABLE VI

USUAL VALUES OF DESIGN CONSTANTS FOR D. C. MACHINES

Rating, Kw.	Diameter, Inches	V f.p.m.	σ Contin. Rating	B	n Arm Reaction	σ 1 Hr.
0–10	3	1900	180	25,000	1200	
	11	2500	400	40,000	1600	
10–50	10	2000	250	35,000	1400	
	15	4000	500	50,000	3000	
50–100	12	3000	300	40,000	2000	
	24	5000	600	50,000	3500	700
100–300	20	3000	400	50,000	3000	
	36	6000	600	60,000	4000	1000
300–500	24	3000	600	50,000	4000	
	45	6000	900	65,000	8000	1000
500–up	32	4000	600	50,000	4000	
		7500	900	70,000	8000	1200

In Table VI an upper and a lower value are given for each design constant for various sizes of machines. Between these limits the values would be considered normal; but in practice machines will be found having values for these constants outside of these ranges. Such machines, however, would be considered special or designed for some unusual specific purpose.

6. Number and Size of Armature Slots.—A rough approximation to practice gives the number of slots as four times the diameter in inches. More specific information on general practice is given in Table VII.

7. Total useful flux per pole in the armature (ϕ) is fixed by the assumed values for the gap density, B, diameter, D, and length, L, of the armature:

$$\phi = \frac{\pi\rho LDB}{p}.$$

TABLE VII

D, Inches	Air Gap, Inches	Number of Slots	Depth of Slots, Inches
5	0.08	25– 40	0.6
10	0.10	30– 60	0.75
15	0.12	40– 80	1.00
20	0.125	60–100	1.25
30	0.15	80–150	1.50
50	0.19	100–200	1.75
100	0.25	150–300	2.50
150	0.35	200–400	3.25

8. Number of turns in series between brushes (S) is obtained from the relation between flux, frequency, and voltage:

$$S = \frac{E \times 10^8}{4f\phi},$$

where f is the frequency as given by $f = \frac{pN}{120}$.

9. Armature Reaction in Ampere-turns per Pole (n).—

$$n = \frac{SI}{p}.$$

The armature reaction should not exceed certain values because high armature reaction is not compatible with good commutation, good regulation, or stability, unless commutating poles or compensating windings are used. On the other hand, low armature reaction makes a large and expensive machine. Proper values for various sizes are shown in Table VI. If n comes out too great at this stage of the design, it may be reduced by increasing the value of N or ϕ. The latter is accomplished by assuming a greater value for B, D, or L.

10. Number of Conductors per Slot (c).—This quantity must fulfill three conditions which are sometimes antagonistic, and usually a compromise must be effected at this point to plan a practical winding for the armature. In choosing a value for c there is first a choice to be made as to whether a multiple or a series winding will be used. With experience this choice can

usually (but not always) be made in advance. Without experience or when the choice is difficult, a value for c and the approximate size of a conductor (q) are worked out for both types of winding and the one giving the number and size of conductors more easily arranged in a suitable slot is selected as the final choice.

Multiple Winding.—Certain preliminary values having been assumed for σ and s, then c is restricted to a value approximately $c = \frac{\pi\sigma pD}{Is}$. If a preliminary value of ϕ and S has been assumed, then c must approximate $c = \frac{2pS}{s}$. The cross-section of a conductor will be roughly $q = \frac{}{2000p}$ sq. ins., assuming 2000 amperes to the square inch.

Series Winding.—Since there are only two paths in multiple, irrespective of the number of poles:

$$c = \frac{2\pi\sigma D}{Is} \qquad c = \frac{4S}{s} \quad \text{and} \quad q = \frac{I}{4000} \text{ sq. in.}$$

11. Form of Armature Winding (for general discussion of forms of armature windings, see page 36).—For either type of drum winding with formed coils, c must be an even number and give a number of conductors which can be conveniently arranged in a slot having a depth of from 3 to 4 times its width. The number of slots, s, may be adjusted to assist in making c fulfill the required conditions, keeping the product $c \times s$ as near the same as possible.

Multiple Winding.—If the number of conductors per slot (c) and their size (q) indicate the desirability of a multiple winding, an arrangement is selected having 2, 4, 6, or 8 coil-sides per slot, and c is assumed at a value that is the nearest multiple of the coil-sides per slot, thus grouping the conductors into coils. The total number of slots is preferably assumed at some multiple of the number of poles, so that each of the parallel circuits in the armature will have exactly the same number of inductors or turns in series and thus give the same voltage and have the same resistance.

Series Winding.—For a four-pole machine, choose preferably a number of conductors per slot divisible by 6 (giving 6 coil-sides per slot) and a number of slots in the series 29, 33, 37, etc., endeavoring to obtain that value of $\frac{sc}{4}$ that comes nearest to the preliminary

value of S, the number of turns in series between brushes. Or, for a four-pole machine choose $c = 2$, for 2 coil-sides per slot, and any odd number of slots; or for $c = 10$ a number of slots in the series 27, 31, 35, etc., arranging a value of $\frac{sc}{4}$ nearest the desired value of S.

For a six-pole machine choose a value of c divisible by 4, giving 4 coil-sides per slot, and a number of slots in the series 26, 32, 38 that gives the nearest value of $s \times c$.

The values for s mentioned give the best arrangements of windings, but there are other possible combinations giving unequal front and back pitches and longer connections.

Revised Value of Turns in Series (S).—The values of s and c having been altered to fit the form of armature winding desired, it is now necessary to revise some of the preliminary assumptions to accord with the actual winding. Thus for a multiple winding the final value of S is $S = \frac{sc}{2p}$ and for a series winding $S = \frac{sc}{4}$. The values of B, ϕ, and n are now different from the original assumptions and may be considered as the second approximations.

12. Size of Armature Conductors.—The current in each conductor is equal to the line current (I) divided by the number of poles in a multiple winding and to I divided by 2 in a series winding. The cross-section of each conductor in square inches is equal to the current per conductor divided by the allowable current density in amperes per square inch. This quantity is found in practice to vary widely, being 1500 in very large machines, 2000 in machines of over 100 kw., 2500 in machines from 50 to 100 kw., 3000 in smaller machines, and 3500 in fractional horse-power motors. If the cross-section needed is greater than 0.02 sq. in., rectangular bars should be considered to be arranged as shown in Fig. 33. The depth of the bars should be three or four times their width in order that the slots may not be too wide and interfere with the path of the magnetic flux. If the cross-section is less than 0.02 sq. in., then standard circular wires would be used. The cross-section in circular mils is (1,270,000 $\times$ square inch). The nearest standard size is chosen from a wire table. No wire larger than a No. 7 B. & S. should be used as it is too stiff to follow the bends of the coils. It is frequently convenient to use two wires in parallel as twin wires for a conductor, the combined cross-

section being of the required value. The twin wires are soldered together into the commutator segments at each end of the coil.

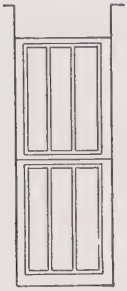


Fig. 33.—Armature slot with six bars.

13. Exact Size of Slots.—The total number of slots is definitely set by the style of winding. The size of the slots depends upon the amount of copper in the form of conductors and the amount of insulation which each slot must contain, and this size is limited by the fact that there must be sufficient material left in the teeth between slots to carry the magnetic flux without excessive densities. The maximum allowable depth of the slots depends upon the diameter of the armature and is roughly given in Table VII. The width of each slot should be from 0.4 to 0.6 of the pitch of the slots, which is the quotient of the periphery of the armature divided by the number of slots.

The ratio of the total cross-section of copper in a slot to the cross-section of the slot itself is known as the **slot factor.** This quantity has values more or less consistent, as shown in Table VIII. The smaller the size of each conductor and the more conductors per slot, the greater must be the insulation used between them; and the higher the voltage of the machine, the greater must be the thickness of insulation. The more insulation used, the less copper can be placed in the slot, and hence the lower the value of the slot factor.

Fig. 34.—Armature slot with two coil-sides each having eight conductors or eight turns per coil.

TABLE VIII

Slot Factor

Rating, Kw.	For 125 Volts	For 500 Volts
0–10	0.28	0.20
10–50	0.35	0.25
50–100	0.40	0.30
100–400	0.50	0.40
400–1000	0.60	0.50

The size and the shape of the slot are governed by the method and the materials used for insulation. The various manufacturing companies use slightly different methods and different forms of material, so that it cannot be said that there is one best way to insulate a machine for a specific purpose. The insulating materials in most general use are listed and discussed in the chapter on "Materials of Design," and the most generally used of all these are treated cloth, also called varnished cambric, and mica paper.

Typical methods of applying these two types of materials to two different types of windings (bar winding and wire winding) are given in the following Insulation Specifications.

MICA ARMATURE COIL AND SLOT INSULATION

Bar winding for 500-volt machine with mica insulation. Six bars per slot with 1 turn per coil. Bars (.11 × .55). See Fig. 33.

Wrap each bar with two layers of .003 mica paper, one-half lapped. (.012 on each side.)

Assemble 3 bars into a polycoil and wrap with four layers of .003 mica paper, one-half lapped. (.024 on side.)

Place leatheroid strip along top of coil on slot portion, thickness .031.

Wrap polycoil with one layer of .007 cotton tape, one-half lapped. (.02 on side.)

Leave clearance on top of slot for binding-wires recessed in grooves in surface of armature.

CALCULATION OF SPACE REQUIRED

Width		Depth	
Each bar	.110	Each bar	.550
2 layers mica paper	.024	2 layers mica paper	.024
		4 layers mica paper	.048
	.134	Leatheroid strip	.031
	3	1 layer cotton tape	.040
3 bars together	.402	One polycoil	.693
4 layers mica paper	.048		2
1 layer cotton tape	.040		
Clearance	.010	Two polycoils deep	1.386
		Clearance	.064
Total width slot	.500	Total depth slot	1.450

Approximate overall dimensions: Width = 3 × .11 + .17 = .50; Depth = 2 × .55 + .35 = 1.45; Slot Factor = 0.50.

CLOTH ARMATURE COIL AND SLOT INSULATION

Wire winding for 125-volt machine with varnished cambric insulation. Four turns No. 12 B. & S. per coil (bare diameter .081) and 4 coil-sides per slot. See Fig. 34.

Wrap a group of four No. 12 wires (double cotton covered diameter .091 forming 1 coil with 1 layer of .005 cotton tape, half lapped. (.01 on side.)

Apply three coats of varnish with alternate drying (.006).

Assemble 2 coils as 1 polycoil, place .031 leatheroid spacer along top of slot portion of polycoil.

Wrap polycoil with 1 layer of .005 cotton tape, half lapped. (.01 on side).

Leave clearance on top of slot for binding-wires in grooves.

CALCULATION OF SPACE REQUIRED

Width		Depth	
Each No. 12 wire d.c.c.	.091	4 No. 12 d.c.c. wires	.364
1 layer of tape	.020	1 layer of tape	.020
3 coats varnish	.012	3 coats varnish	.012
		1 leatheroid strip	.031
Each coil	.123	1 layer of tape	.020
	2		
		Each coil	.448
2 coils wide	.246		2
1 layer of tape	.020		
Clearance	.004	2 coils deep	.896
		Clearance	.104
Total width slot	.270	Total depth slot	1.000

Approximate overall dimensions: Width = $2 \times .091 + .088 = .27$; Depth = $8 \times .091 + .272 = 1.00$; Slot Factor = 0.306.

The actual dimensions of the slots may be found with sufficient accuracy for the calculation of the magnetic circuit of the machine by adding to the width and depth of the cotton-covered conductors, suitably arranged, an allowance for the coil and slot insulation as shown in the examples given above. The usual allowances given in the accompanying table are for voltages of 500 or less.

TABLE IX

ALLOWANCE FOR COIL AND SLOT INSULATION

Type of Slot	Number of Bars or Coil-sides per Slot	Width, Inch	Depth, Inch
Open	2	.09	.30
Open	4	.14	.35
Open	6	.17	.35
Partly closed	4	.15	.40

14. Armature Resistance and Heating.—The mean length of an armature turn is very closely approximated by assuming that the connections at each end have a length equal to the semicircle whose diameter is the winding pitch; thus if $\frac{\pi D}{p}$ is the arc subtended by the sides of a coil, then the length of the end connections at one end would be $\frac{\pi^2 D}{2p}$ and the length of the end connections at the two ends would be $\frac{\pi^2 D}{p}$, which is abbreviated and approximated by the empirical expression $\frac{10D}{p}$.

If fractional pitch or chordal windings are used, this must be multiplied by (k) the per cent pitch expressed as a fraction.

The length of the straight or slot portions of the coil is $2(L + x)$ where x allows for the distance which the coils project from the armature core before starting to bend, and is intended to avoid crowding of the coils and creepage. x should be about one-half inch in low-voltage machines and 1 in. in 500–600-volt machines. Thus the total length of 1 turn is:

$$\text{m.l.t.} = 2(L + x) + \frac{10Dk}{p} \text{ inches.}$$

The resistance of the machine is usually calculated for a temperature of 75° C. in order to allow for the effect of the running temperature upon the losses and efficiency. The equivalent

resistance of all paths in multiple of the armature winding from brush to brush is

$$\text{For a wire winding: } R_a = \frac{12.7S(\text{m.l.t.})}{12mq},$$

where 12.7 is the resistance of 1 ft. of wire of 1 circular mil and q is the cross-section of the wire in circular mils.

$$\text{For a bar winding: } R_w = \frac{.01S(\text{m.l.t.})}{12{,}000mq},$$

where q is in square inches.

The voltage drop in the armature winding is IR_a.

The power lost in the armature copper is I^2R_a watts.

The rise in temperature of the copper in C.° is given approximately for preliminary purposes by

$$T_w = \frac{100I^2R_a}{(a + b)ls},$$

where 100 equals the rise per watt per square inch of coil surface,

b = width of 1 coil in slot portion;
a = depth of 1 coil in slot portion:
l = m.l.t;
s = number of coils.

The weight of copper required for the armature is given by the relation: Pounds = .32 mqS(m.l.t.) where q is in square inches and .32 is the weight of a cubic inch of copper. For round wire the weight per thousand feet can be taken from the commercial wire tables and multiplied by the total number of feet required.

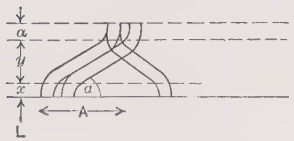


FIG. 35.—Arrangement of end connections of coil showing axial length required.

Overall Length of Armature Windings (Parallel to Shaft).—To determine the space required for the end connections it is conventional to assume that the ends of the coils are bent at such an angle that in the slanting portion the width of each coil (or its geometrical projection) occupies the space of a slot and a tooth. Referring to the diagram (Fig. 35) which is self-explanatory, the calculation is as follows:

$$\text{Let } \sin \alpha = \frac{\text{slot width}}{\text{slot pitch at bottom}}.$$

$$\text{Let } A = \text{one-half pole pitch} + \text{slot width} = \frac{\pi D}{2p} + w;$$

w = slot width;
$y = A \tan \alpha$;
x = straight portion as above;
a = depth of 1 coil.

Then the total overall length of armature winding is

$$= L + 2(x + y + a).$$

15. Design of Commutator. The diameter of the commutator is usually determined by the peripheral speed allowable, which should be about 3000 ft. per minute and should not exceed 4000 ft. per minute unless a special mechanical construction is used. The peripheral speed is figured for the outside diameter of the commutator. Excessive speeds cause chattering of the brushes and bad commutation and also cause a tendency for the commutator bars to get out of true, due to the centrifugal force. The outer diameter of the commutator must, of course, be less than the diameter of the armature.

The total number of segments should be such that the average voltage between segments (volts per bar) is less than 15.

$$\text{Volts per bar} = \frac{(\text{rated voltage}) \times (\text{poles})}{\text{number of segments}}.$$

The number of segments must be a multiple of the number of coils. While physically and mechanically the number of coils may be equal to the number of slots, it is often convenient to divide a coil electrically into two or three parts as respects connection to the commutator. Thus a coil may be a physical entity having sixteen turns in it, but by bringing out a tap between the eighth and ninth turns, or by bringing out the end of the eighth turn and the beginning of the ninth turn to a commutator bar, the number of coils has been doubled so far as commutation is concerned. Such a construction is called a **polycoil.** By this means a greater latitude is possible in adjusting the volts per bar.

The lower limit to the diameter of the commutator is set by the fact that it is impossible to make a commutator which is

satisfactory from a mechanical standpoint if the pitch of commutator segments is less than 0.20 in. This includes the copper segment and the insulation, usually mica. In small low-voltage machines the mica insulation between segments has a thickness of 0.02 in., and in larger machines and in 200–600 volt machines mica having a thickness of 0.03 in. is used.

Dimensions of Brushes.—The cross-section of the contact surface of the brushes is determined by the allowable current density at this point. This value ranges between 40 and 50 amperes per square inch at full or rated load current. Thus the total cross-section of all of the positive brushes is

$$a_b = \frac{\text{rated line current}}{40} \text{ in square inches,}$$

and there is an equal cross-section of negative brushes. For a multiple-wound armature there are as many brush studs or groups of brushes as there are field poles, and half of these studs (taken alternately) carry positive brushes, the remainder carry negative brushes. For a series-wound armature all the positive brushes may be on one stud and all the negative brushes on another stud, or they may be distributed as with a multiple-wound armature.

The width of each brush is limited by the number of commutator segments which it is permissible to cover and the width of these segments. The number of segments which may be covered is limited by the **reactance voltage,** as explained later on. In general, the width of each brush is from two to three times the pitch of the segments.

Total Length of Commutator.—The total length (parallel to the shaft) of the brushes on one stud or the **active length of commutator** is $l_c = \frac{2 \text{ (total surface of positive brushes)}}{\text{(width of brush)} \times \text{(number of studs)}}$. The total length of the commutator is greater than this by enough to allow for clearance between individual brushes and clearance at the ends of about 0.5 in. It is also customary to allow an extra space at the outside end of the commutator for insulating tape or string, to prevent creepage of current. In general, commutators have a gross superficial cylindrical area of from 0.67 to 1.0 sq. in. per ampere of line current.

If the length of brush per stud is greater than 1.5 or 2 ins. it is preferable to use two or more brushes, each in a separate brush-

holder or box with individual springs, in order to obtain a more even pressure and better contact between brush and commutator. Aside from this consideration, the subdivision of the required gross brush area into a number of individual brushes of convenient size is largely a matter of planning to use stock sizes of brushes and brush-holders.

Resistance of Brushes and Brush Contact.—This quantity is a complex variable partaking more of the nature of the counter e.m.f. of an arc than of a resistance. It is really a combination of a counter e.m.f. and a resistance in series. The magnitude of this loss, compared with the other losses in the machine, is not sufficiently great to warrant treating it in a very thorough and elaborate way, except in the case of low-voltage (less than 80 volts) machines; thus the usual practice in design is to treat it empirically.

The A. I. E. E. assigns a total drop of 2 volts in positive and negative brushes if pig-tails are attached and 3 volts if pig-tails are not attached, for all conditions of load. Thus the watts lost on commutator and in the brushes would be found by multiplying the line current for each load by 2 or 3, respectively.

To analyze this loss more scientifically, it may be divided into three parts, each caused by a different phenomenon:

First.—The loss due to brush contact incidental to the passage of the current between the brushes and the commutator. The table gives values for this loss as found in physical tests for a brush pressure of 1.5 lbs. per square inch and a peripheral speed up to 6000 ft. per minute.

TABLE X

Drop Due to Brush Contact

Total Volts Drop, Positive and Negative Brushes

Amperes per Square Inch	20	40	60	80	100
Hard brushes.....	2	2.4	2.8	2.9	3
Medium brushes.....	1.5	1.7	1.8	1.9	2
Soft brushes.....	1	1.1	1.3	1.5	1.6

Second.—The loss due to the passage of the current between the brush proper and the brush-holder and bus ring. Indefinite.

Third.—The increase in the loss at the surface, due to imperfect commutation and uneven distribution of current.

Friction of Brushes on Commutator.—The brushes should press upon the commutator with a pressure of from 1.5 to 2 lbs. per square inch of contact surface, the higher value for higher peripheral speeds. This causes a simple mechanical friction evaluated as

$$\text{Friction, watts} = \frac{(\text{No. of brushes}) \times (\text{Pressure per brush}) V_c k}{45},$$

where V_c = peripheral speed of commutator, feet per minute;
k = coefficient of friction, .25 for carbon, .14 for graphite.

The A. I. E. E. gives conventional values for brush friction at a peripheral speed of 1000 ft. per minute as 8 watts per square inch of brush surface with carbon and graphite brushes, and 5 watts with metal graphite brushes.

Rise in temperature of commutator:

$$T = t \times \frac{(\text{brush } RI^2) + (\text{brush friction})}{\text{surface of commutator, square inches}} = \text{degrees C.},$$

where t is the rise in temperature for a loss of 1 watt per square inch and has values as in the table.

Peripheral Speed, Feet per Minute	t
2000	15°
3000	12°
4000	10°

16. Reactance Voltage.—As explained earlier in this chapter, the reversal of the current in each individual coil as it undergoes commutation causes a local voltage of self-inductance called the **reactance voltage.** It is necessary to calculate the magnitude of this voltage because, if it exceeds certain values, sparking occurs at the brushes; this heats the commutator and, if excessive, injures the commutator and brushes by pitting them, thus limiting the rating and reducing the life of the machine.

The formula for this voltage is

$$e = L\frac{di}{dT} = \frac{sd\phi}{10^8 dT},$$

where s is the number of turns in series between commutator segments, ϕ is the flux interlinked with the short-circuited coil, which reverses with respect to that coil, and T is the time period allowed to reverse the current. ϕ may be due to other coils being commutated simultaneously as well as to the particular coil under consideration. ϕ is the difficult quantity to calculate and to simplify the problem it is convenient to assume it to be resolved into three parts and analyze one at a time.[1]

The three fluxes are (Figs. 36 and 37):

(*a*) The flux in the slot proper surrounding the embedded portion of the coil (ϕ_s).

(*b*) The flux in the air gap passing from tooth tip to tooth tip and also surrounding the embedded portion of the coil, ϕ_t.

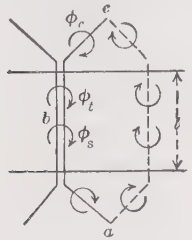


FIG. 36.—Diagram of a coil showing leakage flux in slot, on teeth and around the end connections.

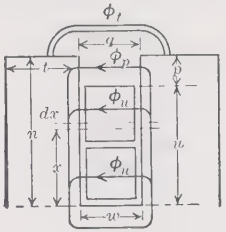


FIG. 37.—Open slot showing paths of the leakage flux across coils, around coils and from tooth tip to tooth tip.

(*c*) The flux surrounding the end connections of the coil outside of the armature core, ϕ_c.

For the sake of simplicity the inductance of a group of conductors in a slot, as *a*, *b*, *c*, Fig. 36, is calculated as a unit because the fluxes surrounding the two halves of a coil are entirely independent of each other and the two inductances are independent and in series, their sum forming the inductance of the whole coil. From the various inductances, the **flux per ampere conductor,** which is a more convenient quantity for application in the practical numerical formula may be derived.

(*a*) *Slot Flux.*—Figure 37 shows a customary form of armature slot with wide opening and the copper conductors in the 2 coil-sides. The slot flux, ϕ_s, due to armature current is divided into

[1] Arnold, Gleichstrom I.

2 parts, ϕ_u crossing the copper and ϕ_p crossing the slot above the copper.

Since in any magnetic circuit we have

$$\text{flux} = \frac{3.2SIA}{\text{length}},$$

when A is in square inches and length in inches, and since in the magnetic circuits shown in Fig. 37 the reluctance of the part of the path in the core and teeth (iron) is very small compared with the reluctance of the part across the slot, it is obvious that the density of the flux, ϕ_u, will vary uniformly from zero at the bottom of the slot to a maximum at the top of the coil-sides, the m.m.f. varying in that manner.

We can set down the value of the flux in the narrow band dx due to the m.m.f. of that portion of the conductors below it $\left(\frac{xcI_c}{u}\right)$ ampere conductors, thus:

$$d\phi = \frac{3.2xcI_c ldx}{uw},$$

where x, u, and w are as in Fig. 37;

c = number of conductors per slot;

I_c = amperes in each conductor;

l = length of 1 side of embedded portion of a coil, inches.

The conductors interlinked with this band of flux are $\frac{xc}{u}$ and the interlinkages are:

$$\frac{xc}{u}d\phi = \frac{3.2c^2I_c lx^2dx}{u^2w}.$$

Summing up all the interlinkages in the portion of the slot occupied by the copper (u) and dividing by 10^8 times the current, we get the inductance

$$L_u = \int_{x=o}^{x=u} \frac{3.2c^2lx^2dx}{u^2w10^8} = \frac{3.2c^2lu}{3w10^8} \text{ henry.}$$

This is the self-inductance due to that part of the slot flux which passes through the copper.

The flux, ϕ_p, passing across the slot above the copper, is of

uniform density and interlinks all of the c conductors. It and the inductance due to it are:

$$\phi_p = \frac{3.2cI_c lp}{w} \quad \text{and} \quad L_p = \frac{3.2c^2 lp}{w10^8} \text{ henry.}$$

The combination of L_u and L_p gives the inductance due to the flux in the slot, ϕ_s,

$$L_s = L_p + L_u = \frac{3.2c^2 l}{10^8}\left(\frac{u}{3w} + \frac{p}{w}\right) \text{ henry.}$$

The equivalent flux per ampere conductor is, therefore,

$$\text{flux} = \frac{\text{inductance } 10^8}{(\text{conductors})^2} = \phi_s = 3.2l\left(\frac{u}{3w} + \frac{p}{w}\right).$$

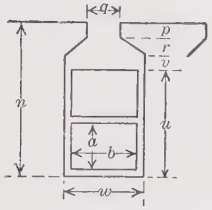


FIG. 38.—Partly closed slot with overhanging teeth showing dimensions needed to calculate the leakage flux.

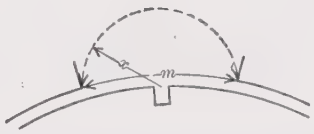


FIG. 39.—Diagram showing hypothetical limit of tooth tip leakage flux.

This is for open slots as in Fig. 37 (where $q = w$). If the slot is partly closed as in Fig. 38, it can be easily shown that the flux per ampere conductor is:

$$\phi_s = 3.2l\left(\frac{u}{3w} + \frac{v}{w} + \frac{2r}{w+q} + \frac{p}{q}\right).$$

In some cases, only the lower coil-side in one slot and the upper coil-side in another slot are commutated at one time. In this case the flux is slightly greater than that given above, but in most machines two coil-sides in one slot, one above the other, are commutated simultaneously and the above formulae apply.

(*b*) The flux from tooth tip to tooth tip is shown in Fig. 37 and may be assumed as following a path consisting of a straight portion, q, and two quadrants of radius x, that is, of a length $(q + \pi x)$ where q is the width of the slot opening (equal to w for an open slot as in Fig. 37) and x varies from zero to a value $\frac{m}{2}$ equal to one-half the distance between the pole tips as in Fig. 39.

In the band dx we have:

$$d\phi = \frac{3.2cI_c l dx}{q + \pi x},$$

and

$$\phi = \int_{x=0}^{x=\frac{m}{2}} d\phi = 2.35cI_c l \log_{10} \frac{\pi m}{2q},$$

and from this, the flux per ampere conductor:

$$\phi_t = 2.35\, l \log_{10} \frac{\pi m}{2q}.$$

(*c*) The end connections may be imagined as segregated from the embedded portions of the coil as shown in Fig. 40, giving a circuit resembling a coil of approximately rectangular shape. In this let l_c be the length of end connection at one end (i.e., one-half the free length of turn) in inches, and let d_c be the diameter in inches of a cross-section of the coil (assumed to be round). The self-inductance of such a circuit is given approximately by the well-known relation:

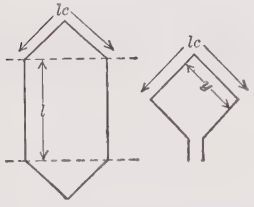


FIG. 40.—Linear dimensions of coils and hypothetical arrangement to deduce reactance of the end connections.

$$L = 2.35c^2 2y 10^{-8}\left(\log_{10}\frac{2y}{d_c} - .20\right),$$

where y is one side of the square and $2y = l_c$.

However, the machine coils are not round in cross-section but rectangular, therefore let U_c be the perimeter (inches) of a section including the insulation between turns, thus $U_c = 2(a + b)$, as in Fig. 38. Then, so far as the path of the surrounding magnetic lines is concerned, the equivalent diameter $d_c = \frac{U_c}{\pi}$. For consistency, we are interested in the inductance of the end connections at one end only, that is, half the inductance of the coil. The formula then becomes:

$$L_c = 1.17c^2 l_c 10^{-8} \log\left(\frac{\pi l_c}{U_c} - 0.20\right),$$

$$l_c = [(\text{m.l.t}) \div 2] - l,$$

calling $\log \frac{\pi}{2} = 0.2$, this may be written more simply:

$$L_c = 1.17c^2 l_c 10^{-8} \log_{10} \frac{2l_c}{U_c} \text{ henry.}$$

From this the flux per ampere conductor around the end connections of one side of one coil is:

$$\phi_c = 1.17 l_c \log_{10} \frac{2l_c}{U_c}.$$

This is the real flux per ampere conductor around the side of a coil in the end windings; but as the coil-sides in the slot separate into two groups as they leave the core, the upper layer going in one direction and the lower layer in the other direction (Fig. 41), there are only one-half as many ampere conductors per group in the end connections as in the slot. Consequently, the equivalent or effective flux per ampere conductor, or permeance, of this path would be one-half of the real value. We have therefore for practical use the equivalent permeance, or flux per ampere conductor, around the end connections of one side of one coil.

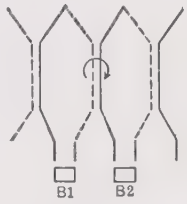


FIG. 41.—Two coils having one side of each in the same slot being commutated simultaneously.

$$\phi_c = .58 l_c \log_{10} \frac{2l_c}{U_c}.$$

Having obtained the equivalent permeance of the path of the leakage flux around one side of a coil, which is the sum of the three equivalent permeances, we note that the two sides of the coil are in series, each interlinked with its own independent flux, and therefore the inductance of the coil as a whole is twice that of one side of a coil and the total permeance for a coil would be $2(\phi_s + \phi_t + \phi_c)$.

Parshall and Hobart have published the results of tests on a large number of machines from which they derive empirical values for these quantities. They state that for the embedded length of conductor the flux per ampere conductor is about 10 lines per inch length of core ($\phi_e + \phi_t = 10l$), and for the free length the flux is about 2 lines per inch length ($\phi_c = 2l_c$). These values are serviceable if the slot proportions are the same as those in the machines tested, but considerable experience is required to prorate

them for slots of different shape. They correspond to open slots with a depth about 3.5 times the width.

If each brush has a thickness equal to twice the pitch of the commutator segments, it will touch three segments most of the time, so that there will be two adjacent coils undergoing commutation at once. There will also be two coils undergoing commutation at the adjacent brush studs of opposite polarity. If the winding has full or normal pitch, two coils short-circuited by adjacent brushes will have one side of each of them in the same slot, as shown in Fig. 41. The number of conductors undergoing commutation simultaneously in one interpolar space, due to the two adjacent brush studs, is therefore:

$$C = (\text{turns per coil}) \frac{(\text{brush thickness})k}{(\text{segment pitch})},$$

where k is 2 for full-pitch windings and must be worked out by diagram for fractional pitch windings as it depends upon the number of coil-sides per slot as well as the pitch and brush thickness. With many bars or coil-sides per slot, k becomes a variable with time and then it is well to determine its maximum value, which, however, never can exceed 2.

The inductance of a short-circuited coil is:

$$L = 2(\phi_s + \phi_t + \phi_c)Ct10^{-8} \text{ henry};$$

t = turns in series short circuited by the brush;

t = turns per coil for a multiple winding;

$t = \frac{\text{poles}}{2}$ times turns per coil for a series winding.

In a *series* winding, L is greater because, in a four-pole machine, for example, there are two coils in series between adjacent commutator segments.

The period of time (in seconds) occupied by a reversal of current (one alternation) is by convention the time required for a segment to pass across the brush. In reality it is slightly greater, being the time elapsing between the arrival of the forward edge of the segment at the brush and the departure of the trailing edge of the segment.

$$\text{Nominal time of reversal} = T = \frac{60 \text{ (brush thickness)}}{12V_c}$$

(Brush thickness in inches),

where V_c = peripheral speed of commutator, feet per minute.

If I_c is the current per circuit $\left(= \frac{I}{m}\right)$, then the average reactance voltage is $E_{av} = \frac{LI}{T}$ volts. The average reactance is a criterion used by many designers. It assumes that the change of current is a straight-line function of time, which, however, is not the case in practice.

In order to be on the safe side, Hobart has proposed that it be assumed that the change of current follows a sine law and that the maximum value of the voltage induced by this rate of change be called the reactance voltage.

In accordance with this hypothesis, the frequency of commutation (cycles per second) is:

$$f_c = \frac{1}{2T} = 120 \frac{12V_c}{(\text{brush thickness})'}$$

From this the value of the maximum reactance voltage is:

$$e_m = 2\pi f_c L I_c \text{ volts.}$$

17. The Magnetic Circuit.—The exact number of turns in series per path (S) was determined in Paragraph 11. If the calculation for reactance voltage shows no reason for changing this value, then the magnetic circuit and excitation may be calculated. The **useful flux per pole** in the armature is given by the equation:

$$\phi = \frac{Em\,60 \times 10^8}{pZN},$$

where E = some chosen value of generated voltage;
p = number of poles;
Z = total number of inductors = $2mS$;
N = revolutions per minute;
m = number of parallel paths in the armature

The flux in the field pole is greater than the flux in the armature, as all the flux that is set up by the field m.m.f. does not reach the armature, some of it leaking across from pole to pole, particularly at the pole tips. This is known as the **leakage flux,** and the ratio of the total flux in the middle of the field pole (leakage+useful) to the useful flux is known as the **leakage coefficient** (ν), which has values from 1.10 to 1.50.

This value may be calculated in detail for any given machine by the method given under "Alternating-current Generators," on page 163; but it is not worth while making the calculation in a d.c. machine, as it is usual practice to provide ample margin in field strength and it is inexpensive to do so. For the purpose of knowing with reasonable accuracy the magnetic densities in the various parts of the field, a value of leakage coefficient is assumed, in accordance with the values given in the table. A long pole pitch and a short armature length cause a low value of the coefficient and *vice versa.*

TABLE XI

Leakage Coefficient

Machine		Coefficient
Bipolar.....	Overtype	1.3 –1.5
Bipolar.....	Cylindrical	1.18–1.3
Four-pole.....	1–10 kw.	1.2 –1.3
Multipolar.....	5 kw.	1.2 –1.4
Multipolar.....	10 kw.	1.16–1.35
Multipolar.....	25 kw.	1.15–1.30
Multipolar.....	50 kw.	1.13–1.25
Multipolar.....	100 kw. and up	1.10–1.20

18. Cross-section of Magnetic Path.—*Flux Densities.*—It has been found advisable, from experience, to operate the various parts of the magnetic circuit at certain conventional densities in order to keep the coreloss and excitation within reasonable limits. A slow-speed or low-frequency (15 to 25 cycles) machine may have densities approaching the higher limits, but a high-efficiency or high-frequency machine should have densities approaching the lower limits.

The proper cross-section (square inch) of each part is found by dividing the flux in that part by an assumed density. Then, by dividing this area by the length along the shaft (L) as found in Par. 5, the other linear dimension is found.

19. Detailed Calculation of the Magnetic Circuit.—*Values of Flux in Different Parts of the Circuit.*—In the armature teeth and air gap the total flux is equal to the calculated value of the useful

flux per pole (Paragraph 17). In the armature core the flux divides into two branches and there is one-half of the useful flux in each branch (see Fig. 42).

TABLE XII

CONVENTIONAL MAGNETIC DENSITIES

Part	Material	Density, Lines per Square Inch
Field yoke	Cast steel	70,000 to 100,000
Field yoke	Cast iron	35,000 to 70,000
Pole core	Sheet or cast steel	70,000 to 100,000
Air gap	Air	40,000 to 70,000
Armature teeth	Sheet steel	90,000 to 125,000
Armature core	Sheet steel	60,000 to 90,000

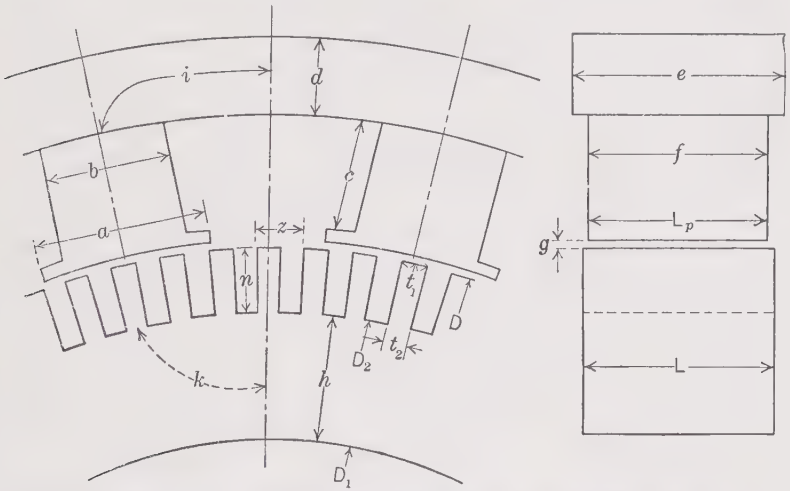


FIG. 42.—Dimensional drawing of a d. c. machine showing dimensions of the magnetic circuit.

The flux in the pole cores is greater than that in the air gap by the amount which leaks from pole tip to pole tip and from pole to pole. As this leakage is distributed all along the radial length of the pole piece from gap to yoke, the density is different in each part of the pole. However, by far the greatest amount of leakage occurs right at the pole tips, as here the leakage path is shortest

and the magnetomotive force is greatest. It is customary and reasonable to consider the flux in the pole core to be the same throughout and equal to the useful flux multiplied by the leakage coefficient (see Table XI).

In the yoke the flux divides again and each branch contains one-half as much as the pole core.

Ampere-turns.—The number of ampere-turns required for excitation for any particular voltage (as, for instance, rated voltage at no load) is calculated by means of a tabulation as given in Table XIII.

TABLE XIII

MAGNETIC CIRCUIT

1	2	3	4	5	6	7	8
Part	Flux	Cross section	Material	Flux Density "B"	Ampere-turns per Inch	Length of Path	Ampere-turns per Pole
Field yoke...	$v\phi/2$	de	.....	.....	.....	i	
Pole core....	$v\phi$	fb	.....	.....	.....	c	
Air gap.....	ϕ	aL_p	.....	.....	$.313B$	g	
Arm. teeth..	ϕ	Tl_0l_1	.....	.....	.....	n	
Arm. core...	$\phi/2$	hl_1	.....	.....	.....	k	

Explanation of Table

Column 2 gives the total flux passing through each particular part of the magnetic circuit.

Column 3 gives the cross-section of each part in square inches, the symbols referring to dimensions taken from a sketch or drawing similar to Fig. 42, as follows:

a = pole arc, inches;
D = diameter of armature at face;
D_1 = diameter of armature at back;
f = total length of pole along shaft;
g = radial length of air gap;

h = radial depth of armature core under slots;
k = length of magnetic path in armature core;
L = total length of armature core;
L_p = total length of pole shoe;
l_i = effective length of steel in armature core;
n = depth of armature slots;
z = pitch of slots at face of armature;
t_1 = width of tooth at face of armature;
t_2 = width of tooth at root.

Column 4 draws attention to the fact that different materials have different permeabilities and magnetization curves.

Column 5 gives the density in lines or maxwells per square inch, obtained by dividing the flux of Column 2 by the area of Column 3.

Column 6 contains the specific m.m.f. in ampere-turns per inch for the particular density and material, as taken from appropriate magnetization curves for the respective materials as given in Figs. 5 and 6 on page 12. For the air gap the number of ampere-turns per inch is directly proportional to the density and is given by the relation $(0.313 \times B)$.

Column 7 gives the length of the respective paths in inches as shown in the drawing. It should be noted that only one-half of the magnetic circuit is calculated as the two halves are identical and each field spool is required to supply only one-half the total m.m.f. of the circuit. It is customary to calculate the ampere-turns per pole or per spool, as all poles and spools are alike.

Column 8 gives the ampere-turns required for each part of the path and is obtained by multiplying the respective values in Column 6 by those in Column 7. The sum of all values in Column 8 is the excitation required on each spool of the field to establish the assumed value of useful flux. At no load this would correspond to a definite voltage as given by the formula of Par. 17.

Field Yoke.—The field yoke carries one-half the gross flux of the pole core. This divided by the cross-section (*de*) gives the density in Column 5. By referring to a magnetization curve of the proper material, the ampere-turns per inch of that material are obtained and set down in Column 6. The length of path per pole, or that whose reluctance is overcome by the m.m.f. of one field coil, is one-half the distance from pole to pole as, *i* in Fig. 42.

There are two such paths in multiple, one to the right and one to the left, each supplied by the same m.m.f. At a point in the yoke midway between the poles, the m.m.f. of the next spool may be said to pick up the flux and carry it the rest of the way.

Field Pole or Magnet Core.—This carries the gross flux, $v\phi$. The material is usually laminated sheet steel in the latest machines, while cast-steel poles are used in some cases. The cast-steel pole is usually circular in cross-section and its area is obvious. For laminated pole pieces, sheets having a thickness of 25 mils (0.025 in.) are used; the cross-section is usually rectangular and the net or effective area of magnetic material is 95 per cent of the total cross-section, that is, the insulation or oxide between sheets takes up 5 per cent of the stacked length. The **effective** area then becomes $.95fb$. The length of the path in the pole is usually taken as equal to c, the length of the core proper, although the real length may be greater than this by the height of the pole shoe. However, the magnetic density in the shoe is so much lower, because of its greater cross-section, that the error is negligible.

Air Gap.—The useful flux, ϕ, crosses the air gap from pole shoe to armature. The purpose of a pole shoe is to make the area of this path as large as possible and larger than the section of the magnet core proper. It is usually rectangular in cross-section, spans an arc about equal to 60 per cent of the pole pitch, but is 0.5 in. less than the armature core in length along the shaft as the flux fans out this much in crossing the gap. As a first approximation the cross-section of the path of the flux in the air gap is taken the same as the area of the face of the pole shoe (aL_p). In most cases this is sufficiently accurate.

If more accurate results are desired it is necessary to consider the following effects (see Figs. 43 to 46, inc.).

(a) Fringing of flux at the pole tips.

(b) Constriction of the flux in the gap due to the armature slots.

(c) Fringing of the flux at the faces of the poles.

(d) Constriction of the flux in the gap due to the air ducts in the armature.

In most cases these four effects neutralize each other, a and c offsetting b and d. Of the four, the effect of the armature slots (b) is the most important quantitatively. The four effects will be considered categorically.

(a) *Fringing at the Pole Tips* (Fig. 43).—Arnold and Thompson

give a method of plotting the flux graphically and calculating the permeance of these extra paths which may be added to the permeance of the main or obvious paths; or the percentage increase in permeance may be applied to the pole arc to obtain an equivalent pole arc which is used in finding the equivalent area and density in the gap. This increase in pole arc is usually equal to

FIG. 43.—Showing spreading or fringing at the pole tips.

once or twice the length of the air gap (g), an equivalent pole arc of $(a + g)$ being the usual approximation. But in many modern machines the pole face has a **chamfer,** that is, the gap is greater at the tips than at the center of the pole. This offsets any effect due to fringing at the tips, and few designers make any allowance for the effect in machines with a chamfer.

(*b*) *Constriction of Flux in Gap Due to Armature Slots.*—This effect was very ably analyzed in a paper by F. W. Carter before the Br. I. E. E. in 1901, and his method is used by most designers

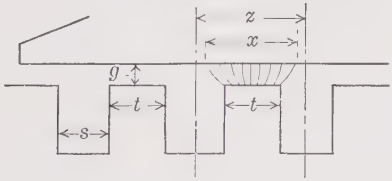


FIG. 44.—Showing fringing at the tooth tips in the air gap.

in the form of the **Carter coefficient** which is a constant, greater than unity, by which the real length of air gap (g) is multiplied to get the equivalent or **effective** gap (g'), thus $g' = Cg$.

This allows for the effect shown in Fig. 44 in which the cross-section of the band of flux from each tooth to pole face is greater than t, the tooth face, on account of spreading, but is not as great as z, the pitch of slots, or tooth plus slot (which is the value assumed when the gap area is taken as equal to the pole-face area), but has some intermediate value shown by x in the diagram,

where $x = t + fs$;
t = tooth face;

s = slot width;
z = slot pitch = $t + s$;
f = empirical constant (see Table XIV).

The apparent reluctance of each band is $\frac{g}{z}$. The real or effective reluctance is $\frac{g}{x}$ or $\frac{g}{t + fs}$ and the ratio of effective to real is $\frac{z}{t + fs} = \frac{g'}{g} = C$, the Carter Coefficient.

The empirical constant, f, is determined analytically by evaluating the reluctances for the two paths for various conditions and is found to depend primarily upon the ratio of the slot width to the gap length, or upon $\frac{s}{g}$. Carter compiled data upon this and found the following relations:

TABLE XIV

Constant for Carter Fringing Coefficient

s/g	0	.5	1.	1.5	2.	3.	4.	5.	6.	7.	8.	9.	10.
f	1.	.95	.84	.78	.71	.62	.55	.49	.44	.40	.38	.35	.32

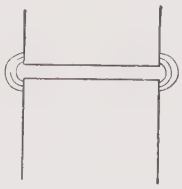


Fig. 45.—Showing fringing at the outside ends of poles and armature.

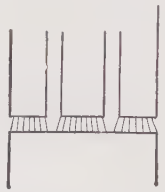


Fig. 46.—Showing fringing at the ventilating ducts of the armature.

(*c*) *Fringing or Spreading of the Flux at the Ends of the Poles* (see Fig. 45).—This may amount to an apparent increase in the length of the pole, L_p, by an amount equal to g or $1.5g$ but this is very small compared to L_p.

(*d*) *Constriction of the Flux Due to the Ventilating Ducts in the Armature* (Fig. 46).—This effect is similar to that (*b*) of the armature slots but of lesser magnitude. Regarding a ventilating duct as similar to a slot (but in another dimension or direction), and the solid part of the armature as similar to a tooth, the ratio of

(duct width) ÷ (gap length) $= \frac{s'}{g}$ is found, and from the table or curve given for slots a value of f which is substituted in the formula:

$$C' = \frac{z'}{t' + fs'}.$$

This is seldom greater than 1.05.

The final adjustment for the four effects is usually made by multiplying the actual gap length by the four coefficients, two of which (a and c) are less than unity and two (b and d) greater than unity. Thus the gross area of pole face may be used as gap area, and refinements accomplished by adjustments in the value of the effective length of gap.

Having found the approximate or the effective pole arc and pole length as above, we obtain the magnetic density in the air gap by dividing the flux, ϕ, by this area. The number of ampere-turns per inch is always $0.313B$ in air. The total ampere-turns per pole for the air gap is obtained by multiplying this value of ampere-turns by g for the approximate case or by $g' = CC'g$ for the more exact calculation.

Armature Teeth.—At any given instant only a portion of the armature teeth carry flux. These are known as the **teeth under a pole** (T), and the number is slightly greater than those actually in the angle subtended by the pole shoe, due to the spreading or fringing at the pole tips as shown in Fig. 43. This spreading is approximately at 45° and increases the width of the flux at the armature surface by an amount equal to the gap length at each tip of the pole. The number of teeth under a pole carrying flux is therefore $T = \frac{a + 2g}{z}$. T may be a fractional number as it is a time average. Since the density in the teeth varies uniformly from a maximum at the root to a minimum at the face, and since the necessary m.m.f. varies at a greater rate, the effective or equivalent section of each tooth is not that which gives the average density but that which gives the average or mean excitation required. This is shown in Fig. 47.

The usual approximation for this is to take a section of the tooth one-third the distance from the narrowest end, and calculate the density here. This cross-section is given by the formula:

$$\frac{t_1 + 2t_2}{3} l_i = t_0 l_i.$$

The factor l_i, **effective length of iron,** is the net length of magnetic material and excludes the space taken by air ducts and the insulation (varnish or oxide) between the sheets of steel, thus:

$$l_i = 0.9 \times (L - \text{space of ducts}).$$

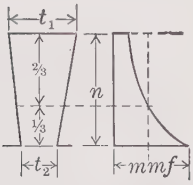


FIG. 47.—Method of determining the mean m.m.f. through the length of a tooth as distinguished from mean density. Equivalent tooth section.

The factor 0.9 is known as the stacking factor and varies with the thickness of laminations used. The standard is 0.9 for sheets having the standard thickness of 14 mils. Thus the equivalent cross-section of the path of the flux in the armature teeth is Tt_0l_i as noted in Column 3. Column 4 is the flux of Column 2 divided by the area in Column 3.

This, however, is the apparent density. If the teeth are worked at high densities (above 120,000), an appreciable amount of flux will pass through the slot in a magnetic path in parallel with the teeth. This makes the real density in the teeth less than the apparent density. To find the real density it is necessary to calculate a table and plot curves for the real and apparent densities as shown in Table below.

1	2	3	4	5	6
Real Density	Ampere-turns per Inch	Density in Air	Apparent Density		
			$s=t$	$s=1.5t$	$s=2t$
120,000	380	1220	121,220	121,830	122,440
130,000	660	2120	132,120	133,180	134,240
140,000	1000	3200	143,200	144,800	146,400
150,000	1500	4800	154,800	157,200	159,600

Columns 1 and 2 are taken from the magnetization curve of the steel used for the teeth. Column 3 is the magnetic density in air for the respective magnetizing forces. If the slot width is s and the equivalent tooth width is t, then the total flux carried by a tooth and a slot is $(120{,}000t + 1220s)$ and the apparent density becomes

$$\frac{120{,}000t + 1220s}{t} = 120{,}000 + 1220\frac{s}{t}.$$

To give an idea of the magnitude of this effect, Table XV gives results for a typical quality of steel laminations expressed in a coefficient of the ratio by which the apparent density may be multiplied to get the real density:

TABLE XV

Apparent Density	Coefficient		
	$s = t$	$s = 1.5t$	$s = 2t$
120,000	0.99	0.985	0.98
130,000	0.984	0.976	0.968
140,000	0.978	0.967	0.956
150,000	0.969	0.954	0.94

Having found the approximate or the real tooth density, depending upon the accuracy desired, the magnetization curve of the material used in the teeth is consulted for the corresponding number of ampere-turns per inch, and this figure is placed in Column 5 of table and multiplied by n, the depth of the slot or tooth, to get the total ampere-turns required for the teeth.

Armature Core.—The axial length of the armature core is usually slightly greater than the axial length of the pole face. A common allowance is twice the radial depth of the air gap (some manufacturers standardize a $\frac{1}{2}$ in. difference in all cases). The core is built up of sheet-steel punchings, 14 mils thick, the punchings having a thin coat of varnish. Ventilating ducts from $\frac{3}{8}$ to $\frac{1}{2}$ in. wide (measured along the shaft) are usually provided every 2.5 to 3 ins. The effective axial length of the armature core is therefore $l_i = 0.9 \times (L - \text{space of air ducts})$. One-half of the useful flux per pole passes through each section of the armature core, dividing under the center of the pole. The cross-section of the magnetic material in the core is equal to the effective length multiplied by the radial depth back of the slots, h. From these data the density is found, and the magnetizing force per inch is obtained from a magnetization curve for the material used. This is multiplied by the length of the path per pole as illustrated by k in the drawing and the ampere-turns for the path in the armature core are set down in Column 8.

By repeating the calculation of this table for two or three

different voltages and corresponding fluxes at the same speed, points are obtained for the calculated or **predicted saturation curve** of the machine. The ordinates are usually volts and the abscissae are ampere-turns per pole, which may be converted to amperes in field by dividing by the number of turns in one shunt-field spool, when known. (See Fig. 91.)

20. Stability Factor.—At this point in the design it is necessary to check the relations to determine whether the machine is likely to be unstable or to require an excessive shift of the brushes if it is not to have commutating poles. If commutating poles are to be used this is not important, but if commutating poles are not to be used certain conditions must be fulfilled in order to insure successful operation. The criterion of these conditions is the **stability factor,** which is the quotient of the number of field ampere-turns required for the gap and teeth (lines 3 and 4 of Col. 8 of Table XIII) divided by the number of **armature ampere-turns beneath the pole.** This quantity is:

$$\frac{(\text{amp.-turns for gap} + \text{teeth})}{(\text{arm. react. amp.-turns}) \times \rho} = \text{stability factor.}$$

The conditions to be fulfilled are that, at maximum operating current in the armature, the armature ampere-turns beneath the pole shall not exceed the field ampere-turns for gap and teeth. This is because the armature reaction tends to distort the flux and the ampere-turns for gap and teeth tend to hold the flux in its proper place. This relation is covered by the empirical rule of making the stability factor at full-load current have a value of 1.3 to 1.5 in generators and 1.5 to 3.0 in motors. A stability factor of 1.5 for full load means that this factor will be unity when the armature carries 1.5 times full-load current or at 50 per cent overload.

The simplest means of raising the stability factor is to increase the length of the air gap, and it is for this reason that the final length of the air gap is not decided upon until it is known that the stability factor is satisfactory.

In a generator without commutating poles, a low stability factor makes a machine which will not commutate well without shifting the brushes a considerable amount. This in turn intro-

duces **back ampere-turns** and causes poor voltage regulation. In a motor with low stability factor there is liability to bad commutation at full load and overloads, and also if the field is weakened for the purpose of increasing the speed. Even in a motor with commutating poles, the speed is likely to be very unstable if the stability factor is low.

21. Final Length of Air Gap.—If the stability is too low with the preliminary design used thus far, it is necessary to make a change by choosing a greater length of air gap, thereby increasing the number of ampere-turns required for the air gap and, in consequence, increasing the stability factor. In a machine without commutating poles the length of the air gap is determined largely by the stability factor desired.

22. Armature Reaction or Interference.—(See page 47.) The current in the armature conductors sets up a magnetomotive force which reacts upon the m.m.f. of the field coils and prevents them from setting up as much flux as if there were no current in the armature. It is therefore necessary to have the field strength at any load greater than at no load by an amount sufficient to overcome or neutralize the armature reaction, or to provide special auxiliary windings or poles for that purpose. There are four methods of estimating the effect of armature reaction and of taking care of it:

(*a*) It is common practice in the manufacture of d.c. generators in quantities to provide a series-field coil of such ample capacity that it is sure to be more than sufficient to overcome all armature interference and set up the desired value of flux. This field is then adjusted in practice by putting an adjustable resistance in series with the shunt field and a suitable fixed value of resistance in parallel with the series field. With this method it is customary to provide a total number of field ampere-turns per pole equal to the net ampere-turns required for the flux as calculated in Paragraph 19 plus a number equal to 40 per cent of the armature reaction at full load as calculated in Paragraph 9. In a generator the excess over that required for the flux is usually put in a series field.

(*b*) An exact method is given by S. P. Thompson and by E. Arnold. The effect of armature reaction is divided into **Back** ampere-turns and **Cross** ampere-turns. The back ampere-turns

per pole are caused by shifting the brushes and are numerically

$$A' = \frac{\text{armature reaction} \times \text{brush shift in segments} \times 2p}{\text{total commutator segments}}$$

p = number of poles.

The cross ampere-turns = armature reaction $- A' = C$.

The effect of the cross ampere-turns is to increase the magnetic density in one pole tip and decrease the density in the other, leaving the density at the center unchanged. On account of the varying permeability due to different saturation in the teeth, the increase cannot be as great as the decrease, thus the net effect is to decrease the average density and the total flux per pole. This is best shown by a diagram making use of a magnetization curve of the machine. It is allowable to use the **no-load saturation** as calculated in Paragraph 19 but it is more exact to construct a special curve between total flux, ϕ, and ampere-turns for gap and teeth, leaving out the ampere-turns required for the yoke, pole core and armature core as these parts are not acted upon by the cross ampere-turns.

Referring to Fig. 48, let OP be the ampere-turns for gap and teeth at the flux densities required to give rated voltage plus the IR drop (in a generator) or, in the less accurate method, the ampere-turns per pole for the whole circuit. The choice depends upon the accuracy desired, the less accurate method being pessimistic. Then Pa represents by proportion the uniform density in the gap with no cross-magnetization. Lay off from P to the left $Pb = \frac{1}{2}\ C$ and to the right, $Pc = \frac{1}{2}\ C$, so that the distance bc represents the cross ampere-turns, C, to scale. Then eb represents the density under the weakened pole tip and dc the density under the strengthened pole tip. Thus the curve, ead, represents the varying density under the pole due to the cross ampere-turns. It will be noted that the area, efa, which represents the loss on one

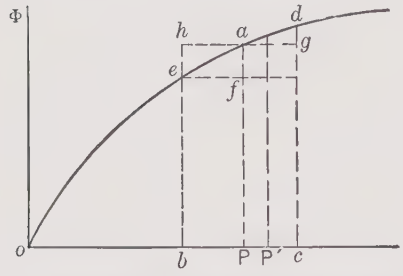


Fig. 48.—Effect of cross ampere-turns which decrease density at one pole tip more than they increase density at the other tip.

side, is greater than the area, *agd*, which represents the increase on the other side. The net reduction in flux is proportional to the quotient of area *beadc*, divided by area *bhagc*. To obtain the required value of flux it is necessary to increase the excitation from *OP* to *OP'*, so that the new irregular area will be equal to *bhagc*. This must be done by trial and error, as the shape of the saturation curve is irregular and the slope constantly changing.

(*c*) The effect of armature reaction is divided into back and cross ampere-turns. The number of field ampere-turns required to overcome the back ampere-turns is:

$$A' = \frac{(\text{armature reaction}) \times (\text{brush shift segments}) \times 2p}{\text{total commutator segments}}.$$

The excitation to overcome the effect of the cross ampere-turns depends upon many variables; but in machines designed in accordance with standard practice it bears a fairly uniform relation to the cross ampere-turns, depending upon the value of the ampere-turns in gap and teeth as given by an empirical relation due to Cramp.

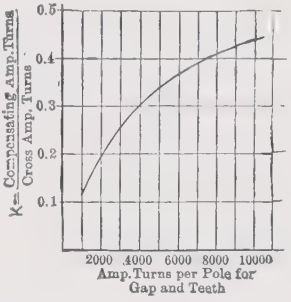


FIG. 49.—Cramps empirical constant for the effect of cross ampere-turns.

Cross ampere-turns = (armature reaction) − (back ampere-turns).

Field ampere-turns to compensate for cross turns:

$$B' = K \times (\text{cross ampere-turns}).$$

Where K has values as shown in Fig. 49.

Then the gross ampere-turns required,

$$F = \text{Net ampere-turns required for flux} + A' + B'.$$

(*d*) Interpoles may be used as described in section 24, in which case commutation will be satisfactory and the brushes need not be moved, but an increase in excitation will be required to give the required terminal voltage.

23. Field Windings.—*Excitation for Various Loads.*—At no load the excitation required would be only that necessary to produce the flux corresponding to the rated terminal voltage. At

any load corresponding to a current I, the excitation in ampere-turns must be equal to the sum of: (1) that required to produce a flux corresponding to $E + RI$, where R is the total resistance of the armature between brushes, the brush resistance, and the resistance of the series field; (2) that required to overcome the back ampere-turns caused by I, and (3) that required to overcome the cross ampere-turns due to I. The process of calculation indicated in sections 19, 20 and 22 for full-load conditions is, therefore repeated for other load conditions, i.e., other values of armature current.

Shunt Field.—The shunt field is required to give a certain number of ampere-turns, usually equal to the no-load ampere-turns. For a factor of safety this number is usually determined by the highest voltage and the lowest speed at which the generator is expected to operate under unusual or abnormal conditions. The lesser excitation for normal operating conditions is obtained in practice by a variable-resistance rheostat in series with the shunt field. A value having been assumed for F, the ampere-turns per pole required of the shunt field, and e, the available voltage for each shunt-field spool (the total voltage divided by the number of the poles), there is a fixed relation between the size of wire required and the mean or average length of the turns on the spools.

Let q_1 be the cross-section of the proper size of wire expressed in circular mils, l_1 be the mean length of shunt turn in feet, and t_1 the number of turns; then the following relations hold:

$$F = i_1 t_1 \qquad i_1 = \frac{e}{r_1} \qquad r_1 = \frac{\rho l_1 t_1}{q_1}$$

hence

$$F = \frac{e t_1 q_1}{\rho l_1 t_1} = \frac{e q_1}{\rho l_1},$$

where ρ is the resistance of copper wire per mil foot at the proper temperature, say $\rho = 12.7$ for 75° C. Then the proper size of wire, regardless of the number of turns, is:

$$q_1 = \frac{12.7 l_1 F}{e} \text{ circular mils.}$$

The proper number of turns, t_1, is determined by the allowable loss of I^2R, which affects the heating of the coils and the efficiency of the machine. The greater the number of turns the less the loss and the less the heating.

To provide against overheating we may calculate the approximate radiating surface of one field spool by assuming the cylindrical surface and ignoring the top and bottom ends as these are usually rendered ineffective for cooling purposes by sheets or collars of insulating material.

For a preliminary sketch, the space required for the shunt-field winding may be taken as a cross-section of 2 sq. ins. for every 1000 ampere-turns, and the radial depth of winding (b_2 in Fig. 50) may be chosen with regard to the available space between

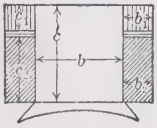


FIG. 50*A*.

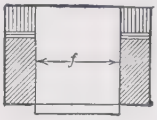


FIG. 50*B*.

FIG. 50.—Cross-section of shunt and series field coils on a pole.

poles. A clear space for ventilation must be left between adjacent spools. In small machines the series and shunt windings are frequently wound on the same spool, the series inside next to the pole and the shunt outside. In larger machines the two coils are separate, the two windings having the same depth, b_2, and appropriate heights c_1 and c_2 as in Fig. 50.

Referring to Fig. 50, let b = pole width, f = pole depth or length, and c_2 = height of shunt spool, then the radiating surface would be:

$$A_1 = 2c_2(f + b + 4b_2) \text{ sq. in. approximately.}$$

The rise in temperature of the coil will be from 45° C. to 60° C. per watt per square inch, depending upon the ventilation (open construction and high peripheral speed of armature correspond to 45°). A loss of from 0.8 to 1.0 watt per square inch is allowable, and from this assumption the allowable watts per spool may be calculated. Dividing this loss in watts by the value of e assumed gives the shunt-field current for the condition assumed. The proper number of turns is derived from the relation:

$$t_1 = F \div i_1.$$

In some machines in which a very high efficiency is desired the watts loss must be made less than that allowed by the heating and then certain definite assumptions are made for the shunt-field loss

(see Table XVI, "Usual Efficiencies and Losses," p. 107). This value of loss is assured by increasing the number of turns without changing the size of wire from that obtained originally. The loss in watts decreases directly as the number of turns increases.

Having thus a preliminary determination of the desired size of wire and number of turns, we can lay out the winding in detail on the drawing board, fitted into the space available and recalculated for the final arrangement. The resistance of the whole shunt field (all poles in series) will be:

$$r_1 = \frac{12.7 t_1 l_1 p}{q_1},$$

where p = number of poles;

q_1 = circ. mils,

$l_1 = 2(f + b + 2b_2) \div 12$ ft.

A collar of about $\frac{1}{8}$ in. is allowed at each end and a metal spool covered with insulation surrounds the pole piece and holds the winding. Single-cotton-covered wire is commonly used in moderate-sized machines.

Series Field.—In a compound-wound generator the ampere-turns required in the series field are equal to the difference between those required at full load and those required at no load. The ampere-turns at full load would be those required to produce the particular generated voltage desired ($E+IR$), which may be greater than the **no-load voltage** if an over-compounded machine is desired, plus those to overcome back and cross ampere-turns of the armature.

When the total or gross ampere-turns excitation required at full load has been determined, the quantity assigned to the shunt field is deducted and the remainder is the quantity used in the design of the series field. It is usual (for emergency conditions) to shunt about 25 per cent of the load current through a shunt resistance connected across and in parallel with the series field; thus the number of turns per pole should be:

$$t_2 = \frac{\text{ampere-turns required}}{0.75 \times \text{load current}}.$$

Series fields are usually made of copper strap and operated at a current density of 1000 amperes per square inch. Therefore the

cross-section of each turn in square inches is taken equal to 0.75 of the full-load current divided by 1000. The thickness of each turn is determined by the value assumed for the thickness of the whole spool (b_1) which is usually set by the design of the shunt winding. The coils are wound with an extra half turn to facilitate connecting in series with the adjacent coils in correct polarity. The relation for determining the thickness of each strap is:

$$b_2 = (\text{thickness of strap} + .016 \text{ insulation})\ (t_2 + 1).$$

The width of strap must be: c_1 = cross-section ÷ thickness. Allowance for a layer of insulation inside and outside adds about 0.375 in. to the depth of winding.

The mean length of turn, l_2 (in feet), is calculated similarly to that for the shunt field and the resistance of the series field is:

$$r_2 = \frac{.01 t_2 l_2 p}{1000 q_2},$$

where q_2 = cross-section of conductor in square inches.

The total drop in voltage in the series field is dependent upon the amount of current shunted through the series-field shunt or upon the equivalent resistance of the two parallel paths, and should be between 0.5 per cent and 1 per cent of the rated voltage (see Table XVI, "Usual Efficiencies and Losses," page 107).

It should be noted that the shunt field carries current and is subject to heating all the time the machine is in operation, while the current in the series field varies with the load and is not the cause of continuous heating. Therefore the heating and ventilation of the series field is not as important as that of the shunt field.

24. Commutating Poles or Interpoles.—While good commutation may be achieved by moving the brushes, this action introduces a back m.m.f. in the armature, which is undesirable, and the proper degree of movement of the brushes varies with the magnitude of the armature current so that only a compromise can be obtained if the brushes are not changed with every change in the load.

Commutating poles give good commutation at all loads in all machines except those for a very high voltage (1500) or for very high speeds. Commutating poles are magnetic poles, placed at the neutral point midway between the main poles, having a flux of such a magnitude and polarity as to generate in the coils short-circuited by the brushes an e.m.f. which is always equal to and

opposed to the e.m.f. of self-induction, and therefore reverses the current in those coils automatically and gives perfect commutation. Instead of moving the brushes in order to bring the particular inductors undergoing commutation to one of the main poles, a part of that main pole is brought to the neutral where the inductors should undergo commutation. Commutating poles do not prevent armature reaction and flux distortion as is sometimes erroneously believed, but they make possible sparkless commutation with the brushes left fixed at the neutral point, which gives minimum demagnetizing effect of the armature.

Commutating poles are used where either the armature reaction or the reactance voltage is great. If the armature reaction at full load is greater than 80 per cent of the field ampere-turns at no load, then commutating poles should be considered. If the reactance voltage, e_m, as calculated by the formula of Paragraph 16, is greater than 2.5 volts, then commutating poles should be used. Also if a motor must run alternately in opposite directions without attention to the brushes, as a railway motor, commutating poles should be used.

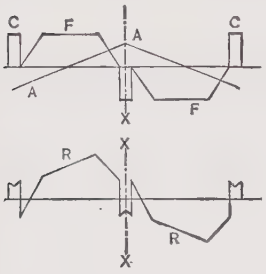


FIG. 51.—Distribution of fluxes in the air gap with commutating poles.

In Fig. 51 let F and A represent the field and armature fluxes separately, as in Fig. 32 (page 51) and let C represent the flux from commutating poles with rated current. When these three fluxes are combined, as at full load, the resultant flux has a distribution shown by R. It will be noted that there remains, at the neutral point, a small flux of the proper polarity to generate an e.m.f. which will reverse the current in the armature coil and give good commutation without the necessity of moving the brushes as was the case in Fig. 32. The commutating pole must have a polarity opposite to that of the armature at that point, and its strength must vary as the strength of the armature magnetization varies, because when there is no current in the armature there should not be any flux at the commutating poles.

The polarity of each commutating pole in a generator should be the same as that of the main pole towards which the inductors at that point are moving, and in a motor each should have the

polarity of that main pole from which the inductors are moving. The windings on the commutating poles are connected in series with the armature and the load so that their strength varies with the armature current. The m.m.f. of each such pole must be greater than the armature m.m.f. per pole in order that it may neutralize or "overpower" the armature magnetism and set up a definite flux of its own in the opposite direction to the armature flux. The polarity of the commutating poles during motor action is different from their polarity during generator action, but if the windings of the poles are connected correctly in series with the armature for one action the conditions will be correct for both actions.

When commutating poles are used the brushes are kept exactly at the neutral point; hence there never can be any back ampere-

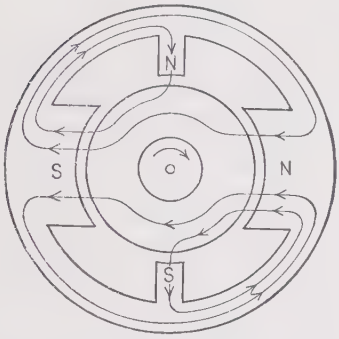


FIG. 52.—Paths of the main flux and commutating pole flux.

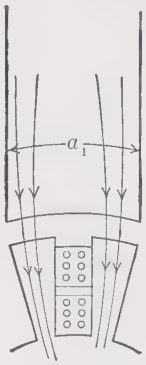


FIG. 53.—Location of the conductors being commutated in the flux from the commutating pole.

turns in the armature, only cross ampere-turns; thus the series field need not be as strong as in ordinary generators. On the other hand, the flux from the commutating poles, passing through the armature core and pole pieces, increases the densities in these parts, distorts the flux in the main pole faces and increases the reluctance of the whole magnetic circuit; thus some series field is necessary to maintain the proper terminal voltage in a generator.

Since these poles do not in any way prevent cross-magnetization and the resulting distortion of the distribution of the main flux in the air gap, there are certain types of machines, such as

those with a high voltage between commutator bars or a low stability factor, which will not commutate successfully even with commutating poles. In such machines a **compensating winding** (q.v.) must be used along with the commutating poles.

Design.—The design of the commutating poles is based upon the simple principle that, if the conductors forming the short-circuited coil are revolved at the peripheral speed of the armature in a commutating field flux of a given density, B, then a voltage will be generated having a value

$$e = Blv10^{-8},$$

where e = the voltage desired;
l = active length of conductors in series (c.m.);
v = peripheral velocity of conductors cm. per second;
B = the density required (gausses).

For good commutation this voltage should be equal to the reactance voltage of the coil undergoing commutation (e_m of Paragraph 16).

A more convenient and more practical method (based on this same formula) is to assume the density B at some conventional value and to calculate the appropriate axial length of pole face which, having this density, would be required to give the desired voltage, thus:

$$L_1 = \frac{e10^8}{kB_1V_a},$$

where e = the e.m.f. desired as determined by the calculation of the reactance voltage, e_m;
k = number of inductors in series in the short-circuited coil = $2t$ (see formula for reactance voltage);
B_1 = 30,000 lines per square inch for most cases and 40,000 in exceptional cases;
V_a = peripheral velocity of the armature in inches per second;
L_1 = axial length of the commutating pole in inches.

The width, a_1, of the commutating pole is made equal to the span of an armature slot and two teeth. The pole core is usually made the same width as the pole face in order to keep the magnetic density in the steel low, so that the flux will vary directly as a linear function of the current. The air gap under the commutat-

ing poles is made from 1.5 to 2 times the length of the air gap under the main poles, for the same reason. If L_1 should come out greater than the length of the armature, then a higher value for B_1 must be assumed.

The flux going across the gap from each commutating pole will be $\phi_1 = B_1 a_1 L_1$. The leakage coefficient for the commutating poles is high because of the restricted space and is in the neighborhood of 1.6 to 1.8, thus this component of the flux in the main yokes is considerable. The ampere-turns m.m.f. required to force this flux across the air gap, through the pole, and to take care of the increased densities in the armature core and field yoke, are calculated in the same manner as for the main magnetic circuit.

Let this value be A'' ampere-turns per pole. Let the armature reaction ampere-turns per pole be B''. Then the total ampere-turns excitation required on each commutating pole at full load is $(A'' + B'')$. This sum, divided by the full-load current of the machine, gives the actual number of turns of wire required upon each commutating pole.

Usually there are as many commutating poles as there are main poles, and with multiple-wound armature windings this is necessary; but with series-wound armatures for four-pole machines (railway motors in particular) only two poles, diametrically placed, may be used satisfactorily. In this case the product of $B_1 \times L_1$ must be twice as great as if four commutating poles were used, because only one side (hence half the inductors) of each coil lies under a commutating pole and has to do double duty. Practically, this is allowed for by using t in the formula in place of $2t$ for k. In all cases the coils of the armature winding must have full pitch, that is, must span a number of slots equal to the slots per pole in order that both sides of each coil shall lie at the neutral point at the same time.

The design of the commutating poles may be reasonably approximated in accordance with empirical methods used in the quantity production of machines whose ultimate individual application is not known beforehand. The windings are designed so generously that they may be adjusted empirically, after installation, by means of taps on successive turns. The width of the poles is made equal to the span of an armature slot and two teeth. The length of the poles is made equal to the length of the main

poles, and the length of the air gap equal to 1.5 to 2 times the air gap under the main poles. The poles are sometimes arranged to be shimmed where they are attached to the yoke frame in order to vary the air gap at will. The winding is designed arbitrarily with 40 per cent more ampere-turns than the armature reaction, and taps are placed on the turns near one end of the coil so that the actual m.m.f. of the coils may be adjusted under working conditions. Sometimes the commutating pole windings are shunted by a resistance, as in the case of the series field, to obtain the adjustable feature; but this is not good practice if sudden changes of load are expected, because when the current increases suddenly the excess will pass through the non-inductive resistance rather than through the inductance of the pole winding and the desired relation between armature current and commutating flux will not exist.

The physical details of the winding on the commutating poles are similar to those of the series-field winding. In small machines the winding is made up of double-cotton-covered or enameled wire of a size that will give a current density of from 1000 to 1500 amperes per square inch at full-load current. In large machines it consists of a score or so of turns of rectangular copper conductor or strap, bent by a special machine into the form of a spiral surrounding the pole core. When this is stiff enough to hold its shape no insulation is used between turns, a clearance of one sixteenth of an inch serving as insulation. Between the pole core and the winding, asbestos insulation is used. All poles are connected in series with the armature, brushes, and series field, if any, and the resistances of these parts are added together for the total resistance of the armature circuit.

Figure 52 shows the paths of the flux of the commutating poles with reference to the main magnetic circuit.

Figure 53 shows a section of the commutating pole, its flux, and the inductors passing through the flux.

Compensating Windings.—Compensating windings are used in those machines in which the problem of commutation is so difficult that commutating poles alone will not give successful commutation. Such is the case in machines for very high voltages (1500 volts) or very high peripheral speeds, or in machines subject to very sudden and large variations in load. In such cases it will be found impossible to design a machine whose armature reaction

is reasonable compared to the field m.m.f. It is therefore necessary to use a high value of armature reaction and neutralize it by means of compensating windings.

The compensating winding is a winding distributed uniformly in the faces of the pole pieces, which, theoretically, neutralizes exactly the m.m.f. of the armature; that is, there is in the face of each pole piece as many ampere-turns as the armature reaction per pole, and the two m.m.f.'s are arranged to oppose each other. This is accomplished by connecting this winding in series with the arma-

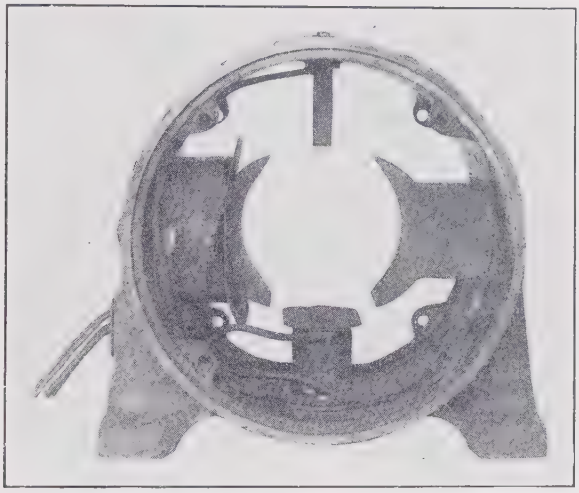


Fig. 54. Photograph of the field of a commutating pole machine with some coils removed to show pole cores.

ture and sending the load current through it. These turns have their magnetic axis in line with that of the armature, i.e., the neutral plane between the poles (where the interpoles are).

Figure 55 gives a diagram of the arrangement of a four-pole machine with armature, main field coils, and compensating windings. Each of the four compensating coils centers about an interpolar space, and half of each winding lies in adjacent pole pieces. Fig. 56 shows the directions of the current streams in the armature and pole faces and indicates the neutralization.

In the discussion of commutating poles it was shown (Fig. 51) that, although these poles gave a desirable commutating flux at the neutral point, the resultant flux (R) around the periphery

remained distorted. As the voltage per coil, or between adjacent

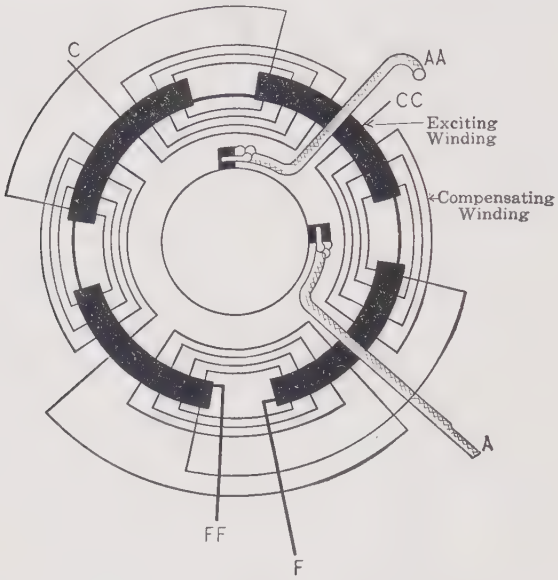


FIG. 55.—Relative position of the shunt or exciting winding and the compensating windings.

commutator segments, is proportional to the magnetic densities indicated by the ordinates of the figure R, it will be seen that since the maximum height of R is greater than the maximum height of F the maximum voltage between bars is increased by the load. With a large value of armature reaction this may become so great as to cause the commutator to flash over because of high voltage between bars, without reference to mere commutation.

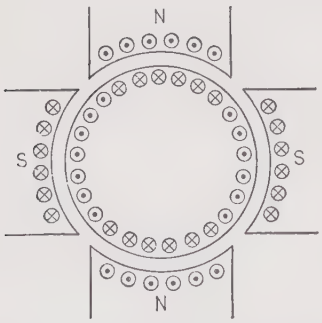


FIG. 56.—Direction of currents and m.m.f.'s of the armature and the compensating windings.

This is shown in Figs. 57 and 58. In Fig. 57 the broken line shows the flux distribution and volts per bar for the condition of no load, and the full line for the condition of full load in a machine

having an excessive armature reaction and *no* compensating winding. It will be noted that at full load the value of the maximum volts per bar is twice that at no load, and that at one point in the cycle the flux in the pole tip and volts per bar are actually reversed. In Fig. 58 the compensating winding has been con-

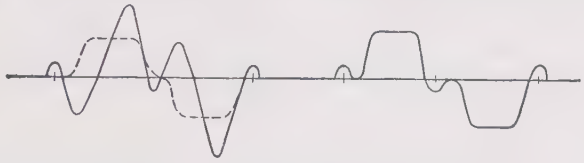


FIG. 57. FIG. 58.

FIG. 57.—Distribution of flux in the air gap of machine having high armature reaction and commutating poles but no compensating windings. (Oscillograph).

FIG. 58.—Distribution of flux in the air gap of the same machine as Fig. 57 but having a compensating winding in action (Oscillograph).

nected in circuit and there is no distortion or excessive voltage. These figures have been taken from oscillographs of an actual machine.

The principle of operation of the compensating winding is shown in Fig. 59, in which F represents the flux, or m.m.f., of the shunt-field winding, A the m.m.f., or flux, due to the armature reaction (as in Figs. 32 and 51), and C the effect of the compensating winding. Since A and C are everywhere equal and opposite, the resulting flux is the same as F and there is no distortion.

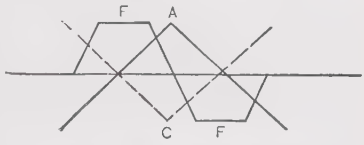


FIG. 59.—Theoretical distribution of the m.m.f.'s of the field, armature and compensating winding.

In practice, the m.m.f. of the compensating winding is usually made to bear the same ratio to the armature reaction as the pole arc does to the pole pitch, that is, 60 to 70 per cent, leaving the reluctance of the interpolar space and the m.m.f. of the commutating poles the duty of preventing the armature from setting up an excessive flux between the main poles. This is expressed practically by saying that the compensating winding should have an m.m.f. per pole of:

$$\text{armature reaction} \times \text{per cent pole arc} = n\rho.$$

If I is the line current and t_c the number of actual turns per pole in the compensating winding, then:

$$t_c = \frac{n\rho}{I} = \text{turns in face of one pole.}$$

These are usually distributed one bar per slot, hence there would be $2t_c$ slots in the face of each pole. In small machines there might be two bars per slot, thus: slots per pole $= t_c$. The winding is usually made of copper strap or bars with from four to eight slots per pole equally spaced.

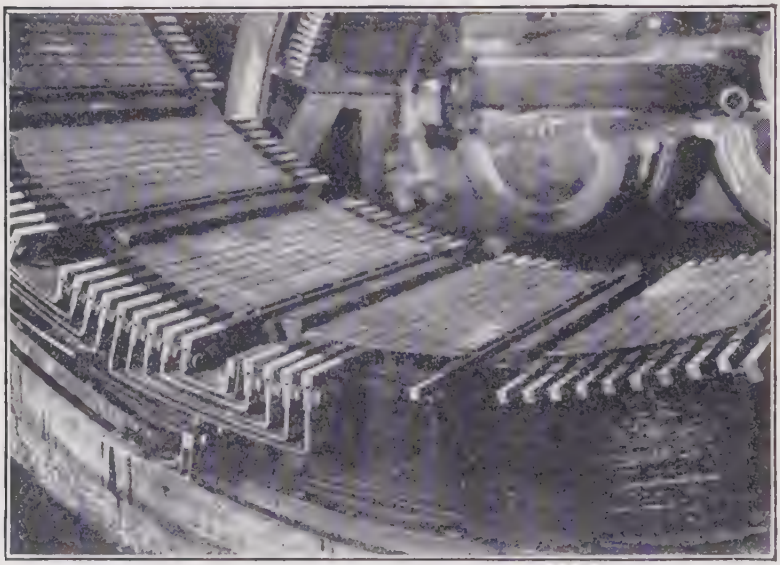


FIG. 60.—Compensating Windings of a d.c. machine being placed in the slots in the pole face.

When a compensating winding is used it is usually in a large special machine, and commutating poles are also employed. In this case the coils on the commutating poles need not have as great a number of turns as when no compensating winding is employed. The total m.m.f. per commutating pole may be calculated as in a preceding section and then reduced by an amount equal to the ampere-turns per pole of the compensating winding, as these two windings cooperate to neutralize the armature reaction.

The cross-section of the bars of the compensating winding is

based on a current density of from 2000 to 3000 amperes per square inch.

Predetermination of Performance from Design

The steps and calculations involved in predetermining the performance and characteristics of a given design are the following:

25. Coreloss.—The coreloss is made up of hysteresis and eddy-current losses. These losses are principally in the armature core and teeth, but if care is not taken in the design and construction there may be a considerable loss in the pole shoes. To avoid excessive losses in the pole shoes the armature should have as narrow slots or slot openings as possible, the pole faces should be **chamfered** (i.e., have a greater clearance between armature and pole at the tips than in the center), and the pole shoes should be of laminated steel rather than solid.

It is a simple matter to calculate the magnitude of these losses in the armature core proper as the flux density and frequency in this part are definitely known, but the loss in the teeth is due, not only to the main flux and the fundamental frequency, but also to the leakage flux and the pulsations due to the passage of the individual teeth past the pole tips. These pulsations also cause the loss in the pole pieces.

The coreloss is best calculated by resort to curves showing the watts lost per cubic inch in hysteresis and eddy currents respectively, at a frequency of one cycle per second for various magnetic densities as in Fig. 7. Let C_h be the value of this constant taken from the curve for hysteresis for the density used in the respective parts, and C_e be the value of the constant taken from the curve for eddy-current loss for the respective parts and densities and proper thickness of steel laminations (usually .014 in.); then:

$$\text{Total loss in core} = V_c C_h f + V_c C_e f^2 \text{ watts};$$
$$\text{Total loss in teeth} = V_t C_h f + V_t C_e f^2 \text{ watts};$$

where V_c and V_t are the volumes of the metal in cubic inches in the core and teeth respectively, and f is the fundamental frequency $= \frac{\text{poles} \times \text{r.p.m.}}{120}$.

In small generators and most motors it is customary to use standard sheet steel having a value of η of from .002 to .004 and

a value of ϵ for eddy-current loss of from .00015 to .00025. In large machines silicon steel is sometimes used. This has a value of η from .001 to .002 and a value of ϵ from .00005 to .0001. In all cases the thickness of sheets used in the armature is .014 in. or No. 29 gauge.

The calculation of the coreloss is one of the most unreliable and unsatisfactory steps in the design of any machine except a transformer, as the value calculated by theoretical principles has to be multiplied by a factor of from 1.25 to 3 (1.5 is most usual) to obtain the value that will be found by test of the finished machine, on account of the extra losses mentioned above. The longer the air gap the less is the magnitude of this effect and the lower the multiplying factor.

In generators and shunt motors the speed and the flux are fairly constant, so that the coreloss is fairly constant and independent of the load. Where greater accuracy is desired the coreloss is calculated for a density caused by a total flux which corresponds to a generated voltage of $E + IR$ for a generator and $E - IR$ for a motor where E is the terminal voltage and I and R are the current and resistance respectively in the armature circuit.

26. Friction and Windage.—These losses can be calculated for a particular line of machines by an empirical formula based upon the particular mechanical construction of that line, but no method can be stated which is applicable to all types of construction. The true friction (in the bearings) varies directly as the speed and is not difficult to calculate, but it is small compared with the more complex windage loss which varies as the square of the speed. The windage includes unavoidable air friction due to the rotation and also the power to force the ventilating air through the ducts of the machine, which is often made a considerable amount to keep the machine cool. Therefore the windage depends upon the individual preference of the designer. Direct-current machines have also the friction of the brushes on the commutator as given in Paragraph 15. Usual values for the total friction and windage losses (including brush friction) in direct-current machines are given in Table XVI on page 107. In generators and shunt motors this loss may be assumed as constant and independent of the load as the speed is constant.

Stray-power Loss.—This term is sometimes used to designate the sum of the coreloss, friction, and windage.

27. Excitation Loss or RI^2 in Shunt-field Circuit.—Since, in a self-excited machine, the loss in the field rheostat is charged against the machine, the total loss in the circuit is equal to the voltage across the circuit multiplied by the current in the field circuit. In most machines this voltage is the same as the armature voltage, but in compound-wound machines with long shunt connection it is slightly different. As the current in the field is adjusted arbitrarily with the rheostat to some desired value, it is customary to calculate the excitation loss for that value of field current required at rated load. The loss is usually constant (independent of the load) and only subject to the arbitrary variations due to hand regulation.

28. Armature Circuit RI^2 Loss.—In this category are included the loss in the copper of the armature, the series field, the interpole windings, and the commutator and brushes. Except for the loss in the brushes, these are easily calculated by taking the sum of the resistances (taken at 75° C. as per A. I. E. E. Standards) of the armature, series field, and interpole windings (if any), and multiplying the sum by the square of the armature current corresponding to the particular load for which the efficiency is being calculated. The loss in the brushes is determined by multiplying the current by the conventional value of the voltage drop in the brushes (2 or 3 volts as in Paragraph 15).

A reasonably accurate and more convenient method is to assign a value of resistance to the brush contact (by dividing the conventional 2 volts by the rated current), add this resistance to the other resistances, and use this sum as the equivalent resistance of the armature circuit, thus lumping the brush loss with the other losses.

29. Stray Load Losses.—This is the name given to certain miscellaneous losses that occur only when the machine is loaded. They are due mainly to the leakage flux and to the distortion of the main flux. They take the form of eddy-current losses in the conductors, in the pole faces, and in certain mechanical parts that are too close to the leakage flux. They usually follow a law of proportionality to the square of the armature current and hence may be represented by an **effective resistance.** The predetermination of these losses is impossible and it is very difficult to measure them in a completed machine. The A. I. E. E. states that for machines of 200 h.p. and greater, a stray-load loss of

1 per cent is a reasonable conventional value, but for machines of less than 200 h.p. no value has been assigned. Designing engineers frequently assume an effective resistance of the armature proper 20 per cent greater than the true resistance at 75° C.

30. Efficiency.—The efficiency is the ratio of the output in watts to the sum of the output and all the losses. It is predetermined by calculating the value of each loss separately for an assumed value of load output in a generator. In a motor it is better to assume inputs and calculate the separate losses corresponding to the assumed input. This is because in a generator the variable losses vary according to a known law with respect to the output, and in a motor to the input.

Let P represent an assumed output in watts, usually $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, full and $1\frac{1}{4}$ times rated output for a generator.

Let A = the coreloss;

B = the friction and windage;

C = the shunt-field circuit loss;

D = loss in windings of armature, series field and interpoles and the stray-load loss;

E = loss in brushes and contact;

then the per cent efficiency is $\frac{100P}{P + A + B + C + D + E}$.

In a constant-potential, constant-speed machine (generators and shunt motors) the efficiency is a maximum for that load at which the variable losses $(D + E)$ are equal to the sum of the constant losses $(A + B + C)$. In a generator we may find the current corresponding to the output at maximum efficiency by dividing the sum of the constant losses by the resistance of the armature circuit and then extracting the square root of the quotient.

Usual Efficiencies and Losses.—An idea of a reasonable value for the efficiency of d.c. machines of various sizes (generators and motors) and the usual distribution of the losses is given in Table XVI. It should be noted that the efficiency of a machine of any particular size may vary throughout a wide range depending upon the speed, weight, and type.

TABLE XVI

Efficiency and Losses at Rated Load, Per Cent

Rating, Kw.	Efficiency	Total Friction	Excitation	Coreloss	Total Arm. Circuit
1	80	6	6	4	4
5	84	5	4.2	3.2	3.6
10	86	4	3.6	3	3.4
20	88	3	3	2.8	3.2
50	90	2.6	2.2	2.2	3
100	91.5	2.2	2	1.7	2.6
200	92	2.2	1.8	1.6	2.4
500	93	2	1.6	1.4	2

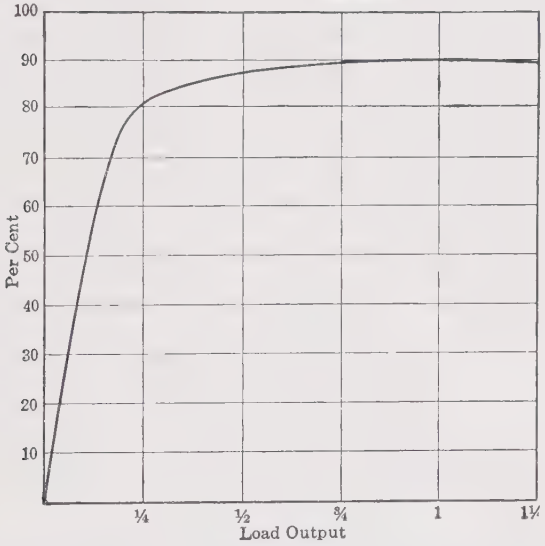


Fig. 61.—Load and Efficiency of a d.c. machine showing efficiency for various fractions of load or output up to 1.25 times rated load.

31. Regulation.—Regulation is not very important in a d.c. generator as compounding by means of a series field provides a means of obtaining any desired voltage at full load. Most generators are designed and constructed with a series field of such strength that they will give 15 per cent greater voltage at full load than at no load. If less compounding effect is desired, a certain portion

of the load current is shunted around the series field through strips of German silver connected in parallel with the series field and adjusted by trial.

The inherent regulation of a shunt generator may be predetermined as follows: Having determined the total m.m.f. required to give the proper voltage at the terminals and overcome the drop in the armature circuit ($E + IR$), and the demagnetizing effect of the armature as in Paragraph 22, *b* and *c*, we consult the no-load magnetization curve and note the voltage corresponding to this m.m.f. Let this be E_0. This is the value to which the terminal voltage will rise when the field is adjusted for the proper voltage, E, with full load on and the load is then removed without altering the field. Then the per cent regulation is:

$$\frac{E_0 - E}{E} \times 100.$$

32. Heating.—The rise in temperature of the field coils is found as in Paragraph 23. A rough estimate of the rise in temperature of the armature conductors was determined in Paragraph 14, and of the commutator in Paragraph 15.

The most rational and exact method of calculating the rise in temperature of the armature winding and core is that given by Arnold. With a correct choice of the empirical constants to suit the particular design, very reliable results are obtained.

The radiating surface of the armature core is taken as the sum of the outer cylindrical surface, the two annular surfaces at the ends, and one-half of the annular surfaces in the air ducts. Only one-half of the duct surfaces is taken because, on account of the narrowness of the ducts, these surfaces are only one-half as effective in dissipating heat as the external surfaces.

As a preliminary approximation it is assumed that the heat energy dissipated by this surface consists of all of the coreloss and that portion of the armature RI^2 which occurs in the portion of the conductors embedded in the armature core represented by the length L, Fig. 62, and that the RI^2 in the end connections is dissipated from the surface of the end windings.

The loss dissipated by the core is:

$$P_t = \text{coreloss} + R_a I^2 \left(\frac{2L}{mlt}\right),$$

where I = total armature current;
R_a = resistance of the armature;
L = total length of core between heads;
mlt = mean length of an armature turn.

The effective radiating surface of the armature core is:

$$A_t = \pi DL + \frac{\pi}{4}(D^2 - D^2_1)(2 + d),$$

where D = outside diameter of the armature, inches;
D_1 = inside diameter of the armature;
d = number of air ducts.

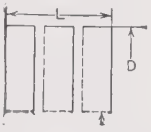


FIG. 62.— Cross-section of armature core showing ventilating ducts and cooling surface.

The rise in temperature of the surface of the core above the surrounding air (ambient temperature) is:

$$T_t = \frac{P_t}{A_t} t \text{ degrees } C,$$

where $t = 30$ for narrow machines or machines with high peripheral speeds (above 5000 ft. per minute) and $t = 40$ for long machines or those of lesser speeds. (See note on p. 111.)

A part of the heat due to the RI^2 flows from the embedded portion of the copper through the slot insulation to the iron of the core, if the temperature of the copper is higher than that of the iron (i.e., at overloads). Sometimes, however, at light loads for instance, the temperature of the iron is higher than that of the copper and the flow of the heat is in the reverse direction. This condition is taken care of by the following method.

Let Q_1 represent the rate of flow of heat energy (in watts) from copper to iron of the core through the insulation of the slots. Q_1 will be positive if it flows from copper to iron and negative if it flows from iron to copper, the direction of flow depending upon whether T_c (the temperature rise of the copper) is greater or less than T_t. The remainder of the heat energy in the copper is dissipated from the free portions of the windings through the insulation on the end connections to the surrounding air. This air is in relative motion due to the rotation of the armature and to its construction, which permits air to flow radially outward through the interstices of the windings. Let this heat power be Q_2.

If the total RI^2 in the armature winding be represented by P_2 we have $P_2 = Q_1 + Q_2$, or the total copper loss is equal to the sum of that going from copper to iron and that going out of the end connections. On the other hand, if heat is coming to the copper from the iron, then P_2 = the difference between that going out by the end connections and that coming in from the iron.

The permeance of the heat path from copper to iron through the insulation is:

$$M = \frac{U_s L s}{k_1 d_1},$$

where U_s = perimeter of the slot in inches = $2(u+w)$. Fig. 37;
L = length of core;
s = total number of slots;
d_1 = thickness of insulation between copper and iron;
k_1 = an empirical constant, being the difference in temperature per inch length of path for a flow of 1 watt per square inch section of path. See note.

The heat power transferred from copper to iron or *vice versa* is:

$$Q_1 = M(T_c - T_i).$$

(If T_i is greater than T_c this gives heat transferred from iron to copper.)

The permeance of the heat path from the copper through the insulation of the end connections to the surrounding air consists of two paths in series: first, from copper through the insulation to the surface of the coils (true conductance of heat); and, second, from the surface by radiation and convection to the air. These two reluctances are:

$$\frac{k_2 d_2}{U_c l_c Z} + \frac{k_3}{U_c l_c Z},$$

where k_2 = heat resistivity of materials of insulation on the end connections, same as k_1 if same material. See note;
d_2 = thickness of insulation on end connections, inches; usually less than d_1;
U_c = perimeter of a coil or group of coils bound together having one outside surface = $2(a+b)$. Fig. 63;
l_c = length of end connections of one-half of a coil;
= $(0.5mlt - L)$;

Z = total number of coil-sides or groups of coil-sides. Usually four coil-sides are bound together in the slot but are spread out on leaving the slot to form the end connections. Hence the necessity of using s and Z in the different equations.

k_3 = rise in temperature per watt per square inch of surface of coils;

$$= \frac{170}{1. + .00025V};$$

V = peripheral velocity of the surface of armature, feet per minute.

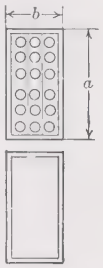


Fig. 63. — Cross-section of a coil showing the cooling perimeter.

The heat power transferred from the copper to the air is proportional to the permeance of this path and to the difference in temperature between the copper and the air, thus

$$Q_2 = NT_c,$$

where N = the permeance $= \dfrac{U_c l_c Z}{k_2 d_2 + k_3} = \dfrac{1}{\text{reluctance}}$.

The total RI^2 of armature $= P_2$ is to be dissipated through the two paths of permeances, M and N, in parallel, thus:

$$P_2 = Q_1 + Q_2 = M(T_c - T_i) + NT_c.$$

In this, T_c, the rise in temperature of the copper above the surrounding air, is the only unknown quantity. Solving for this, we have the practical relation:

$$T_c = \frac{P_2 + MT_i}{M + N} \text{ in degrees C.}$$

Discussion of the Constants.—t, the rise in temperature in degrees C. per watt per square inch of the surface of the armature core (iron) above the surrounding air, or the specific drop in thermal potential at the surface, is less for short armatures than for long armatures, that is, the ends are more efficient than the cylindrical surface. A high peripheral speed is more effective in cooling the armature and hence allows a lower value of t. t follows a law somewhat like that for k_3:

$$t = \frac{130}{1 + .0005V}.$$

k_1 and k_2 are constants for the drop in thermal potential for 1 watt per square inch through a one-inch thickness of the insulation on the coils. They have values as follows:

TABLE XVII

		Mean
Varnished cambric	160 to 300	230
Mica paper	250 to 350	300
Linen tape varnish	265 to 350	310
Micanite	350 to 500	425

The higher values apply in the case of coils that have been formed by hand and have not been thoroughly impregnated, thus leaving air-pockets in the material. When the coils have been impregnated in a vacuum and shaped in molds under pressure, the heat-conducting qualities are better and the lower values apply.

Values for the thermal conductivity and resistivity of various insulating materials will be found in the following references:

Atkinson and Fisher, A. I. E. E., Vol. 32, 1913, p. 325.
Langmuir, I. and Dushman, A. I. E. E. Vol. 32, 1913, p. 303.
Smithsonian Inst. Tables
Taylor, A. S. M. E. Vol. 41, 1919, p. 605.
Walker, M. Br. I. E. E. Vol. 48, 1912, p. 674.
Knowlton, E., G. E. Review, May and June, 1916.

EXAMPLE

Item.

1–2. A 100-kw. 240-v., d.c. machine for 600 r.p.m. $I = 420$ amp. at rated load.

3. Assume 6 poles from Table V.

4. Assume $\rho = .6 \quad \sigma = 600. \quad B = 45{,}000.$ Table VI.

$$\text{Then } D^2L = \frac{610 \times 10^9 \times 100}{.6 \times 600 \times 45{,}000 \times 600} = 6300.$$

For $v = 5000 \qquad D = 32$ ins.
$v = 4000 \qquad D = 26$ ins.
$v = 3740 \qquad D = 24$ ins.
choose $D = 24$ ins. for round numbers

$$L = \frac{6300}{24 \times 24} = 10.9 \text{ say } 11 \text{ ins.}$$

5. Choose air gap = 0.15 in.

6. From Table VII choose about 108 slots of depth 1.5.

7. $\phi = \dfrac{\pi \times .6 \times 11 \times 24 \times 45{,}000}{6} = 3.74 \times 10^6.$

8. $f = \dfrac{6 \times 600}{120} = 30. \qquad S = \dfrac{240 \times 10^8}{4 \times 30 \times 3.74 \times 10^6} = 54.$

9. $n = \dfrac{54 \times 420}{6} = 3780.$

10–11. Total inductors: $Z = 54 \times 2 \times 6 = 648$ Multiple winding.

$$c = \frac{\pi \times 600 \times 6 \times 24}{420 \times 108} = 6 \quad \text{and} \quad c = \frac{2 \times 6 \times 54}{108} = 6.$$

12. $q = \dfrac{420}{2000 \times 6} = .035$ sq. in. or 45,000 c.m.

This is too large for a round wire. Choose six bars per slot.

13. Pitch of slots at face $= \dfrac{\pi \times 24}{108} = 0.70$ in.

Make slot .35 and tooth .35.

From Table IX insulation on two sides should be 0.17, leaving for copper .35 − .17 = .18 or three bars wide each of .06.

Depth of each bar $= \dfrac{q}{.06} = \dfrac{.035}{.06} = 0.6$ in.

From Table IX slot depth should be $0.6 \times 2 + .35 = 1.55$.
Each slot $1.55 \times .35$ with six bars $(.6 \times .06)$.

14. Mean length of a turn, m.l.t. $= 2 \times (11 + .5) + \dfrac{10 \times 24}{6} = 63$ ins.

Resistance of armature $= \dfrac{.01 \times 63 \times 54}{12{,}000 \times 6 \times .036} = .0125.$

Voltage drop $= 420 \times .0125 = 5.25$ or 2.2 per cent.
$I^2R_a = 420^2 \times .0125 = 2200$ watts.

Rough temperature of copper $= \dfrac{100 \times 2200}{(.35 + .65) \times 63 \times 108} = 34°$ C.

15. Design of commutator:
Choose diameter less than 24 ins., say 18 ins. for round number.

$$v_c = \frac{\pi \times 18 \times 600}{12} = 2820 \text{ ft. per minute.}$$

Since there are 108 slots choose 108 coils of three turns in series each and 108 segments. Pitch of segments $= \dfrac{\pi \times 18}{108} = .522.$

Average volts per bar $= \dfrac{240 \times 6}{108} = 13.3.$ Contact surface of positive brushes $= \dfrac{420}{40} = 10.5$ sq. ins. Brush width to be about 1.5 segments or 0.75 in. Length of brushes per stud $= \dfrac{2 \times 10.5}{.75 \times 6} = 4.7$ ins., say 5

ins. Total length of commutator will be 6 ins. Commutator surface per ampere $= \dfrac{\pi \times 18 \times 6}{420} = 0.80$. Use 4 brushes per stud each $1.25 \times .75$. Assume 2 volts drop in all in brushes, then the watts loss $= 2 \times 420 = 840$. At 8 watts loss per square inch at 1000 ft. per minute, the brush friction would be $= 8 \times 1.25 \times .75 \times 24 \times \dfrac{2820}{1000} = 505$. Rise in temperature of commutator $= 14 \times \dfrac{840 + 505}{\pi \times 18 \times 6} = 55°$.

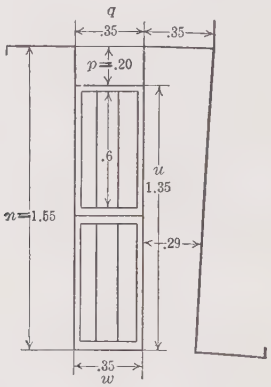


FIG. 64.—Dimensions of armature slot and tooth for machine calculated as example. For magnetic and reactance calculations.

16. Reactance voltage:

See Figure 64.

We find from sketch of slot,

$n = 1.55$ $\quad l = 11$ $\quad t = 3.$

$u = 1.35$ $\quad m = \dfrac{\pi \times 24 \times .4}{6} = 5.05$ $\quad I_c = 420 \div 6 = 70.$

$w = .35$ $\quad U_c = 2(.65 + .35) = 2.$

$p = .20$ $\quad l_c = \dfrac{63}{2} - 11 = 20.5.$

$q = w = .35.$

$$\phi_s = 3.2 \times 11 \times \left(\frac{1.35}{3 \times .35} + \frac{.20}{.35}\right) = 66$$

$$\phi_t = 2.35 \times 11 \times \log \frac{\pi \times 5.05}{2 \times .35} = 35$$

$$\phi_c = .58 \times 20.5 \log \frac{2 \times 20.5}{2} = 15$$

$$116$$

$2(\phi_s + \phi_t + \phi_c) = 232.$

$$c = 3 \times \frac{.75 \times 2}{.522} = 8.6$$

$$L = 232 \times 8.6 \times 3 \times 10^{-8} = .000059 \text{ henry}$$

$$f_c = \frac{2820 \times 12}{120 \times .75} = 376$$

$e_m = 2\pi \times 376 \times .000059 \times 70 = 9.7$ volts.
Commutating poles will be necessary.

17. Magnetic circuit:

$$\phi = \frac{240 \times 6 \times 10^8 \times 60}{6 \times 648 \times 600} = 3.72 \times 10^6.$$

Leakage coefficient = 1.15.
Gross flux = $3.72 \times 10^6 \times 1.15 = 4.28 \times 10^6$.

18. Yoke: Flux in each part = 2.14×10^6.
Assume B = 80,000 in cast steel, Table XII.
Area = 26.8 sq. in.
Assume e slightly greater than L, say 13.5 then $d = 2$ and area = 27. (See Fig. 42.)

Pole core: Flux = 4.28×10^6. B = 90,000. Area = 48.
Assume f slightly less than L, say 10.5 then $b = 4.5$
Stacking factor = .95

Air gap or pole face: $a = \frac{\pi \times 24 \times .6}{6} = 7.55.$

$L_p = f = 10.5$ laminated pole piece.

Armature teeth: pitch at face = .70.
tooth at face = .70 − .35 = .35.
diameter at bottom = $24 - 2 \times 1.55 = 20.9 = D_2$

$$\text{pitch at bottom} = \frac{\pi \times 20.9}{108} = .61.$$

tooth at bottom = .61 − .35 = .26.

$$t_o = \frac{.35 + 2 \times .26}{3} = .29.$$

$$T = \frac{7.55 + 2. \times .15}{.70} = 11.2$$

effective length of iron with 3 ducts of $\frac{1}{2}$ in.
$l_i = [11 - 3 \times .5] \times .9 = 8.55.$

Armature core: Assume B = 70,000, flux = 1.86×10^6.
Area = 26.6. $l_i = 8.55$; then $h = 3.1$.
Inside diameter,
Armature = $24 - 2 \times 1.55 - 2 \times 3.1 = 14.7$.
Call it 15 ins. = D_1, and $h = 2.95$ final.

Assume $c = 5$ ins., then outer diameter yoke equals
$24 + 2 \times 5 + 2 \times 2 = 38$.

19.

1	2	3	4	5	6	7	8
Part	ϕ			$B \times 10^{-3}$	H	Length	Amt. t
Field yoke...	2.14×10^6	$2 \times 13.5 = 27$	CS	79	30	10	300
Pole core....	4.28×10^6	$10.5 \times 4.5 \times .95 = 45$	LS	95	45	5	225
Air gap.....	3.72×10^6	$7.55 \times 10.5 = 79.2$	A	47	14,700	.15	2200
Arm. teeth...	3.72×10^6	$11.2 \times .29 \times 8.55 = 27.8$	LS	133	740	1.55	1150
Arm. core...	1.86×10^6	$2.95 \times 8.55 = 25.3$	LS	73	15	5.5	82
Grand total..	.....	.....	.....	.....	.....	.....	3957

Length of path in yoke $= \dfrac{\pi \times 38}{6 \times 2} = 10.$

Length of path in pole core, assumed at 5 ins., about equal to $b = 4.5$.

Length of path in armature core $= \dfrac{\pi \times 20.9}{6 \times 2} = 5.5.$

The density in the teeth is rather high. This might be corrected by using a little higher copper density which would give a smaller bar and smaller slot, or the length of armature, L, might be increased. Since the I^2R loss in armature is already rather high, the latter alternative would be better in this case.

20. Stability factor $= \dfrac{(2200 + 1150)}{3780 \times .6} = 1.48$ satisfactory.

21. Final length of air gap, 0.15, satisfactory.

22. Armature interference, method (c). Since commutating poles are to be used, the brushes will always be on neutral, there will be no back ampere-turns, and the cross ampere-turns at rated load will be: $n = 3780$. Ampere-turns, gap and teeth = 3350. K, from Fig. 49, would be 0.27. $B' = .27 \times 3780 = 1020$. Ampere-turns required at full load, $F = 3960 + 1020 = 4980$.

23. Shunt field: designed for about 4000 ampere-turns per pole. Minimum volts per pole available $= \dfrac{240}{6} \times .8 = 32$. Minimum distance between poles $= \dfrac{\pi \times 24.3}{6} - 4.5 = 8.2$ ins., ample space. Rough cross-section of shunt coil $= 2 \times \dfrac{4000}{1000} = 8$ sq. ins. Assume section is 4×2 hence $c_2 = 4$ and $b_2 = 2$. Rough mean length turn equals $2(10.5 + 4.5 + 2 \times 2) \div 12 = 3.16$ ft.

$$q_1 = \frac{12.7 \times 3.16 \times 4000}{32} = 5000 \text{ c.m.}$$

The nearest standard size is No. 12 with 6529 c.m., diameter d.c.c. = .091 in.

Rough calculations show that a 2 per cent loss in fields would cause excessive heating, so assume a loss of 1 per cent, 1000 watts or 167 watts per spool.

$$i_1 = \frac{167}{32} = 5.2 \qquad t_1 = \frac{4000}{5.2} = 770.$$

Turns per layer $= \frac{4}{.091} = 44$. Layers $= \frac{770}{44} = 18$. Then actual $b_2 = .091 \times 18 = 1.64$ ins. $A_1 = 2 \times 4 \times (10.5 + 4.5 + 4 \times 1.64)$ $= 172$ sq. ins. Rise in temperature $= 50 \times \frac{167}{172} = 48°$ C.

$l_1 = 2(10.5 + 4.5 + 2 \times 1.64) \div 12 = 3.05$ ft. Resistance of 6 spools in series $= \frac{12.7 \times 770 \times 3.05 \times 6}{6529} = 27.6$ ohms.

Series field:

From No. 22, ampere-turns required at full load $= F = 4980$
Ampere-turns at no load 3960

Series field $a - t$ $= 1020$

$$t_2 = \frac{1020}{.75 \times 420} = 3.25 \text{ say } 3.5$$

$$\text{cross-section} = \frac{420 \times .75}{1000} = 0.315 \text{ sq. in.}$$

$$\text{Thickness of strap} = \frac{1.64}{4} - .016 = .39 \text{ in.}$$

Width of strap $= .315 \div .39 = 0.8$ in.

Use 3.5 turns of thin strips 0.8 in. wide to make up a pliable strap about 0.4 in. thick.

Then $c = c_1 + c_2 = .8 + 4. = 4.8$ ins. or 5 ins. with collars.

$$r_2 = \frac{.01 \times 3.5 \times 3.05 \times 6}{.32 \times 1000} = .002.$$

24. Commutating poles.

$e_m = 9.7$ volts. $\qquad V_a = \frac{\pi \times 24 \times 600}{60} = 750.$

$$L_1 = \frac{9.7 \times 10^8}{6 \times 30{,}000 \times 750} = 7.2 \text{ ins., say } 7.5.$$

$a_1 = 1.05$ Gap under pole $= .15 \times 2 = .30$.

$\phi_1 = 30{,}000 \times 1.05 \times 7.5 = 237{,}000.$

Ampere-turns for gap $= 30{,}000 \times .313 \times .3 = 2800$

$$\text{Density in pole} = \frac{237{,}000 \times 1.6}{1.05 \times 7.5 \times .9} = 53{,}000$$

Ampere-turns for pole core 8×4 = 32

A'' = 2832

B'' = 3780

Ampere-turns per commutating pole = 6612

Turns per pole = $6612 \div 420 = 15.7$, say 16.

Conductor = $\frac{420}{1000} = .42$.

Conductor thickness = $4 \div 16 = .25$.

Conductor width $.42 \div .25 = 1.66$.

Say copper strap $1.5 \times .25$ wound on edge.

m.l.t. = $2(7.5 + 1.05 + 2 \times 1.5) \div 12 = 1.94$ ft.

resistance = $\frac{.01 \times 16 \times 1.94 \times 6}{.375 \times 1000} = .005$.

25. Coreloss. $D = 24$ $D_1 = 15$ $D_2 = 20.9$

Vol. of core = $\frac{\pi}{4}(D_2^2 - D_1^2) \times 8.55 = 1410$.

Vol. of teeth = $1.55 \times \frac{.35 + .26}{2} \times 8.55 \times 108 = 440$.

Core hysteresis = $1410 \times .015 \times 30 = 630$.

Eddy = $1410 \times .000055 \times 30^2$ = 70

700

Teeth hysteresis = $440 \times .039 \times 30$ = 520

Teeth eddy = $440 \times .00018 \times 900$ = 70

590

Total coreloss = $700 + 590 = 1290$.

26. Friction and windage:

Brush friction from No. 15 = 505 watts. Assume total friction as 2.2 per cent or 2200 watts.

27. Excitation = $240 \times 5.2 = 1250$ watts.

28. Armature circuit:

Resistance of armature proper	.0125
Resistance of brushes = $2 \div 420$	.0048
Resistance of series field	.0020
Resistance of commutating field	.0050
Resistance of armature circuit	.0243

29. Stray-load losses; ignore.

30. Efficiency.

30.

Load	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$
I^2-load	210	315	420	525
I^2-armature	215	320	425	530
Coreloss	1290	1290	1290	1290
Friction	2,200	2,200	2,200	2,200
Excitation	1,250	1,250	1,250	1,250
Armature circ.	1,120	2,470	4,470	6,750
Losses	5,860	7,210	9,210	11,490
Output	50,000	75,000	100,000	125,000
Input	55,860	82,210	109,210	136,490
$\frac{\text{Losses}}{\text{Input}}$	.105	.088	.084	0.084
Efficiency	.895	.912	.916	.916

32. Heating:

$$P_i = 1290 + \overline{425^2} \times .0125 \times \frac{22}{63} = 2080$$

$$A_i = 830 + 275 \times 5 = 2200$$

$$t = \frac{130}{1 + .0005 \times 3760} = 45$$

$$T_i = \frac{2080}{2200} \times 45 = 43°$$

$$U_s = 2(1.35 + .35) = 3.4 \qquad s = 108 \qquad L = 11 \qquad d_1 = .07$$

$$k_1 = 250 \quad M = \frac{3.4 \times 11 \times 108}{250 \times .07} = 232.$$

$$U_c = 2(.75 + .35) = 2.2 \quad Z = 216 \quad d_2 = .05 \quad k_2 = 250$$

(see Fig. 64)

$$l_c = \frac{63}{2} - 11 = 20.5 \qquad k_3 = \frac{170}{1 + .00025 \times 3760} = 88$$

$$N = \frac{2.2 \times 20.5 \times 216}{250 \times .05 + 88} = 98.$$

$$P_2 = 2250 \qquad T_i = 43$$

$$T_c = \frac{2250 + 232 \times 43}{330} = 37° \text{ C.}$$

This machine could be improved by:

(*a*) Reducing reactance voltage by using a lesser value of σ involving a greater length.

(*b*) Reducing density in teeth by using smaller bars and a greater length of L.

(*c*) Saving copper by allowing higher temperature resulting from smaller bars and higher copper density.

Examples of Design of Continuous-current Machines

The following tabulation gives data on several continuous-current machines. The first part gives the dimensions of the various parts, and the second gives the calculated electrical and magnetic characteristics of the machines as discussed in the body of the text.

Column 1. An ordinary compound-wound generator to be driven by an induction motor. It has two commutating poles.

Column 2. An ordinary compound-wound generator to be driven by a waterwheel and used as an exciter. No commutating poles.

Column 3. A high-speed compound-wound generator with commutating poles, forming part of a motor-generator set.

Column 4. A high-speed compound-wound generator for railway work, having commutating poles and compensating windings in the faces of the main poles. The latter consist of eight slots in each pole face, each slot 1.5 in. deep by .55 in. wide with an opening of .0625 in. Each slot contains one bar ($1.3 \times .5$) making four turns per pole or 5216 compensating ampere-turns per pole to overcome the cross ampere-turns of the armature.

Column 5. A compound-wound motor of 15 h.p., having commutating poles.

Column 6. An adjustable-speed shunt-motor with commutating poles, having a speed and rating range from 17.5 h.p. at 750 r.p.m. to 21 h.p. at 1800 r.p.m. The change in speed is accomplished by adjusting the shunt-field current.

Column 7. A series motor for industrial purposes.

Column 8. A series motor for industrial or railway purposes.

TABLE XVIII

	1	2	3	4	5	6	7	8
	Generator	Generator	Generator	Generator	Motor	Motor	Motor	Motor
Poles	4	6	6	6	4	4	4	4
Rating	5 kw.	100 kw.	500	750	15 h.p.	17.5	37.5	75
R.p.m.	1180	750	900	720	1200	750	535	500
Volts	125	125	275	575	115	240	220	220
Sh., *Se* or *CW*	*CW*	*CW*	*CW*	*CW*	*CW*	*Sh*	*Se*	*Se*
Armature diameter, face	7.75	24	31.5	36	10.5	12.5	14	16
Armature diameter, back	3	14.5	19.6	22	4	4.75	6	6
Armature, total length of core	5	10	13.87	17.125	4.5	5.5	9.25	9.25
Armature, air ducts	0	0	5 $\frac{3}{8}$	7-$\frac{3}{8}$	1-.5	1-.5	1-.625	2 $\frac{1}{2}$
Slots, number	33	81	108	108	45	35	31	45
Slots, dimensions	1.25×.28	1.16×.47	1.12×.36	1.35×.45	$1\frac{1}{4}\times\frac{3}{8}$	1.4×.37	$1\frac{1}{4}\times\frac{5}{8}$	$1\frac{1}{8}\times\frac{1}{2}$
Conductors per slot	18	4	2	4	16	16	10	6
Conductors, size	(.051×.161)	(.15×.45)	(.36×.4)	(.15×.50)	No. 8	(.055×.25)	($\frac{3}{8}\times\frac{1}{2}$)	($\frac{1}{8}\times\frac{3}{8}$)
Series turns per path	148	27	18	36	89	139	77	67
Winding type	*SD*	*MD*	*MD*	*MD*	*SD*	*SD*	*SD*	*SD*
Air gap min.	.125	.25	.188	.25	.188	.135	.25	.125
Pole arc	4	7.5	11.5	13	6	5.85	7.625	8.75
Pole length	4.75	9.25	13.5	16.5	4.25	4.75	9	9.875
Pole core width	2	5.25	8.5	10	4	4	5.25	5.75
Pole core radial length	3	6	10	10	4	4.5	2.625	4
Yoke dimensions	6×.75	2×11	4×15	4×20	1.5×8	2×10	1.25×9	2×18
Shunt spool turns	960	670	900	1250	1106	1500		
Shunt conductor size	No. 21	No. 10	No. 10	No. 13	No. 16	No. 18		
Series spool turns	8.5	4 5	15	5.5	8.5	.....	38	18
Series conductor size	No. 8	$2\frac{1}{4}\times\frac{1}{4}$	2.5×.75	3.25×.4	$.\overline{229}^2$	.....	1.25×.081	3.66×.064
Commutator *d* and *l*	5.7×3	18×13$\frac{1}{4}$	20.6×13	25×15	6.5×3	8.5×3.5	11.2×5.7	13×6.63
Commutator segments	99	162	108	216	89	139	155	135
Studs and brushes	4×2	6×6	6×10	6×9	4×2	4×2	2×3	2×4
Brush dimensions	1×.375	$1\frac{1}{4}\times\frac{7}{8}$	1.25×1.25	1.25×.75	1×.625	1.25×.5	1.5×1	1.25×1

Commutating pole length	4.75	.....	13	15.5	4.25	4	.....	10
Commutating pole arc	1.25	.....	2	1.5	1.19	1.5	.....	1.5
Commutating, turns per	53	.....	4.5	4.5	43	45	.....	30
Commutating, conductor size	No. 5	.....	1.25×.75	1.125^2	No. 00	No. 4	.....	3.66×.064
Commutating, number poles	2	.....	6	6	2	4	.....	4
Commutating air gap	.125	.....	.375	.312	.188	.188	.....	.188
Rating	5	100	500	750	15	17.5	37.5	....
Volts	125	125	275	575	115	240	220	....
Amperes	40	800	1830	1304	113	64	140	275
ρ Pole arc per cent	.66	.61	.7	.69	.7	.59	.69	.70
V Armature, feet per minute	2400	4750	7400	6800	3300	2460	1960	2100
σ Ampere conductors	490	570	670	830	605	455	495	730
B Kilo-max. gap	32	45	57	52	32	65	58	56
ϕ Mega-max. pole	0.54	3.1	8.7	11.1	0.81	1.61	4	4.9
n Ampere-turns pole	1485	3600	5500	7850	2590	2220	2700	4600
v Leakage coefficient	1.25	1.11	1.18	1.15	1.25	1.3	1.2	1.2
Excit. no load, ampere-turns	1600	3800	6700	6000	2100	3700	6700	4800
Stability factor	1.28	1.65	1.6	1.02	1.15	2	3	1.18
Excit. full load ampere-turns	1800	4700	9100	6900				
Volts per bar	4.7	4.6	15.2	15.9	5.3	6.9	5.7	6.5
Reactance volts	4.7	2.85	7.4	10.5	4.4	3.9	1.6	3.2
Brush friction	35	409	3800	1900	80	111	115	136
Other friction	125	1040	5400	13,900	385	330	485	....
Coreloss	80	2300	8300	7800	215	361	600	500
Armature resistance 75° C.	.233	.0035	.0018	.0064	.051	.146	.075	.036
Brush resistance	.043	.0021	.0010	.0013	.016	.028	.011	.0064
Series field resistance	.043	.0015	.0002	.0013	.015	....	.041	.01
Commutating field resistance	.047	.....	.0004	.0025	.009	.061	.....	.016
Shunt field resistance	79	15.4	47.5	93	35.6	76	....	
Efficiency at rating	82	92	94	93.4	84	85	88	90
Slot factor	.42	.50	.52	.49	.45	.585	.31	.50
Current density armature	2440	2000	2930	2700	2160	2346	2900	2900
Current density, sh. f.	2540	700	920	1170	1000	2360		
Rise in armature winding, ° C.	32	47	52	44	49	36		
B under commutating pole	32	.....	20	25	39	24	.....	23

CHAPTER III

THE SALIENT-POLE ALTERNATING-CURRENT GENERATOR

Classification.—Alternating-current generators may be classified according to several different distinguishing characteristics. The following classification considers these characteristics in the order of their prominence.

Synchronous Generators.—In the synchronous type the action of inducing the electromotive force results from the relative motion of the armature conductors and a constant magnetic field produced by exciting coils in which a continuous or direct current flows. The frequency of the alternating electromotive force depends directly upon the number of field poles and the angular velocity of the revolving part.

Induction Generators.—In the induction type the magnetic field is of the rotating polyphase type and is produced by polyphase alternating currents flowing in the same windings with the load current. The mechanical construction is usually that of an induction motor with a short-circuited polyphase winding on the revolving member. The frequency depends upon the characteristics of the external circuit. In practice the frequency is determined, or **set,** by a synchronous machine, either generator or motor, in the external circuit. This **frequency setter** is necessary to supply the exciting current of the induction generator; that is, by means of the frequency setter the power factor of the total load is adjusted to equal the inherent power factor of the generator.

Advantages and Disadvantages of Induction Type.—A minority of the units of a station may be of the induction type with advantage, as they will cause less disastrous effects in case of a short circuit on the system. Their instantaneous short-circuit current is less. A certain number of the units must be of the synchronous type to set the frequency and supply the exciting current for the induction machines. Consequently, if there is a large proportion

of induction machines, the synchronous machines operate at a poor power factor unless there is much capacity effect in the load system, such as synchronous motors or line capacity.

Revolving-field and Revolving-armature Types.—In the revolving-field or revolving-armature type there are two, or some multiple of two, poles, each pole having its own coil for the direct-current excitation, and the flux in each pole is to all intents and purposes constant in value. The relative movement of the poles and armature conductors causes the variation in flux interlinkages both in direction and intensity.

The early machines were constructed with an armature revolving inside of a stationary field. As sizes and voltages increased it was found that a more effective use of the material could be obtained by making the armature the external member, and that the insulation of the high-voltage member was better preserved if that member was kept stationary.

At the present time all generators of large capacity (500 kilowatts and greater) and for high voltages (600 volts and up) are made of the revolving-field type.

Inductor type.—In the inductor type the direct-current excitation is concentrated in one (usually stationary) coil and the variation in magnetic flux is obtained by revolving a spider with bare projecting poles which alters the reluctance of the magnetic path. Thus the flux threading any particular armature coil is always in the same direction, but varies in intensity or quantity.

Salient-pole Type.—The majority of alternating-current generators are built with a revolving field consisting of a circular rim, to which are attached definite pole pieces projecting outward, each pole piece having its own exciting winding in the form of a spool surrounding the body of the pole piece. The revolving field is surrounded peripherally by the stationary armature, or **stator,** which carries the armature winding in slots.

This type is illustrated in Figs. 65 and 66 and is used in all slow- and moderate-speed machines to be driven by waterwheels, by belting, or by direct connection to reciprocating engines.

Turbo-generator or Round-rotor Type.—(Fig. 67.) This type is intended to be direct-connected to steam turbines operating at angular velocities of from 1200 to 3600 r.p.m. and at peripheral velocities of from 15,000 to 27,000 ft. per minute. On account of the great centrifugal forces due to these high peripheral speeds,

special forms of construction must be used in the rotating member. The body of the revolving field is made either of a solid steel forging turned to a cylindrical shape or of a cylinder built up of thick disks of steel held together by through-bolts. Slots are milled in the periphery of this cylinder and contain the field winding, consisting of many form-wound coils held in the slots by strong wedges.

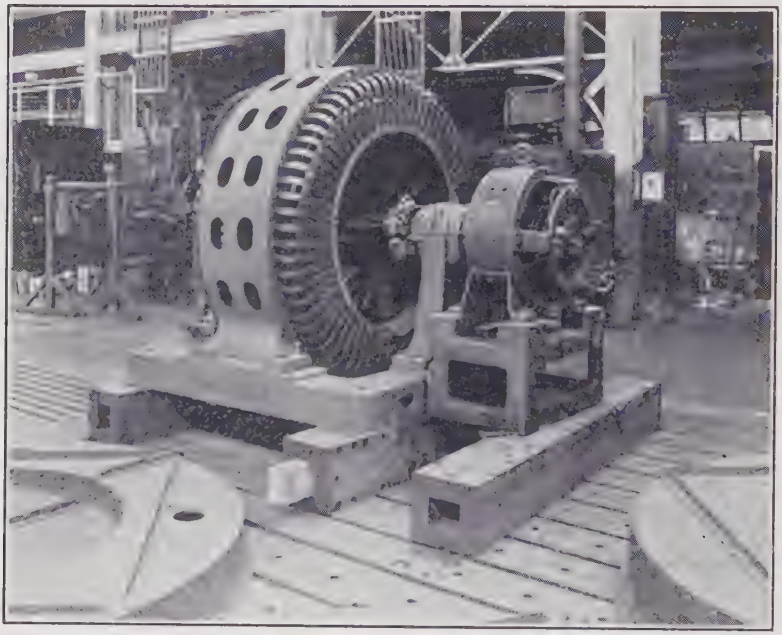


FIG. 65.—Salient pole alternating current generator with revolving field and exciter on the same shaft.

Single Phase, Two Phase or Quarter Phase and Three Phase.—The single-phase generator is usually about 30 per cent heavier and more costly than a polyphase generator of the same rating. By changing the internal armature connections, a polyphase machine may frequently be reconnected to be a three-, two- or single-phase machine.

As transmission by three-phase currents is more economical of copper than by two-phase or single-phase currents, all power-transmission lines are three-phase. Consequently, unless there is

some local condition requiring single- or two-phase, the three-phase generator is preferable.

Two-phase and three-phase generators of the same capacity and voltage strain are of practically the same dimensions, weight, and cost.

Separately Excited and Self-excited Types.—For years attempts were made to develop a satisfactory and simple self-excited alternator. Some of the machines designed were successful,

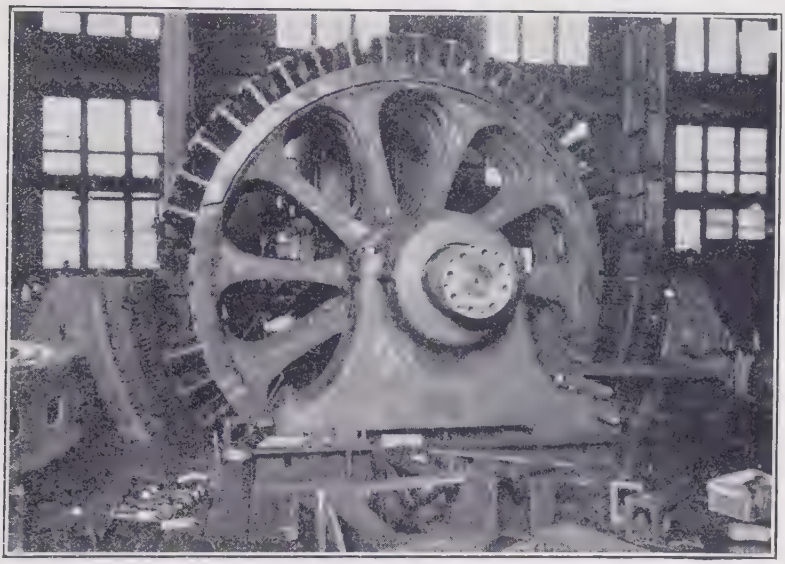


FIG. 66.—Revolving field structure showing pole pieces.

but not simple. The object was to obtain a constant voltage on the load by means of automatic self-excitation. However, with the perfection of the automatic voltage regulator the need for automatic self-regulation ceased, and there is now very little demand for self-excited alternators.

The separately excited alternator must be supplied with direct current at 125 or 250 volts, from a separate machine called an **exciter.** There may be one exciter for several alternators or an individual exciter for each alternator, and the individual exciter may be on the same shaft with the prime mover and alternator.

Methods of Rating.—All a.c. generators are rated in kilovolt-amperes (kv.-a.) and, unless otherwise specifically stated, are

rated at that kv-a. which they will give continuously with a rise in temperature not exceeding certain values depending upon the character of the insulating materials used (see "Standardization Rules," A. I. E. E.).

Machines, or parts of machines, insulated with mica, asbestos, or similar heat-resisting material are allowed a rise in temperature of 70° C. if measured by thermometer, or 75° if measured by increase of resistance. This includes most turbo-alternators,

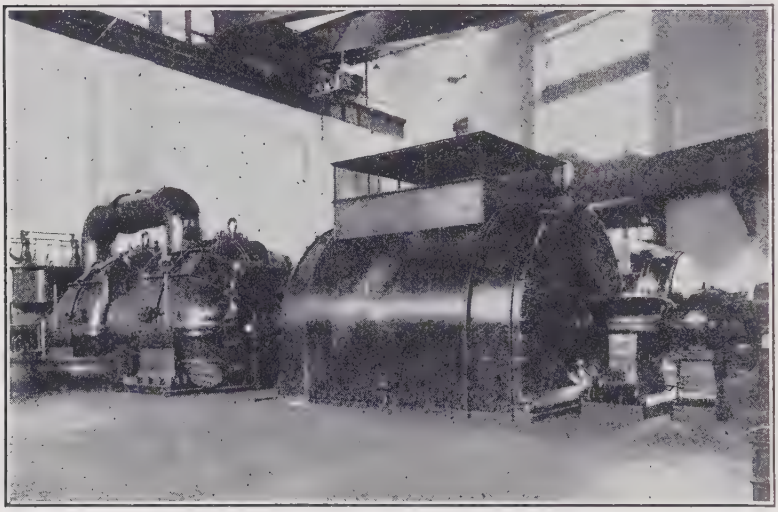


Fig. 67.—Turbo alternator (in foreground) driven by a steam turbine.

both armature and field, and the fields of many other alternators.

Machines, or parts of machines, insulated with varnished cambric or impregnated cotton or linen, are allowed a rise of 50° C. if measured by thermometer, or 55° if measured by resistance. This includes the armatures of most slow- and moderate-speed machines.

Most machines have their fields designed sufficiently liberally to enable them to give their rated kv-a. at 80 per cent power factor with rated voltage, and this is stated in the specifications. It should be noted that the heating of the *armature* depends upon the kv-a. and not upon the kilowatt load. The lower the power factor of the inductive load connected to the alternator, the greater the heating of the *field coils* for the same kv-a. output.

Voltage.—Alternators are now built to generate voltages up to 13,000 and 15,000 volts between lines, either single phase or polyphase. Above that voltage the extra cost of the insulation and the danger of damage from discharges cause it to be less expensive to install transformers with a lower voltage alternator. Some engineers consider the limit of economical voltage of generators to be even lower than 13,000 volts.

Standard voltages for alternating-current generators are as follows:

2200 volts, used in small installations where the power is distributed at 2200 volts, namely for lighting, or in larger stations where practically all the power is transmitted at higher voltages by means of transformers stepping up from 2200 volts to a transmission voltage such as 110,000.

6600 and 13,200 volts, for stations in large cities where most of the transmission is by means of underground cables.

11,000, for certain special cases, such as for single-phase alternating-current railroad operation.

Frequency. –The frequency depends upon the speed of rotation and the number of poles. If the speed of rotation of the revolving part is given in revolutions per minute the frequency is:

$$f = \frac{\text{r.p.m.}}{60} \times \frac{\text{number of poles}}{2}.$$

In the early alternators it was found much more economical to run at high speeds, and thus high frequencies, such as 133 and 125 cycles per second, were customary; but, as systems increased in size and complexity, electrical difficulties arose as a result of these high frequencies and the tendency has been to reduce the frequency till now we have 60, 50, 40 and 25 cycles per second as usual frequencies, of which 25 and 60 are standard in this country. In Europe, 50 cycles is used instead of 60. Of late there has been an attempt to have 15 or 16 cycles standardized for railway work.

A frequency of 25 cycles is preferable where there is a very long transmission line, on account of the lower inductive voltage drop; also where there is much synchronous machinery, such as synchronous motors and rotary converters, as these are more stable and better adapted to parallel operation at low frequencies.

A higher frequency (60 cycles in the United States and 50 in Europe) is preferable for electric lighting, as the light is steadier.

The higher frequency is also preferable where many transformers are used, as these are cheaper and more efficient at the higher frequencies.

Because of the higher speed for a given number of poles a 60-cycle generator of a given rating is usually smaller and lighter than a 25-cycle generator of the same rating.

On account of the definite frequencies and the fact that there must be an even number of poles, there is a limited number of combinations of poles and speed which may be used. Therefore the prime mover must be designed so that its normal speed corresponds to a definite relation between poles and frequencies, namely

$$\text{r.p.m.} = \frac{\text{cycles} \times 120}{\text{number of poles}}.$$

Phase and Line Voltages and Currents.—For moderate voltages, up to 3000 between lines, the method of connection is decided by such considerations as the giving of a convenient number for the conductors per slot for the required voltage; but for higher voltages the Y connection is usually chosen as it causes less strain on the machine insulation for a given voltage between lines.

Single-phase.—In a single-phase generator the voltage per phase is the same as the voltage between lines, and the current per phase is the same as the current per line, the product of voltage and current giving the volt-ampere rating of the machine.

Two-phase or Quarter-phase.—In a properly designed machine of this type, each phase supplies half the rating, thus the voltage and current per phase in the machine are respectively the same as the voltage and current per phase on the line. The current is equal to $1000 \times$ (kv-a.) $\div 2 \times$ volts per phase.

Three-phase.—Machines of this type may be connected either Y or Δ. In a Y-connected machine the current per phase is the same as the current in each line and is equal to $1000 \times$ (kv-a.) $\div \sqrt{3}$ (volts between lines), while the voltage per phase is (volts between lines) $\div \sqrt{3}$.

In a Δ-connected machine the line current is equal to $1000 \times$ (kv-a.) $\div \sqrt{3} \times$ (volts between lines) and the current per phase is equal to (line current) $\div \sqrt{3}$, while the voltage per phase is equal to the voltage between lines.

PRINCIPAL PARTS

The stator frame is usually of cast steel in a box or hollow form and is the mechanical structure of the machine serving to support the stator. Incidentally, it is usually made hollow and serves to direct the ventilating air.

The stator core is a ring of laminated steel with slots on the inner periphery and carries the armature winding in the slots. The core is usually made up of sheets of steel .014 in. thick, punched in sectors of a circle with the joints staggered as they are piled up in layers.

The armature winding may be single-phase, two-phase, or three-phase, the last being the most common. It consists of form-wound coils placed in slots and connected up in accordance with principles explained under the heading "Armature Windings." The coils may be insulated with varnished cambric or mica paper and tape.

The field poles are usually of laminated steel punched .025 in. thick with flaring pole shoes. They are usually of rectangular cross-section and bolted to the rim, thus holding the spool in place. The pole face is usually beveled or chamfered in order to give the flux wave a desirable shape.

The field coils consist of many turns of wire, or copper ribbon wound on edge, with insulation between turns. All spools are connected in series and designed so that the specified exciter voltage will give the required field current and a margin for emergencies.

The field yoke is of cast steel in the form of a rectangular ring or rim of reasonable magnetic qualities, as it forms the yoke of each magnetic circuit. It must have ample mechanical strength to stand the centrifugal force.

Collector rings, of which two are required, are placed on the shaft and are required to carry the excitation current for the revolving field spools. Metal graphite brushes resting on the rings form the connecting link.

The spider, shaft, bearings, pedestals, and **bed-plate,** are the usual mechanical accessories required for the completion of the machine.

Armature Windings.—The armature winding is for the purpose of connecting a sufficient number of inductors in series to give the required voltage. It may be connected for one, two or three

phases, and each phase may have series or multiple circuits within it. While many forms of armature windings have been and may be used, there are only two forms that are in practical use in the United States at the present time, the **chain winding** and the **barrel** or **multiple-drum** winding. Of these the chain winding is only

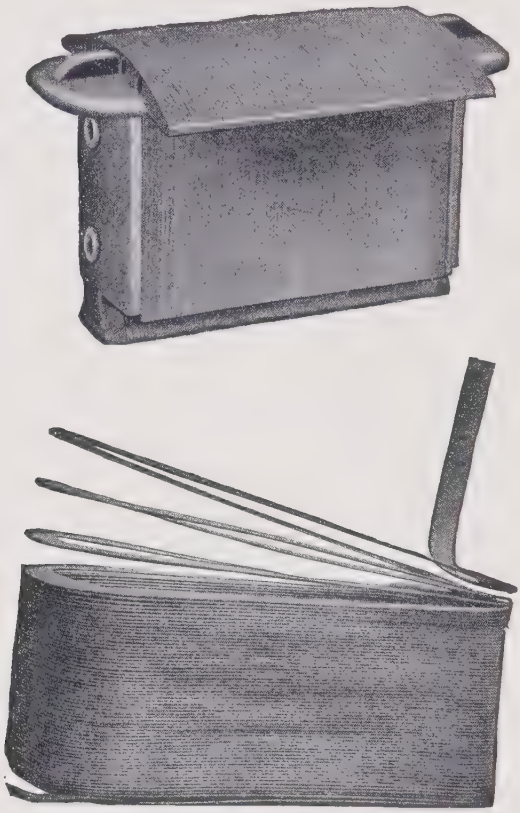


FIG. 68.—Field pole of sheet steel and coil of copper strap wound on edge.

used in a few old machines of moderate size, whereas the barrel winding is used in the large majority of present day machines.

Chain Winding.—This is for machines whose voltage is high for their ratings, because this type of winding is easier to insulate for high potentials. This winding is characterized by having a number of coils equal to half the number of slots; that is, there is

only one coil-side in each slot. The coils may be either form-wound and insulated, in which case slots with open faces are required; or they may be wound by hand, in place, in which case partly or entirely closed slots may be used.

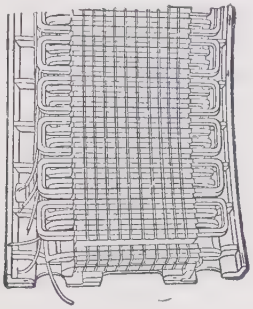


FIG. 69.—Illustration of a single layer or chain winding for an a.c. machine.

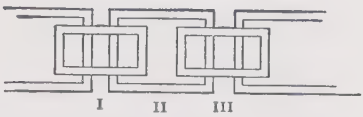


FIG. 70.—Principle of the arrangement of a chain winding.

There are at least two kinds of coils in each machine. These are characterized by the shape of their end connections, as these end connections lie some in one plane and some in another. The three-phase winding with six slots per pole will require four different shapes of coils. Figs. 69 and 70 illustrate the chain winding.

In the chain winding there is one group of coils per phase per pair of poles as in Fig. 71, and if similar terminals are labeled as the beginning of each phase, then the connection is simple. For a Y connection, three similar terminals, the three beginnings for instance, are connected together for a common neutral and the three ends become the terminals of the machine as in Fig. 72. For a Δ connection, the end of I is connected to the beginning of II, the end of II to the beginning of III, etc. as in Fig. 73, and the three junction points are tapped for the leads of the machine.

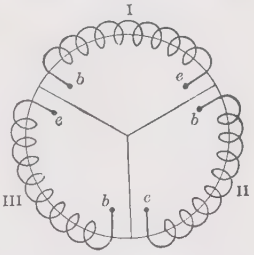


FIG. 71.—Simple Gramme Ring or 120° three-phase winding.

The **barrel winding** is sometimes called the **lap winding** and is very generally used at present, particularly in large machines for any reasonable voltage. Only one shape of coil is required and

all coils are interchangeable. It is very much like the multiple winding for a d.c. armature. There may be any number of coils per pole and any number of phases. The winding is in two layers, one side of each coil lying in the bottom layer and the other side of the same coil in the top layer. With two coil-sides per slot, one top and one bottom, the slots must be open, but with four or six coil-sides per slot, the slots may be partly closed. This type

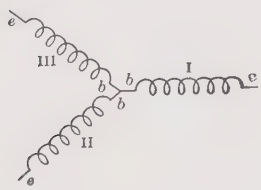


FIG. 72. — A Y-connection of the winding of Fig. 71.

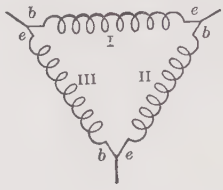


FIG. 73.—A delta connection of the winding of Fig. 71.

of winding is satisfactory for low voltages and very convenient if there are a large number of slots per pole or if large currents are used.

In machines of large sizes, those of thousands of kilowatts, it is also easy to obtain high voltages with a barrel winding.

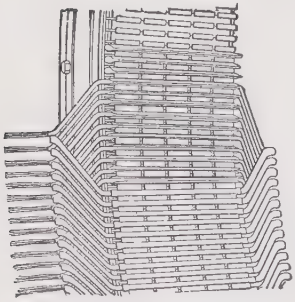


FIG. 74.—Illustration of a double layer, a barrel or a multiple winding for an a.c. machine.

The coils are connected in groups of two, three, etc., per pole per phase, depending upon the slots per pole per phase. The neighboring coils of one phase in consecutive slots under one pole must be connected in series, giving what is called a phase group. The various phase groups under the different poles may be connected in series or in multiple at will, depending upon the voltage or current of the machine.

In the usual multiple-drum or barrel winding, illustrated in Fig. 74, there are three groups of coils or phase belts for each pole, giving really six phases in the two poles, and successive groups are only 60° apart.

Figure 75 gives a cylindrical development of such a winding

for a two-pole machine having 12 slots, 6 per pole, or 2 slots per pole per phase. It will be noted that there are 12 coils. One side of coil No. 1 lies in the top half of slot No. 1 and is represented by a full line. The other side of coil No. 1 lies in the bottom of slot No. 7 and is represented by a broken line. Coils No. 1 and No. 2 must be connected in series and so must be coils No. 7 and No. 8, but these two groups may be connected either in series or in multiple with each other with due precaution for the fact that they lie under poles of opposite magnetic polarity and therefore generate

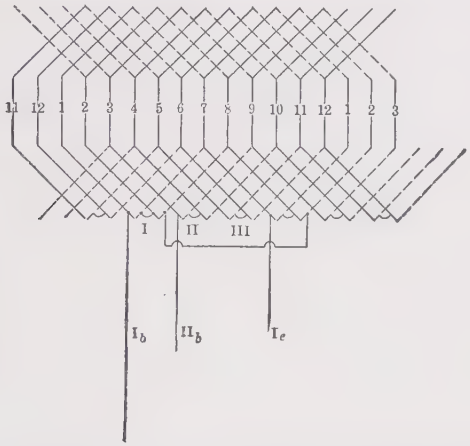


FIG. 75.—Cylindrical development of a three-phase barrel winding showing polar connection of phase I.

voltages in exactly opposite directions. These four coils form one phase. Coils 3, 4, 9 and 10 form a second phase and coils 5, 6, 11 and 12 a third phase.

In the usual three-phase machine the barrel winding gives what are really six phases as there are six groups of coils per pair of poles and successive groups are only 60 electrical degrees apart. Assuming a two-pole machine for three-phase, with six such groups as in Fig. 76, then groups Nos. 1 and 4 may be connected in series, with due regard to polarity, for one phase. Groups 2 and 5 form second phase and groups 3 and 6 a third phase.

In order to obtain a logical 120° separation we should label groups 3 and 6 as phase II and groups 5 and 2 as phase III, but this is not the conventional shop practice. Fig. 76 is labeled in

accordance with conventional practice and it will be noted that if the three beginnings (*b*) are connected together we get an unsymmetrical system of voltages only 60° apart as in Fig. 77*a*.

But if phase II is reversed by connecting the end (*e*) to the beginnings of I and III, we get a true 120° three-phase relation as in Fig. 77*b*.

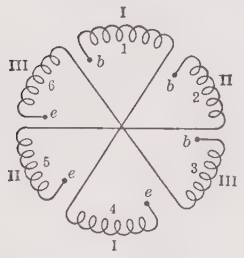
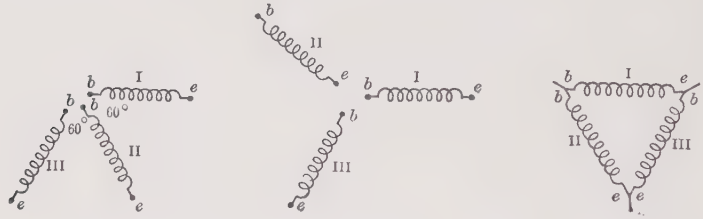


FIG. 76.—Diagram of a three-phase barrel winding for a two-pole machine with phases labeled in 60° sequence as is conventional.

For a Δ connection we should connect these phases as in Fig. 78, and as we follow around through the triangle we find II is reversed. If the labels on II and III had been interchanged it would have brought the beginning of III where the end of II was, and no reversing of phases would have been necessary. (See Fig. 79.) This would be more logical and simpler in the vector diagram (Fig. 80) but the other method is simpler when dealing with the actual windings and is a shop convention.

Figure 81 is a picture of a barrel winding in the process of construction, and Fig. 82 is a typical winding diagram showing the connections for a three-phase, four-pole machine, having 24 slots, or two slots per pole per phase. This is for a barrel winding with



77 *A* 77 *B* FIG. 78.

FIG. 77*A*.—Vector showing incorrect connection of conventional winding.
FIG. 77*B*.—Correct connection of conventional winding for a Y.
FIG. 78.—Vector showing correct connection of a conventional winding for a delta connection.

all four groups in each phase in series. Each phase is labeled and these three phases may be treated as in the simpler diagrams of Figs. 76, 77 and 78.

This diagram (Fig. 82) is typical of all barrel windings as, if the number of slots per pole were greater, for instance, twelve, it would simply require that four consecutive coils be connected

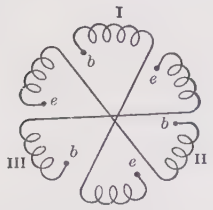


Fig. 79.—Logical labeling of phases at 120° apart.

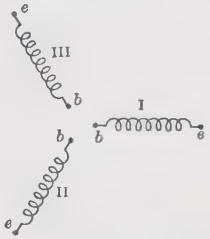


Fig. 80.—Logical connection of winding of Fig. 79.

in series as a phase belt as there would be four slots per pole per phase instead of two. If the number of poles were greater, eight poles, for instance, the diagram would merely repeat what is already shown by having eight groups per phase instead of four.

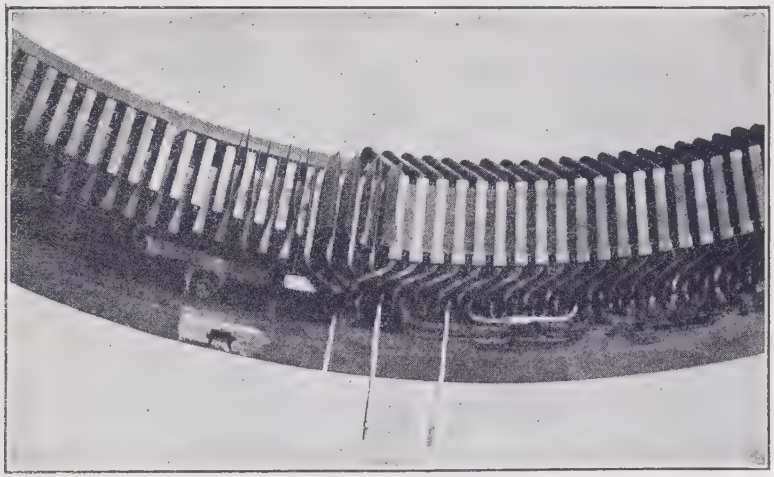


Fig. 81.—A barrel or two-layer winding in construction.

With the four-pole machine, we have the four polar groups per phase, and as the diagram shows, these may be all connected in series, for instance, for 440 volts. But any two may be connected in series and this circuit connected in multiple with the remaining

two in series, and we have a series-multiple connection for 220 volts. Finally, if all four of the groups are connected in multiple, we have a multiple winding for 110 volts.

Δ or Y Connection.—In very high voltage machines the Y connection is almost universal, as, because of the possibility of

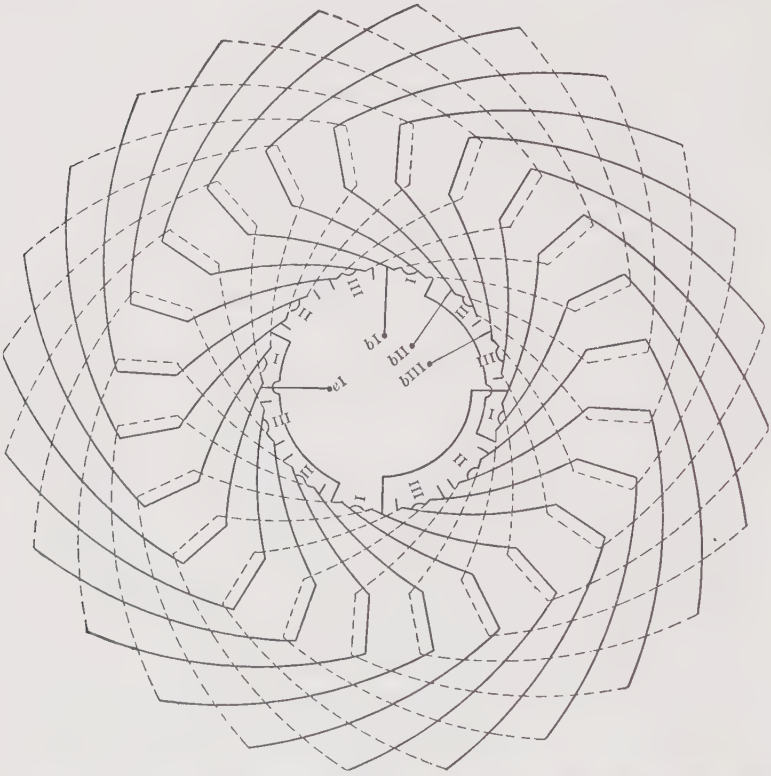


Fig. 82.—Typical winding diagram for a three-phase barrel winding with the connections of only one phase completed.

grounding the neutral point, a much lower voltage strain is placed on the insulation of the machine. Thus, wherever insulation problems are difficult the Y connection is used. However, there are many cases of machines of reasonable voltage, 2200 and less, in which the insulation problem is not paramount, and in some of these the need of large current causes difficulties. In such cases the delta connection is desirable as, for a given current in the line,

the current per circuit inside the machine is less. In other words, the change from a Y to a Δ connection in a three-phase machine is based on the same philosophy as the change from a series to a multiple circuit in any ordinary d.c. armature. However, this possibility gives one more element of elasticity in the design of three-phase machines, and makes possible a larger choice of voltages and currents for a given size of wire in the winding than is usual. Thus we have in the example quoted above, for one size of conductor:

4 circuits in series, connected Y	440 volts	100 amperes
4 circuits in series, connected Δ	255 volts	173 amperes
2 series, 2 multiple Y	220 volts	200 amperes
2 series, 2 multiple Δ	127 volts	346 amperes
4 multiple Y	110 volts	400 amperes
4 multiple Δ	63 volts	692 amperes

It should be noted that the current capacity of these windings is inversely proportional to the voltage rating, which leads to the final conclusion that for very heavy current we should desire a delta connection with all circuits in multiple.

Voltage Relations

Distribution Constant.—If each coil spans the same arc of the armature as the arc between the center of the pole pieces, it is said to have full or 100 per cent pitch. In this case the inductors forming the two halves of a turn generate similar voltages, and the voltage of the coil is the arithmetical sum of the voltages of the separate inductors. This voltage will have a shape similar to that of the flux distribution at the pole face. The voltages of the coils in successive slots, however, reach a maximum at different times and therefore these voltages are out of phase with each other and cannot be added arithmetically. If, as in Fig. 82, there are six slots per pole and six coils per pole, the successive coils differ in phase by one-sixth of the pole pitch. This is $\frac{1}{12}$ of a cycle, or 30 electrical degrees. Hence, if connected in series the resultant would be 1.93 times the voltage of one and not twice. Thus, if there are several slots per pole per phase, the coils in them must be connected in series and the resultant voltage obtained by vector combination. For nine slots per pole or three slots per pole

per phase, spaced 20 electrical degrees apart, the calculation would be as in Fig. 83.

Let e be the virtual volts per coil, then the resultant would be:

$$e \cos 20 + e + e \cos 20 = .943\ e + e + .943\ e = 2.886\ e.$$

This is taken care of in practical calculations by a **distribution constant,** k_1. The distribution constant is the ratio of the resultant

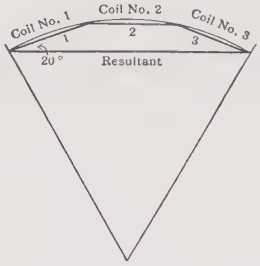


FIG. 83.—Vector diagram for the distribution constant for three-slots per pole per phase in a three-phase machine.

e.m.f. divided by the arithmetical sum. Thus in the example above:

$$k_1 = \frac{2.886e}{3e} = .962.$$

Values for the distribution constant, k_1, are given in the following table:

TABLE XIX

DISTRIBUTION CONSTANT

Value of k_1

Slots per Pole	Single-phase	Two-phase	Three-phase
1	1.00		
2	.707	1.00	
3	.666	.....	1.00
4	.650	.925	
6	.640	.912	.966
8	.640	.905	
9	.640	.....	.962
12	.640	.900	.960
24	.635	.900	.958
∞	.632	.900	.955

The connecting in series of the coils in successive slots under one pole piece smooths out the voltage wave by combining waves of slightly differing phase; the more coils in series per pole the better the wave shape. With a wave shape of flux having a crude rectangular form, the voltage per coil would have the same shape; but even with this crude shape, if an infinite number of

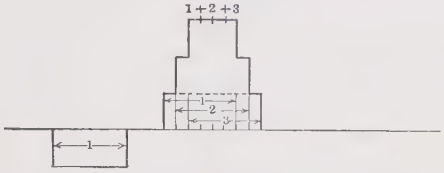


FIG. 84.—Wave shape for one inductor and for three inductors, in different slots, in series.

coils were connected in series the resulting voltage of the series circuit would be a sine wave.

This is best seen in Fig. 84, in which the distribution of the flux wave is shown, which would also be the voltage per inductor or per turn with full pitch.

When three of these, differing slightly in phase, are combined as in the figure, it will be seen that the wave shape is considerably

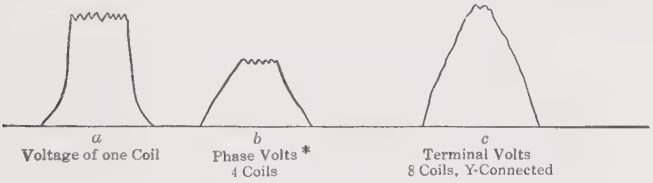


FIG. 85.—Oscillographs of wave shapes.

a. One coil alone. *b*. Four coils in different slots in series. * (Different scale from *a*.) *c*. Two phases in series forming a Y and giving eight slots in series. (Same scale as *b*.)

improved. This is a concrete example of the case of a machine having nine slots per pole, therefore 20° between slots and three slots per pole per phase in a three-phase machine.

Figure 85 shows the effect in an actual machine as pictured by an oscillograph record, showing first the wave shape for one full pitch coil, second for four coils in series in a machine having four slots per pole per phase, this being the voltage per phase,

and finally the voltage between terminals for a Y connection, this being the resultant of eight coils in series. The saw-teeth or ripples in the top of the wave are due to the eight teeth which lie under a pole at a given instant.

In many cases it is desirable to have the coils span an arc less than that of the pole pitch; thus in a machine having nine slots per coil one side of a coil might be in slot No. 1 and the other side of the pole in slot No. 7. This would be a pitch of $\frac{6}{9}$ or $66\frac{2}{3}$ per cent. The reasons for using a **fractional pitch** are: first, to reduce the

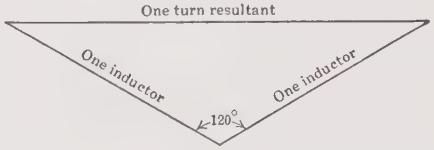


FIG. 86.—Vector diagram for pitch constant for a pitch of 67 per cent.

length of the useless copper in the end connections; second, to reduce the resistance and consequent loss; third, to improve the shape and character of the wave shape of e.m.f.; and fourth, to reduce the **leakage reactance.**

If a coil has a pitch of $\frac{2}{3}$ the two sides are only 120 electrical degrees apart instead of 180. Thus the voltages of any two inductors forming a turn are not in phase and we get a resultant less than the sum of the two. Fig. 86 is the vector diagram for this case, giving a resultant equal to 1.73 times the voltage of one side. The **pitch constant,** k_2, is used in calculations to allow for this:

$$k_2 = \cos \frac{180 - 180 \times \text{pitch}}{2} = \sin(90 \times \text{pitch fraction}).$$

k_2 has values as in Table XX.

TABLE XX

Pitch		k_2
.50	90°	.707
.67	120°	.866
.75	135°	.93
.83	150°	.97
1.00	180°	1.00

Coils with a fractional pitch have the effect of smoothing out the wave and the teeth ripples, and eliminating certain harmonics. Thus a pitch of $\frac{2}{3}$ will practically eliminate the third harmonic. (See paper by C. A. Adams, A. I. E. E.) By a careful design of the pole face, the use of a long air gap, of many narrow slots per pole, and a fractional pitch coil, it is possible to make the wave shape of the voltage practically a sine form, which is the ultimate desideratum.

If a wave having higher harmonics is impressed upon an inductive circuit, the inductance tends to smooth out the harmonics in the current wave. Conversely, an anti-inductive load exaggerates the harmonics.

If there are so few slots and the wave shape of the flux is so bad that the resulting voltage is not a sine wave, then the effective value of the voltage does not bear the usual ratio to the maximum. For this very unusual case another constant would have to be employed, the **form-factor constant,** and this would have to be determined by a detailed construction of the flux wave shape and resulting voltage wave shape.

To sum up, the practical formula for the relation between virtual voltage per phase and gross flux per pole is

$$E = 4.44 k_1 k_2 f S \phi 10^{-8},$$

in which E = virtual voltage per phase;
k_1 and k_2 = constants as in table, distribution and pitch;
f = frequency, cycles per second;
S = total turns in series per phase;
ϕ = useful flux per pole, maxwells.

Armature Reaction

When a current flows in the armature of a polyphase synchronous machine, the armature winding becomes the seat of a magneto-motive force, which is constant in magnitude and revolves at synchronous speed with respect to the armature, and thus is stationary with respect to the field coils. This reacts on the field m.m.f. and either diminishes or increases the useful flux and in either case distorts it. If the current in the armature is in phase with the generated e.m.f. it causes a **cross-magnetizing** force acting axially midway between pole pieces as in the case of a d.c. machine having the brushes on the geometric neutral (q.v.)

As the phase of the armature current changes, the direction of the axis of this armature m.m.f. shifts, and with lagging armature current in a generator a component is introduced which opposes the field m.m.f. Conversely, in a generator, if the armature current leads the generated e.m.f. this component assists the field m.m.f. This is quite similar to the effect of moving the brushes of a d.c. generator with rotation (lagging) or against rotation (leading).

The usual distribution of polyphase windings around the air gap gives a wave shape of m.m.f. which roughly approaches a sine wave. If the air gap is long, or reasonably long, and there are a considerable number of slots per pole, the m.m.f. becomes a sine wave with some higher harmonics of negligible amplitude. It is therefore the custom, and is reasonably accurate, to assume that the m.m.f. of the armature has a sine-wave distribution; and only in the case where the slots per pole are few and the opening of the slots wide compared to the teeth does this introduce an appreciable error. The following treatment is based on the assumption of the sine-wave distribution.

In any a.c. machine the maximum instantaneous value of the armature m.m.f. per phase is $\sqrt{2}SI$, where S is the turns in series per phase and I is the virtual value of the current per phase. The instantaneous value is thus $\sqrt{2}SI \cos wt$ where wt is a function of time and frequency (2πft.). In a three-phase machine the m.m.f.'s of the three phases are displaced 120 electrical degrees in space and $\frac{1}{3}$ cycle or 120 degrees in time. The instantaneous value of the m.m.f. of phase I (Fig. 87) is $\sqrt{2}SI \cos wt$, and its component acting along an axis arbitrarily placed wt degrees from I is $\sqrt{2}SI \cos^2 wt$. The instantaneous value of the m.m.f. of phase II is $\sqrt{2}SI \cos (wt + 120)$, and its component along $x - x$ is $\sqrt{2}SI \cos^2 (wt + 120)$. The instantaneous value of the m.m.f. of phase III is $\sqrt{2}SI \cos (wt + 240)$, and its component along $x - x$ is $\sqrt{2}SI \cos^2 (wt + 240)$. The resultant m.m.f. of the three phases is the sum of these three components or $\sqrt{2}SI \times 1.5$ and is constant in magnitude but revolves with the angular velocity $2\pi f$ or at synchronous speed.

Stated in other words: If the vector m.m.f.'s, I, II, III, are assumed to vary with time, the resultant of these instantaneous values will be constant in magnitude, equal to 1.5 times the maximum of one, and will revolve in space. (Fig. 87.)

Since the field structure revolves at synchronous speed with respect to the armature and since the armature m.m.f. revolves at the same speed, this m.m.f. is stationary with respect to the field coils except for minor changes due to a change in the phase of the armature current with respect to the generated e.m.f.

If the field structure has no polar projections or interpolar spaces (i.e., as in the round rotors of turbo-generators) the direction of the armature m.m.f. would be such that the angle between this m.m.f. and that of the field is 90° when the armature current is in phase with its voltage; $90 + \phi$ when the current lags ϕ degrees behind the e.m.f.; and $90 - \phi$ when the current leads by ϕ degrees.

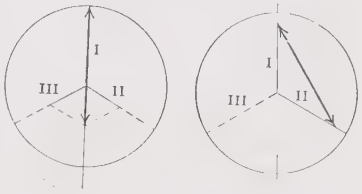


FIG. 87.—Vector combination in time and space of the m.m.f.'s of three phases to show resultant.

With the salient-pole type of alternator having interpolar air spaces, the reluctance of the path offered to the armature m.m.f. is intentionally much greater than that offered to the field m.m.f. The result is that the effect of the armature m.m.f. is minimized. This renders accurate calculations complicated and difficult, but the error introduced by using the simple geometrical relation of the general theory is on the safe side, as the results of calculations will indicate a field current greater than that which will be required in the practical operation. If a machine has fractional pitch or several slots per pole per phase, the resulting m.m.f. is not as great as the sum of the m.m.f.'s of the individual turns, as in the case of the resultant voltage and by the same ratio.

The formulae for the **armature reactions per pole** of polyphase machines are:

For a two-phase machine $n = \dfrac{\sqrt{2}SIk_1k_2}{p}$;

For a three-phase machine, $n = \dfrac{1.5\sqrt{2}SIk_1k_2}{p}$,

where n = armature reaction per pole, ampere-turns;
S = turns in series per phase;
I = effective value of armature current per phase;
p = number of poles;
k_1 and k_2 = constants as in tables XIX and XX.

The armature reaction of a single-phase generator is not constant in magnitude but pulsates in value between a maximum of $\sqrt{2}SI$ and zero. It is mathematically represented by two m.m.f.'s, each equal to one-half of the above value and each rotating in opposite directions at synchronous speed. One of these hypothetical components acts like the m.m.f. in a polyphase machine, and the other causes a pulsation in the main magnetic flux which introduces undesirable effects and extra losses. An **amortisseur,** or squirrel-cage winding, in the poles is desirable to reduce these effects.

The appropriate value of the armature reaction ampere-turns per pole at rated current varies with the size and type of machine. It may increase with the capacity of the machine, with the peripheral speed, and with the pole pitch, but should decrease with higher frequencies.

Where good inherent regulation is desired the armature reaction must be low, but for machines to be controlled by voltage regulators a high value is permissible. Reasonable values for the armature reaction of salient-pole polyphase generators would be:

TABLE XXI

Rating in Kv-a.	Armature Reaction	
	25 cycles	60 cycles
100–1000	1500	1000
1000–2000	2000	1500
2000–5000	4000	3000

DESIGN OF A SALIENT-POLE GENERATOR

The procedure in designing an alternating-current generator for a given power output, voltage, and frequency to fulfill given requirements regarding efficiency, heating, and regulation, is partly analytical and partly empirical. The method of procedure is to lay out a preliminary design from rough calculations, calculating the performance of this design, modify the preliminary design where this calculation indicates, and recalculate the performance until a design is arrived at which meets the given require-

ments. The design of turbo-generators of the round-rotor type will be found under a separate heading.

The method is based on conforming to the general practice of keeping certain cardinal constants within well-recognized limits. These limits are indicated in the accompanying paragraphs, and the general proportions of several designs are given in the tables of examples of design and performance.

Definitions.—The following terminology is common among designing engineers and is used in the discussion here.

Pole Arc.—The arc subtended by one pole face from tip to tip, measured along the periphery of the armature. In the following discussion it will be expressed in inches. It is usually between $\frac{6}{10}$ and $\frac{2}{3}$ of the pole pitch.

Pole Pitch.—The arc measured along the periphery of the armature from the center of one field pole (*N* pole) to the center of the next field pole (*S* pole); in the following discussion it will be expressed in inches.

Ampere Conductors per Inch of Armature Periphery.—The product of the number of conductors per inch, measured along the periphery of the armature, by the effective amperes flowing in each conductor. This is a characteristic of electrical machines which carries much information for the designing engineer. Represented by σ.

Slot Pitch.—The distance in inches measured along the armature periphery between the centers of two adjacent slots, or the perimeter of the armature divided by the total number of slots.

Coil Pitch.—(See "Armature Windings.") The number of slots spanned by a coil; that is, if a coil has one side in slot No. 1, and the other side in slot No. 7, the pitch of a coil is 7 minus 1 = 6. A coil that spans an arc exactly equal to the pole arc is said to have a full pitch, but if it spans a lesser arc, it is said to have a fractional pitch. For example, if there are six slots per pole and a coil has one side in slot No. 1 and the other side in slot No. 5, it is said to have a fractional pitch of $\frac{4}{6}$ or $\frac{2}{3}$.

Leakage Factor.—The ratio of the flux per pole which enters the armature core and interlinks the conductors to the total flux in the yoke (including the leakage flux) which is produced by the field winding. (See page 163.)

Statement of Problem.—The design of a generator usually starts with a statement of the required output in kilovolt-amperes

or kilowatts at a certain power factor, the speed in r.p.m., and the voltage between terminals. It is much more usual to use the Y connection in the case of three-phase generators, and this assumption will be made in the following discussion. As by far the greater number of generators are of the three-phase type rather than two-phase or single-phase, this is the type used throughout the following discussion. A two-phase machine is designed in exactly the same manner, with merely minor changes resulting from the connection of the windings in the two circuits instead of three; but the design of a single-phase machine is quite a different matter and must be treated separately.

Having been given the kv-a. rated output desired of the machine, and the voltage between the terminals, and having assumed a Y connection, we may find the voltage per phase and current per phase as directed in a preceding paragraph (page 130).

Suitable Speed and Poles.—All commercial machines in the United States operate at one of three frequencies, 25, 50, or 60 cycles per second, in order that they may be connected in standard commercial circuits. Therefore the frequency of the required machine would be very definitely specified at the beginning. On account of the definite relation between frequency poles and speed, given by the well-known relation $pN = 120f$, in which p is the number of poles, N the revolutions per minute, and f the frequency in cycles per second, there are only certain speeds at which a generator of a given frequency may be operated; and this, in the case of 60-cycle machines, would be found in the following sequence:

For 2 poles.....3600 r.p.m.
For 4 poles.....1800 r.p.m.
For 6 poles.....1200 r.p.m., etc.

If, as is usually the case, a speed is specified, the number of poles is definitely set by this relation, and if the speed specified does not fit into the above sequence it must be made to do so by requiring the prime mover to adapt itself to one of these definite speeds. Thus the problem of choosing the speed and number of poles is very definite.

Diameter of Armature.—This dimension is usually most influenced by the mechanical design which it is desired to use. For peripheral speeds from 2000 to 5000 ft. per minute, no unusual precautions need be observed to take care of the centrifugal forces, and in this class are what are called the slow-speed machines.

Above 5000 and up to 10,000 ft. per minute, the centrifugal forces are such as to require a special design of the salient-pole type of revolving field, but yet are not great enough to require the special round-rotor construction. With peripheral speeds from 15,000 to 25,000, a special construction common in turbo-generators is required, and this construction so changes the electrical design that the subject is treated in a separate chapter. To sum up, the peripheral velocity would be either between 4000 and 5000, for the normal machines, or between 9000 and 10,000, for the specially constructed salient-pole machines. Having assumed the peripheral speed which it is thought desirable to use in view of the mechanical construction anticipated, the diameter of armature D in inches then becomes $D = \frac{12V}{\pi N}$ where V is the peripheral speed in feet per minute.

Length of Armature.—There is a definite relation, based on fundamental principles, between the cardinal constants of any electrical machine. This relation has been discussed in a previous chapter. For the particular type of machine here discussed, and for the particular purpose of determining the suitable length of armature between heads, this formula takes the form:

$$L = \frac{144P_0 10^9}{\rho\sigma BVDk_1k_2},$$

in which P_0 = output in kilovolt-amperes;
ρ = ratio of pole arc divided by pole pitch, usually .65.
B = average magnetic flux density per square inch in air gap;
σ = ampere conductors per inch of periphery, as described above;
V = peripheral velocity in feet per minute at gap;
D = diameter of armature in inches;
L = length of armature core between heads in inches;
N = revolutions per minute;
k_1 and k_2 as given on pages 140 and 142.

All of the constants in the right-hand member of this equation are either known or should have values within recognized limits and are therefore subject to choice by experience. The choice of the values for these constants may be made from the following tables, based upon modern practice.

Ratio of pole arc to pole pitch (ρ) is covered by two antagonistic phenomena; for the sake of small magnetic leakage and good form factor of voltage wave, a low value is desired; for the sake of low reluctance to the main flux and economy of material, a high value is desired. The usual values lie between .6 and .65.

Magnetic flux density (B) is limited by the exciting ampere turns required to produce it, length of gap, and sometimes by the heating resulting from coreloss. In general, the larger the machine the higher may be the flux density in the gap. Usual values of B are given in the accompanying table:

TABLE XXII

Cycles	B Lines per Square Inch in Gap
25	44,000 to 58,000
60	36,000 to 50,000
125	30,000 to 40,000

Ampere conductors per inch of periphery (σ) is limited by the heating resulting from high current densities. If ventilation is good, insulation thin, or if the slots are very deep, as in armatures of large diameters, the values of σ may be high. The table gives values of σ for continuous rating, with a rise of temperature of 50° C. Naturally, with special means of ventilation σ may be greater than the values here given, as for example, in turbo-alternators with forced ventilation.

TABLE XXIII

σ AMPERE CONDUCTORS PER INCH

Diameter of Armature, In.	Less than 2000 Volts	2000 to 7000 Volts
0 to 30	300–400	
30 to 100	400–600	
100 to 200	600–1000	600–800
200 and up	.....	800–1000

By substituting values chosen from these tables, with due reference to the size and character of the machine, the equation becomes definite with the exception of the winding constants, k. As

the type of winding to be used is not yet known at this stage of the design, these constants may be assumed as .96 and 1.0 respectively, this assuming a full pitch winding. But if it is desired to leave sufficient margin in the design to make it possible to take advantage of fractional pitch, it may be well to assign k_2 a value of .9 for 60-cycle machines, and .8 for 25-cycle machines. It is the custom to use a smaller fraction of pitch in low frequency machines than in those of higher frequency. Thus, by the assumption of certain constants which in themselves presume certain well-recognized proportions of design, it is possible to arrive at a reasonable value for L, a second important dimension of the design which may be used as a preliminary figure in the following calculations. But the designing engineer always keeps in mind the fact that after his first calculation he is going to make changes in these principal constants, partly for some purposes of adjustment and partly for the importance of the characteristics of the machine.

Number and Size of Slots.—It has been the custom for years to make the number of slots per pole an even multiple of the number of phases, thus, for a three-phase machine, 3, 6, 9, etc.; this is not necessary, but merely convenient for making a simple winding. Lately the expedient is frequently employed of purposely making the number of slots per pole prime to the number of phases in order to obtain a good wave form of voltage, i.e., suppress harmonics. Of course the total number of slots on the armature must be a multiple of the number of phases in order that the phases may have equal voltages.

An example would be a three-phase, four-pole machine having 42 slots or 10.5 slots per pole and 3.5 slots per pole per phase, but 14 slots altogether in each phase. Assuming 42 coils in a lap winding, the coils would be grouped in accordance with the following tabulation:

	Coils per Phase			Coils per Pole
Phase	I	II	III	
Under pole 1	4	3	3	10
Under pole 2	3	4	4	11
Under pole 3	3	3	4	10
Under pole 4	4	4	3	11
Total	14	14	14	42

All the coils of each phase would have to be connected in series because the voltages of the separate groups differ slightly in magnitude and time phase. But this arrangement could be used as a unit in an eight-, twelve-, or sixteen-pole machine and the windings of each phase of each four-pole unit be connected either in series or multiple with the units of other groups of four-poles.

However, for the purposes of the present discussion the more simple winding having the slots and coils per pole a multiple of the number of phases will be assumed. In order to proportion the space along the periphery of the air gap rationally between that allotted to copper (slots) and that allotted to iron (teeth), the number of slots and depth of slots will vary quite logically with the diameter of the armature. If too many slots are used, space is wasted owing to the increased space for slot insulation. If too few slots are used the wave shape and leakage reactance are bad. If the slots are too shallow, valuable space is again wasted in slot insulation; and if the slots are too deep, the leakage reactance is high, the slots encroach upon the space required by the core, and a greater external diameter of punching is required. Reasonable values taken from standard practice are given in Table XXIV.

TABLE XXIV

SALIENT POLE GENERATORS

Diameter, Inches	Number of Slots	Depth of Slots	Length of Gap
10	36–48	1–1.25	.094
20	48–72	1.5–1.7	.125
30	60–96	1.6–1.8	.15
50	72–120	1.8–2	.20
100	120 144 168	2 –2.5	.25
150	210 240 270	2.5–3	.375
200	300 324 348	3–4	.5
300	348 360 420	4–6	.75

The preliminary value of the number and the depth of the slots would be estimated along the line of reasoning suggested by the table. The width of the slots is roughly about the same as the width of the teeth at the face, and this value would be ordinarily selected as a preliminary figure to be adjusted later under compulsion of necessity. In small, low-speed machines, it is sometimes

necessary to make the width of slot greater than the width of the teeth, or in other words, more than one-half the slot pitch, while in large machines, and particularly those of high peripheral speeds, the width of the slots would usually be less than the width of the teeth and somewhat in the neighborhood of 40 per cent of the slot pitch.

Total Useful Flux per Pole in the Armature (ϕ). This quantity has already been definitely fixed by the assumed values assigned to the gap density (B), diameter (D), and length of armature (L). Thus,

$$\Phi = \frac{\pi DLB\rho}{p}.$$

Number of Turns in Series per Phase (S).—The preliminary value of this quantity has now been fixed by the assumptions made and is obtained by solving the primary equation for the relation between the electromotive force and flux.

$$S = \frac{E \times 10^8}{4.44 k_1 k_2 f \Phi}.$$

The value of S depends upon whether a Y or a Δ winding is to be used, as this affects the value of the voltage per phase. In general, a Y connection is assumed, particularly in high-voltage machines, but in low-voltage machines the Δ connection is acceptable, and sometimes the use of a Δ connection gives an arrangement of conductors in the slots which is much more convenient than the Y connection. An objection is raised against a Δ connection, particularly in large machines, because in certain conditions of wave shape which favor the generation of a third harmonic in the electromotive force, a considerable third harmonic current may circulate in the three windings as the Δ connection forms a definite short-circuit for the third harmonic.

Number of Conductors per Slot (c).—The value for this quantity is approached by finding a value for the number of **effective** conductors per slot. The **effective conductors per slot** is a term much used by designing engineers to indicate the number of conductors there would be in the slot if there were no multiple circuits in the machine, i.e., if all the conductors and turns in each phase were connected in a series circuit. It is equal to the

number of actual physical conductors, wires, or bars in the slot divided by the number of multiple circuits. It is of value in indicating the number of times the line or phase current passes through a given slot; thus, if there are two effective conductors per slot the ampere conductors in the slot will be two times the phase current. If, however, there were two bars per slot and the winding had two circuits in multiple, each bar would carry only one-half of the phase current and consequently the ampere conductors per slot would be equal to the phase current and we should have one effective conductor per slot.

It is obvious that each one of the S turns in series per phase represents two effective conductors, one at each side of the coil; consequently, if S is the number of series turns per phase, then $2S$ will be the number of effective conductors per phase, and in a three-phase machine $6S$ will be the total effective conductors on the armature. Dividing this by the number of slots assumed gives the preliminary value of the effective conductors per slot.

The next step is to adjust from effective conductors per slot to practical conductors per slot in order to make an arrangement of wires or conductors which will fit suitably into the shape of the slot and into the two or four coil-sides into which the conductors are grouped for the purposes of the winding. It is permissible to have as many circuits in parallel per phase as there are poles. Consequently, if the number of effective conductors per slot is so small as to be inconvenient we may use a larger number of wires by having two circuits in parallel, four circuits in parallel, etc. It then becomes a matter of cut-and-try to adjust a number of wires per slot which will give a suitable shape of slot and a total number of wires per slot which approximates the product of effective conductors per slot and circuits in parallel. In doing this it is also necessary to have in mind the approximate size of each conductor, which is determined in the following paragraph.

Size of Conductors.—The size of each conductor is a compromise between a size that will give a reasonable current density in the conductor and one that will fit into a reasonable slot. The current density in the copper conductor should be from 2500 amperes per square inch in small machines to 2000 in medium-sized and moderate-voltage machines, and 1500 in high-voltage machines. By means of these figures a preliminary value for the size of conductor and the circuits in multiple may be determined,

which, however, will have to be checked later to determine whether the heating is reasonable and the efficiency satisfactory. The conductors are arranged in the slots so that the slots are usually about four times as deep as they are wide. The size of the slots should be such as to allow a space for insulation over and above the space occupied by the conductors and such cotton covering as they may have.

Size of Slots.—Knowing the number and the size of the conductors which it is desired to put into each slot, the designer next determines if it can be done, and if not, what is the next best arrangement.

The first rough approximation is obtained by means of a convenient quantity, known as the **slot factor,** which is the ratio of the total cross-section of the copper in a slot to the gross cross-section of the slot (depth × width, ignoring overhanging teeth). The slot factor varies with the size of the slot, the size of the machine, and the voltage for which the winding is insulated. It has values as given in Table XXV.

TABLE XXV

Slot Factor

Voltage	10–1000 Kv-a.	Larger than 1000 Kv-a.
500	.45	.60
1000	.45	.50
2000	.40	.46
4000	.35	.42
6000	.30	.40
10000	.25	.35

A cross-section for the slot (in square inches) is thus found by the formula:

$$\text{Slot area} = \frac{\text{number of conductors} \times \text{square inches of each}}{\text{slot factor}}.$$

Dividing this area by the depth of the slot chosen from Table XXIV gives the desired width of the slot. If this gives a tooth which is too narrow, a second trial must be made by assuming a greater depth. If this again should give too narrow a tooth it must be assumed that too high values have been used in choos-

ing the empirical constants, B and σ, used in determining L, the length between heads.

The next step is to lay out the arrangement of the conductors in the slot in more or less detail. For this purpose an understanding of the usual methods of making coils, insulating them, and placing them is necessary. The most usual method is to wind the coils up individually on forms or models, insulate them, press them to exact shape and dimensions in a hot mold, and then place them in the slots one by one. The lap winding with two coil-sides per slot and totally open slots is so generally used in the United States that it will be the only method considered here.

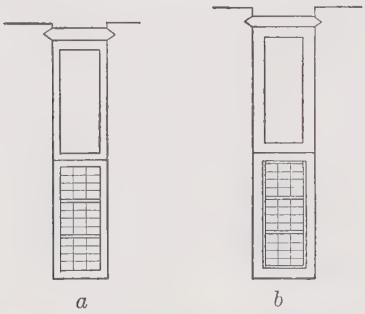


FIG. 88.—Cross-section of slot showing coils and wedge to scale, (*a*) for 2200 volts and (*b*) for 6600 volts.

As typical cases have been selected a winding for 2200 volts with **varnished cambric** insulation and a winding for 6600 volts with **composite mica** insulation. Windings for 125 and 250 volts would be very similar to those for d.c. machines (q.v.).

Armature Coil and Slot Insulation.—*2200-volt Cambric Insulation* (see Fig. 88*a*).—Stranded cable with two coil-sides in open slots, three turns per coil, each turn consisting of 12 strands, laid 3 wide and 4 deep. Each strand to be .12 × .20 in. (See cut.) Each strand is insulated with a double cotton covering and the 12 strands formed into a rectangle as one conductor.

Each conductor or turn is wrapped with one layer of .025 varnished cambric one-half lapped for insulation between turns.

The three conductors are stacked, and around the assembled group are wrapped two layers of varnished cambric (.0275 in.), one-half lapped.

Apply one layer of .007 in. cotton tape one-half lapped. Give six applications of varnish with alternate dryings.

The whole coil is dipped and impregnated with insulating compound, then dried.

In the slot is placed a layer of slot armor consisting of .015 in. horn fiber, and the two coils are laid in, one on top of the other, the slot armor bent over the top, and a wooden wedge driven in.

CALCULATION OF SPACE REQUIRED

Depth		Width	
Each strand, d.c.c.	.128	Each strand	.208
	4		3
Four high	.512	Three wide	.624
Insulation on each turn	.050	Insulation on each turn	.050
	.562		.674
	3		
Three conductors	1.686		
Two layers cambric	.110	Two layers cambric	.110
One layer cotton tape	.054	One layer tape	.054
Each coil	1.850		
	2		
Two coils	3.700		
Slot armor	.030	Slot armor	.030
Wedge and space	.312		
Total depth	4.042	Total width	.868

$$\text{Slot factor} = \frac{(.12 \times .20) \times 72}{4.05 \times .87} = 0.49.$$

6600-volt Mica Insulation.—(Fig. 88*b*). Stranded cable with two coil-sides in open slots, three turns per coil, each turn consisting of 12 strands, laid 3 wide and 4 deep. Each strand to be .12 × .20 in. when bare.

Twelve strands are assembled and wrapped with one layer of varnished cambric, one-half lapped for insulation between turns.

The three conductors are stacked, and around the assembled group are wrapped 4 layers of mica tape one-half lapped.

Wrap with 3 layers of cloth tape one-half lapped.

Tape with one layer of cotton tape one-half lapped. Give six applications of varnish with alternate dryings.

The whole coil is impregnated in an insulating compound in a vacuum.

In the slot is placed a layer of .015 in. slot armor. The coils are placed inside the slot armor, the armor is folded over the top and lapped, and a wedge is driven in.

CALCULATION OF SPACE REQUIRED

Depth		Width	
Each strand d.c.c.	.128	Each strand	.208
	4		3
	.512		.624
On each turn, v.c.	.050	On each turn, v.c.	.050
	.562		.674
	3		
	1.686		
Four layers mica tape	.100	Four layers mica tape	.100
Three layers cloth	.165	Three layers cloth	.165
One layer cotton tape	.025	One layer cotton	.025
One coil	1.976		
	2		
Two coils	3.952		
Slot armor	.030	Slot armor	.030
Wedge and space	.312		
Total depth	4.294	Total width	.994

$$\text{Slot factor} = \frac{(.20 \times .12) \times 72}{4.3 \times 1} = 0.40.$$

For a preliminary check on the size and shape of the slots, an allowance may be made for the insulation over and above the space occupied by the conductors and their cotton covering, as in table XXVI.

TABLE XXVI

Slot Insulation

Voltage	Insulation, Inches	
	Depth	Width
0–500	.35	.14
2200	.625	.18
4400	.75	.25
6600	1.00	.32
13000	1.50	.55

For instance, the necessary vertical depth of slot for a machine insulated for 2200 volts may be found by multiplying the overall diameter of the cotton-covered wire or depth of bar, by the number of conductors in a vertical row and adding 0.625 to the product.

In this allowance for vertical space is included the space for the wedge required to hold the coils in the slot. It is customary to arrange the conductors in the slot so that the depth of the slot will be four or five times as great as the width.

If it should happen that the slot is so large that the tooth is very much restricted, it is necessary to start a new design by assuming a lower value of ampere conductors per inch of periphery or of density in the air gap, which in turn will relieve the saturation in the teeth by allowing a narrower slot or less flux per tooth respectively. However, without experience, this difficulty may not be obvious until the calculation of the magnetic circuit is made.

Armature Winding.—With the number of slots and number of effective conductors per slot fixed by the preceding calculations, it is possible to lay out the armature winding. The number of turns in series per phase will have to be readjusted to fit the number of slots per phase and effective conductors per slot. The number of parallel paths will be chosen so that the size of each physical conductor or wire is not clumsy, and in most cases a winding of the multiple-drum type with a Y connection will be found most satisfactory.

The final value of the turns in series per phase will be

$$S = \frac{\text{conductors per slot} \times \text{total slots}}{2 \times nm},$$

where conductors per slot means the actual physical bars or cables per slot;

n = number of phases;
m = number of parallel circuits per phase.

The effective conductors per slot will be

$$c = \frac{\text{conductors per slot}}{m} = \frac{2nS}{\text{total slots}}.$$

For the details of the armature winding diagram, see page 131.

It is customary to assume a pitch of coils for preliminary calculations of about $\frac{2}{3}$ in 25-cycle machines and $\frac{5}{6}$ in 60-cycle machines.

Exact Value of Useful Flux per Pole.—To generate the required voltage at the specified frequency with the winding chosen, there will be required a useful flux per pole, entering the armature and cutting the conductors, whose value is given by the relation:

$$\phi = \frac{E \times 10^8}{4.44 k_1 k_2 f S} \text{ maxwells,}$$

where E = the virtual voltage per phase;
$k_1 k_2$ = winding constants as on page 140;
f = the frequency;
S = the turns in series per phase.

Preliminary Dimensions of Magnetic Circuit.—The diameter of the armature and the axial length of the pole face for the preliminary design are fixed by the preceding calculations. There remain to be determined the pole arc, the air gap, the dimensions of the armature teeth, the length and radial depth of the armature iron, the dimensions of the field poles, and the dimensions of the field yoke. Preliminary values of these quantities may be arrived at as indicated below:

Pole Arc.—The pole arc is usually from 60 per cent to 65 per cent of the pole pitch. The face of the pole is chamfered so as to make the clearance between it and the armature face greater at the tips than in the center. Usually the length of gap at the tips is 1.5 times the length at the center. This is done on revolving-field machines by making the face of the pole an arc of a circle whose radius is less than the radius of the armature. (See Fig. 89.)

Air Gap.—The minimum clearance between armature core and field poles (or air gap at the center of the pole) in modern machines has usual values as given in table XXIV. The average length of air gap resulting from the chamfer of the pole face is about 25 per cent greater than this minimum.

Armature Teeth.—The dimensions of the armature teeth depend upon the number and dimensions of the slots, which in turn must be such as to contain the necessary conductors. Another important limitation to the width of the teeth, however, is the value of the magnetic flux density therein. Let z = pitch of slots, a = pole arc, g = average radial depth of air gap. Then the effective number of teeth under each pole is $(a + 2g) \div z$. The effective width of the tooth is its width at a section one-third

the distance from its minimum width towards its maximum width. The effective axial length of the teeth is approximately equal to 0.75 of the axial length of the armature L determined above. From these data the effective cross-section of the teeth perpendicular to the flux lines is determined, and this divided into the total flux per pole, ϕ, gives the average flux density in the teeth. The average flux density in the teeth in modern alternating-current generators ranges from 90,000 to 110,000 lines per square inch. The higher values are for 25-cycle machines.

Armature Core.—The core is usually built up of sheet-steel punchings, 14 mils thick, the punchings having acquired previously a thin coat of oxide. Ventilating ducts from $\frac{3}{8}$ to $\frac{1}{2}$ in. in width (measured along the shaft) are usually provided, these ducts dividing the core into sections from 2 to 3 ins. thick (measured along the shaft). The effective axial length of the armature iron is therefore the total length of the armature core less the space occupied by the air ducts and insulation between punchings. This latter is about 10 per cent of the net length after deducting the space occupied by the air ducts.

One-half the total flux per pole passes through each section of the armature core. The flux density in the armature core of modern alternating-current generators ranges from 50,000 to 70,000 lines per square inch. Let B_a = the value of the flux density chosen, l = effective axial length of armature iron, ϕ = flux per pole. Then the necessary radial depth of the armature core is $h = \phi \div 2B_a l$.

Field Poles.—The axial length of the pole face, f, is about 0.5 in. less than L the length of the armature. The pole arc, a, has already been determined. The width, b, of the magnet core is set by the flux the core must carry and the magnetic density, which is customarily between 90,000 and 100,000 maxwells per square inch. The poles may be of cast steel, but usually are constructed of laminations of steel .025 in. thick, punched to the proper shape. The effective length of such a construction is 95 per cent of the total length.

The flux in the magnet core is greater than the useful flux in the armature because the leakage flux, as well as the useful flux, passes through the pole core. The ratio of the total or gross flux to the useful flux is known as the leakage factor (v). A method for calculating the exact value of the leakage factor is given on

page 163, but it is not necessary to resort to that refinement at this stage of the design as the proportions of the magnetic circuit have not yet been definitely chosen. For this stage of the design, the following approximate values of the leakage coefficient may be used.

TABLE XXVII

Pole Pitch, Inches	Leakage Coefficient v
6	1.4
12	1.3
18	1.25
24	1.20
36	1.15

A good width of the magnet core would then be:

$$b = \frac{v\phi}{.95f \times 90{,}000}.$$

The width, b, is less than the pole arc, a, in order to allow a space for the exciting coil or spool. The radial length of the winding space, c, can be made about equal to the pole arc, a, for preliminary purposes and then adjusted later to accommodate the field spool. The radial depth of the pole shoe at the tip (c_2) is governed by the necessity of holding the spools against the centrifugal force. In slow-speed machines (5000 ft. per minute) it never needs to be greater than $\frac{1}{2}$ in. The chamfer (c_3), as stated before, is usually about 50 per cent of the air-gap length.

Field Yoke.—The field yoke consists of a ring of cast steel having a rectangular cross-section. The outside diameter of the ring is less than the diameter of the armature at face, by twice the radial height of the pole pieces and air gap. That is, it is equal to $D\text{-}2(c + c_2 + c_3 + g)$.

Its axial length, e, is an inch or two greater than the length of the pole piece in order to allow a good seat for the coils or spools. In a cast-steel yoke the magnetic density should be between 70,000 and 80,000 maxwells per square inch. The total flux in any one part of the yoke is one-half that in the pole piece, as the

flux divides at this point; the radial depth of the yoke should therefore be:

$$d = \frac{v\phi}{2e \times 70{,}000}.$$

Frequently it is necessary to make the depth greater than this to provide sufficient mechanical strength or to provide fly-wheel effect.

Leakage Factor (v).—The leakage factor, i.e., the ratio of the total flux produced by the field m.m.f. to the useful flux that enters

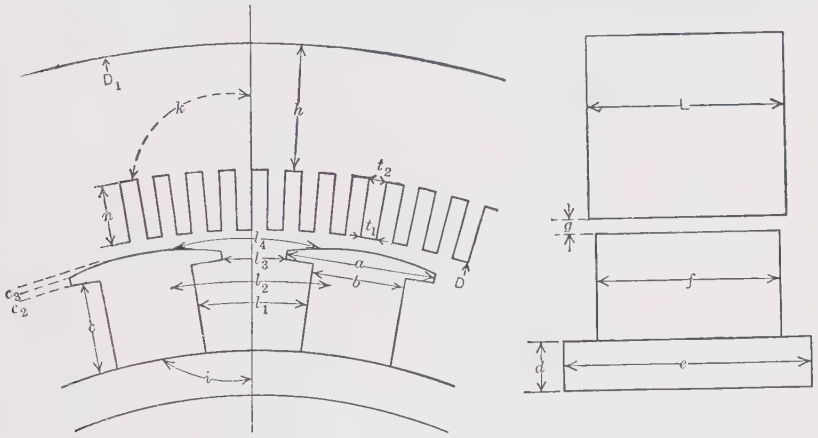


Fig. 89.—Dimensional drawing of machine showing cross-section and length of magnetic circuit.

the armature, may be determined by calculating the permeance of the path of the armature flux and the permeances of the paths of the various leakage fluxes. The sum of the permeances of the main and the leakage paths divided by the permeance of the main path is the value of the leakage factor, v.

Referring to Figs. 89 and 90 the average radial length of the air gap is approximately $1.25g$ and the reluctance of the air gap is therefore $\frac{1.25g}{af}$. The reluctance of the iron part of the magnetic circuit is about 20 per cent of that of the gap in 60-cycle machines and 40 per cent in 25-cycle machines; hence the approximate value of the permeance of the path of the main flux is:

$$P_0 = \frac{af}{1.5g} \text{ for 60 cycles and } P_0 = \frac{af}{1.75g} \text{ for 25 cycles.}$$

The leakage flux between poles emanating from or entering each side of a pole has a path of length, l_1, and a cross-section of cf square inches. The permeance of this path to the neutral plane midway between poles is $\frac{2cf}{l_1}$. There are two of these paths in multiple, one in each direction from opposite sides of each pole. The total permeance of these paths is $\frac{4cf}{l_1}$. If a uniform m.m.f. acted on this path at all points, the flux would be proportional to this permeance; but since the m.m.f. varies from 0 at the yoke to the full m.m.f. of a spool at the pole shoe, the average m.m.f.

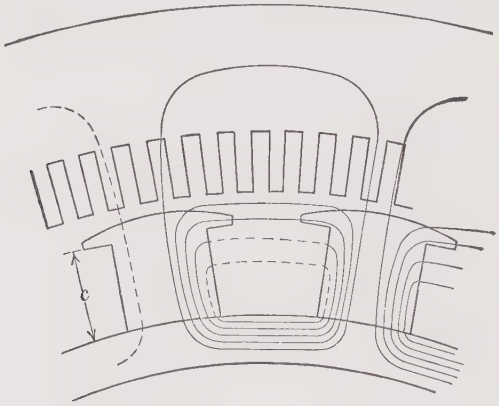


FIG. 90.—Paths of main or useful flux and the various leakage fluxes in the field.

is one-half of the m.m.f. per pole. The leakage across the path may therefore be said to be proportional to the total m.m.f. per pole and to one-half of this permeance. Hence the effective permeance of this path is:

$$P_1 = \frac{2cf}{l_1}.$$

The same reasoning applies to the flux which leaks out from the end surface bc (Fig. 89) of the poles, giving the effective permeance of this path

$$P_2 = \frac{2bc}{l_1 + \frac{b}{2}} = \frac{2bc}{l_2}.$$

The leakage between the pole tips is due to the total m.m.f. per pole, and the effective permeance is the same as the real permeance:

$$P_3 = \frac{4c_2 f}{l_3}.$$

The leakage at the faces $c_3 f$ the poles along the chamfer is also due to the total m.m.f., and the permeance of this path:

$$P_4 = \frac{4c_3 f}{l_4},$$

$$l_4 = \text{pole pitch} - \tfrac{1}{2}a.$$

The leakage factor is then:

$$v = \frac{P_0 + P_1 + P_2 + P_3 + P_4}{P_0}$$

For rough values, see page 162.

Predetermination of Performance

From the preceding calculations a preliminary drawing of the machine may be laid out. (Fig. 89.) The next step is to calculate the performance, that is, the excitation, regulation, efficiency, and heating. The calculations for a two-phase and a three-phase machine are very similar, and are given in the following paragraphs. The calculations for a single-phase machine are quite different and must be treated separately.

Magnetization Curve.—The first step is the calculation of the magnetization curve, a curve showing the relation between the voltage per phase (or between terminals), with no load and at normal speed, and the field excitation expressed either in ampere-turns per pole or in field current. (Fig. 91.) The designing engineer prefers to use voltage per phase and ampere-turns per pole. To do this the useful flux per pole corresponding to any given value of the no-load voltage is first calculated and then the field ampere-turns per pole required to produce this flux are determined. This is also done for a sufficient number of other values of voltage to give a magnetization curve up to 20 per cent above the rated voltage of the machine.

The useful flux per pole is given by the relation explained in a previous section (page 143):

$$\phi = \frac{E \times 10^8}{4.44 k_1 k_2 f S}$$

E is taken first as the voltage per phase corresponding to the rated voltage of the machine.

Ampere-turns.—The ampere-turns required to produce the useful flux, ϕ, may now be calculated by the following systematized procedure:

TABLE XXVIII

CALCULATION OF THE MAGNETIC CIRCUIT

SALIENT POLE A.C. GENERATOR

1	2	3	4	5	6	7	8
Part	Material	Flux	Cross-section	Flux Density B	Ampere-turns per Inch	Length of Path	Ampere-turns per Pole
Field yoke		$.5v\phi$	de			i	
Pole piece		$v\phi$	$0.95bf$			c	
Air gap		ϕ	af		$0.313B$		
Arm. teeth		ϕ	Tl_0l_t			n	
Arm. core		$.5\phi$	hl_t			k	
Grand total ampere-turns per pole							

SYMBOLS

All dimensions in inches.

a = pole arc;
b = width of pole core;
c = radial length of pole core;
d = radial depth of yoke;
D = diameter of armature at gap;
D_1 = diameter of armature at back;
e = axial length of yoke;
f = axial length of pole;
g = minimum length of air gap, radial clearance;
h = radial depth of armature core;

i = length of flux path in yoke;
k = length of flux path in armature core;
L = total length of armature core;
l_i = effective length of iron in armature core;
n = depth of armature slots;
z = pitch of armature slots at gap;
T = teeth per pole carrying flux;
t_0 = effective width of armature tooth;
w = width of armature slots;
v = leakage factor.

Column 1 gives the name of each separate part of the magnetic path. It should be noted that the m.m.f. of one pole is required to overcome only the reluctance of one pole piece, one air gap, once through the teeth, and half the length of the paths in the yoke and core; that is, the calculation is for one-half of the complete magnetic circuit.

Column 2 gives the materials used in the different parts so that the appropriate magnetization curves will be used.

Column 3 gives the total flux passing through each particular part of the circuit.

Column 4 gives the cross-section in square inches of each part of the circuit, the symbols referring to dimensions taken from a drawing similar to Fig. 89.

Column 5 gives the magnetic densities in maxwells per square inch, obtained by dividing the respective fluxes in Column 3 by the areas of Column 4.

Column 6 contains the specific m.m.f. in ampere-turns per inch for the particular material and density. The values are taken from appropriate magnetization curves as given in Chapter I. For the air gap the value of the ampere-turns per inch is always obtained by multiplying the density by the constant 0.313.

Column 7 gives the lengths of the respective paths in inches as shown on the drawing.

Column 8 gives the total ampere-turns required for each part and is obtained by multiplying the respective values in Column 6, by those in Column 7. The sum of all the values in Column 8 is the excitation, in ampere-turns, required in each field spool or coil to establish the assumed value of useful flux. This corresponds to a definite terminal voltage for conditions of no load.

Field Yoke.—The field yoke carries one-half the gross flux of the pole core. As it is usually of solid cast steel its cross-section is $d \times e$, and the density is obviously $\frac{v\phi}{2de}$. The value of ampere-turns per inch for this density is obtained from magnetization curve of the proper material. The mean length of path in the yoke should be obtained by laying it out on a drawing as in Fig. 89. This may be approximated by taking one-half the distance between pole centers along the outer perimeter of the yoke.

Field Pole or Magnet Core.—The density in the pole is not uniform throughout its whole length, on account of the leakage. But as the greatest amount of leakage is at the pole tips no great error (and that error on the safe side) is made by assuming that the flux in the pole is $v\phi$ throughout and calculating the density on this basis. The length of path is usually taken as c, the length of the core proper, although this is not strictly correct, but the density in the pole shoe of length c_2 is so low that the excitation required for this part is negligible.

Air Gap.—The value of the flux in the gap is taken the same as the useful flux, ϕ. The purpose of the pole shoe is to make the area of this path as great as possible and larger than the section of the pole core. It is always rectangular in section $(a \times f)$ and the cross-section of the path of the flux in the air gap is taken to be the same as this. For greater accuracy the refinements explained in the calculation of the magnetic circuit of a d.c. machine may be applied. (See page 80.)

Some designing engineers take these refinements into account empirically by multiplying the ampere-turns, found by the simple calculation, by a constant (0.9 to 1.1) determined by tests on similar machines already built.

The length of the path in the air gap is taken as the mean of the gap length. The air gap is a minimum at the center of the pole arc and a maximum at the tips of the poles. This maximum is usually twice the minimum, and the shape such that the average length of the path in the gap is 1.25 times the minimum. (It should be noted that this is the clearance between the arcs of two circles of different radii and different centers.)

Armature Teeth.—(For details see under d.c. generator, page 83.) The flux is assumed to spread out from the pole tips at an angle of 45°, thus reaching the armature along an arc of length

$(a + 2g)$. The teeth under a pole carrying flux would be $T = \frac{a + 2g}{z}$. The effective length of iron in the core is $l_i =$ (L − space for ducts) × 0.9. The effective cross-section of each tooth is taken at a point one-third the distance from the narrowest end towards the widest and is $t_0 = \frac{2t_1 + t_2}{3}$. Thus the cross-section of path in the teeth is Tt_0l_i. The material is usually silicon steel and the length of path is n, the same as the depth of slot. If the apparent density thus obtained is greater than 120,000, the exact density and m.m.f. should be calculated as on page 85.

Armature Core.—The flux divides in the core, there being two paths in multiple. The core is made up of laminations in the same piece with the teeth, and has the usual air ducts. The effective length is l_i, the same as for the teeth. The other dimension is the radial depth of the core, h, and the cross-section is hl_i. The length of path, k, is approximated by sketching on a drawing. It is a little greater than one-half the pitch of the poles at this diameter.

Magnetization Curve.—A number of points, for various voltages up to 120 per cent of rated voltage, are calculated and these are plotted between "voltage between terminals" as ordinates and "ampere-turns per pole" as abscissae, as shown in Fig. 91.

Armature resistance.—The length of wire in one phase of the armature winding is calculated from the mean length of one turn and the number of turns in series. A turn consists of two straight portions embedded in the slots and projecting an inch or so beyond, and the end connections at the two ends. The length of the straight sides is $(L + x)$ where x is the excess straight length of the coil over the length of the core L ($x = 1''$ or $2''$, see Fig. 93). The mean length of a turn is (see page 63 of d.c. design).

$$mlt = 2(L + x) + \frac{10Dk}{\text{poles}} \qquad \text{inches,}$$

where k is the fractional pitch expressed as actual span of coil in slots divided by the number of slots per pole.

The number of turns in series per phase, S, has been determined. Let q be the cross-section (sq. in.) of one physical conductor, m be the number of such conductors in multiple in one phase, and

.01 be the resistance at 75° C. of 1000 ft. of copper conductor having a cross-section of one square inch. It is assumed, by convention, that 75° C. is the operating temperature. Then the actual resistance of one phase of the armature winding is:

$$r = \frac{.01S \times (mlt)}{12{,}000qm}.$$

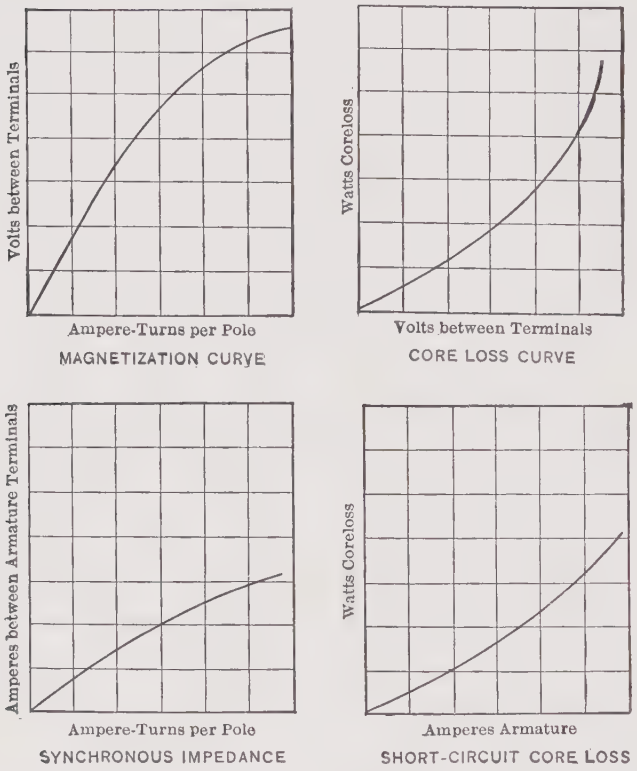


FIG. 91.—Example of a "no load" magnetization or saturation curve and other usual tests.

If pressed cable is used, the length of each strand is 7 per cent greater than the actual length of the conductor and the resistance must be increased by this percentage. The resistance thus calculated is the ohmic or d.c. resistance. The effective resistance to alternating current is about 15 per cent greater, because of the eddy currents and load losses.

In a Y-connected machine the resistance between terminals is twice the resistance per phase, and in a Δ-connected machine the resistance between terminals is two-thirds of the resistance per phase.

For reasonable heating, the I^2R lost in each coil at rated load should be about 0.25 watt per square inch of surface of the coil itself. (Surface = $[u + 2w) \times$ (m.l.t.)].

Reasonable values for the resistance of the armature of salient-pole generators are given in the table on page 183. The resistance is expressed as a percentage of phase voltage $\frac{100Ir}{E}$.

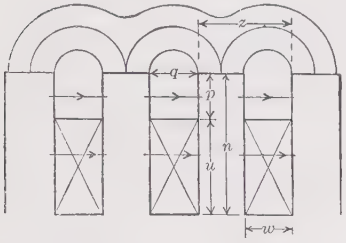


FIG. 92.—Diagram of armature slots showing tooth-tip flux and dimensions used in the calculation of slot leakage flux.

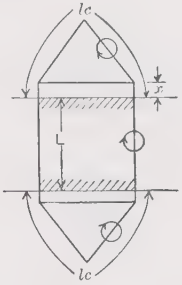


FIG. 93.—Plan of a coil showing dimensions used.

Armature Leakage Reactance.—The current in the armature conductors sets up a magnetic flux which encircles a group of coil-sides of one pole and phase, flowing along the periphery of the armature and in the air gap, through the teeth, back along the armature core, and through the teeth again. (See Fig. 92.)

A similar encircling of a group of coils by leakage flux also takes place around the end connections in the air. (See Figs. 92 and 93.)

For purposes of mathematical analysis, all of these fluxes are divided into five groups as follows:

1. That crossing the slot and passing through the windings. In this case each line of flux interlinks a different number of conductors and the average interlinkages must be calculated.

2. That crossing the straight portion of the slot above the windings and between the parallel sides of the slot. All of this flux interlinks all of the conductors in one slot.

3. (If the slot has a constricted opening or overhanging teeth.) That passing across the narrowed portion of the slot at the tooth tip and interlinking all the conductors in that slot. (Special case, see Fig. 38, page 71.)

4. That passing along the air gap from tooth tip to tooth tip. This is known as the belt leakage as it is caused by the combined m.m.f. of the several slots and some of it interlinks several slots.

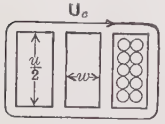


FIG. 94.— Cross-section of a phase belt of 3 coilsides showing perimeter followed by magnetic flux.

5. That surrounding the end connections of a group of coils per pole per phase and interlinking all the coils of this group. This is known as the end-connection leakage.

The first four fluxes flow partly in air and partly in iron, but the reluctance of the part of the path in iron is so small compared to that in air that it may be neglected.

[1] The first three fluxes are lumped together and expressed as an equivalent flux which gives the same interlinkages with the conductors in the slot. (See deduction under "Reactance of D.C. Machines.) This equivalent flux per ampere conductor is:

$$P_s = 3.2L\left(\frac{u}{3w} + \frac{p}{q}\right).$$

With partly closed slots, the more elaborate formula given on page 71 must be used.

The flux per ampere conductor on the tooth tips and in the gap set up by the s_1 slots per pole per phase is calculated by a method similar to that used for d.c. machines, except that, since several slots are involved, the permeance of the path across each slot is calculated by integrating between q as the minimum length of path and the semicircle of length $\frac{\pi z}{2}$ as a maximum. Then the effect of the neighboring slots is taken into account by an empirical constant, A. This gives the formula for the tooth-tip leakage flux per ampere conductor:

$$P_t = \left(2.35 \log_{10} \frac{\pi z}{2q} + A\right)L.$$

The constant A takes account of the fact that in addition to

[1] See Arnold, Wechselstrom Technick, Vol. I.

the flux per slot there is a belt of flux interlinking several slots. The value of A depends upon the number of slots per pole per phase (s_1) and has values thus:

$s_1 = 1$	2	3	4	5	6	7	8	9	12	15	18
$A = 0$	.35	.7	1.1	1.7	2.1	2.7	3.2	3.8	5.8	7.1	8.8

The flux per ampere conductor around the end connections is calculated in the same way as for d.c. machines, except that the coils lie in groups of s_1 (the slots per pole per phase) alongside each other and all s_1 coils carry the same current, and hence react upon each other magnetically, each contributing to the m.m.f. of the group.

If the winding is of the single-layer type (one coil-side per slot) these end connections have the same number of conductors in each as are in the slot. But in the modern two-layer winding with two coil-sides per slot, one coil-side goes to the right as it leaves the slot and the other coil-side turns to the left. Hence the end connections have only half as many conductors as the slot and the equivalent flux is one-half that of a single-layer winding, thus:

$$P_c = 1.17\, s_1 l_c \log_{10} \frac{2l_c}{U_c} \text{ for a single-layer winding,}$$

$$P_c = 1.17\, \frac{s_1}{2}\, l_c \log_{10} \frac{2l_c}{U_c} \text{ for a double-layer winding,}$$

in which l_c is the length of end connections at one end and U_c is the perimeter of a group of s_1 coil-sides (Figs. 93 and 94).

$$l_c = \frac{\text{m.l.t.}}{2} - L,$$

$$U_c = u + 2s_1 w.$$

Let p = number of poles;

c = effective conductors per slot = $\dfrac{\text{conductors per slot}}{\text{paths in parallel}}$;

f = frequency;

k_1 = winding distribution factor;

k_2 = pitch factor.

then the reactance in ohms per phase is:

$$x = 2\pi f p s_1 c^2 (P_s + P_t + P_e) k_1 k_2 10^{-8}.$$

For purposes of comparison the percentage Ix drop is more significant. This is $\frac{Ix}{E} \times 100$ in which I and E are the current per phase and voltage per phase respectively, and x is the reactance per phase. This percentage is usually between the limits of 8 to 15 per cent in 25-cycle machines and of 10 to 20 per cent in 60-cycle machines.

Armature Reaction Ampere-turns per Pole.—Let S = number of turns in series per phase; I = effective value of armature current per phase at rated load; p = number of poles; and k_1 and k_2 the winding distribution and pitch constants. Then the armature ampere-turns per pole effective in producing the armature reaction m.m.f. are:

$$n = \frac{\sqrt{2} S I k_1 k_2}{p} \text{ for a two-phase machine,}$$

$$n = \frac{1.5\sqrt{2} S I k_1 k_2}{p} \text{ for a three-phase machine.}$$

For usual values for a salient-pole machine, see page 146.

Synchronous Reactance.—The effects of the armature leakage reactance and of the armature reaction upon the terminal voltage of a generator at a given field excitation are of like character, since the component of voltage induced in the armature winding due to each of these causes is in quadrature with the current. The **synchronous reactance** of a generator is the equivalent reactance which would produce the same voltage effect as the combined effect of the leakage reactance and armature reaction. Assume the machine operating on short circuit, giving the current I per phase (Fig. 95). Then the armature reaction m.m.f. will be n, in phase with I. The leakage reactance drop will be Ix at right angles to I. The effect of the armature resistance, Ir, is very small and may be neglected for simplicity. Then, since the axes of the armature winding and of the field winding are at right angles when the generated e.m.f. is a maximum, the flux to produce Ix and the field m.m.f. (m) to produce the flux will be in the direction of ϕ, and the m.m.f. may be represented by m. Thus the total

m.m.f. on the field required under these conditions will be $F = m + n'$ where n' is the m.m.f. required to overcome the back-m.m.f. of armature reaction (since, when the current lags 90° behind the generated e.m.f., the armature m.m.f. is opposed to the field m.m.f.). To be logical, n' should be greater than n by the leakage coefficient, v. Now, if the short circuit were removed and the field strength held constant, a voltage E_0 would be generated. The synchronous reactance is the name given to the ratio of this voltage to the current, $x_0 = \frac{E_0}{I}$. It is a measure of the apparent voltage consumed in the armature.

The value of m corresponding to Ix must be found on the magnetization curve of the machine, as must also the voltage, E_0, corresponding to the excitation F ampere-turns per pole.

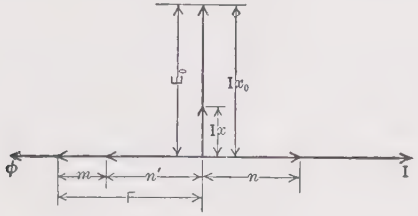


FIG. 95.—Vector diagram of synchronous impedance by m.m.f. method.

Regulation.—The regulation of a generator is defined as the ratio of the difference in terminal voltage at no load and at full load, to the voltage at full load, for a constant field excitation corresponding to full-load conditions. Expressed as a percentage, the regulation is $100(E_0 - E) \div E$ where E is the terminal voltage at full load and E_0 is the terminal voltage at no load for the same field excitation.

Several different methods may be employed for calculating the excitation required to give rated volts at the terminals when the generator is loaded. The following are the most usual.

Magnetomotive Force Method.—This method is only approximate, giving optimistic results, but is valuable as bringing out clearly all the factors which enter into the phenomenon. It is therefore of educational importance.

Let E (see Fig. 96) be the terminal voltage per phase at the given load; I the current per phase at the given load; cos ϕ the

power factor of the load; r the resistance per phase; and x the leakage reactance per phase of the generator. Then Ir, parallel to I, will be the resistance drop inside the armature, and the vector combination of E and Ir gives E_r (not shown), which is the real generated voltage in the armature under load conditions. All other voltages are hypothetical ones representing fluxes and m.m.f.'s which exist but produce components of voltage that neutralize each other.

Let Ix, perpendicular to I, be the drop in voltage due to leakage reactance. The vector combination of E, Ir and Ix gives E_v the **virtual voltage** (Steinmetz). This is a measure of the gross flux existing:

$$E_v = \sqrt{(E \cos \phi + Ir)^2 + (E \sin \phi + Ix)^2}.$$

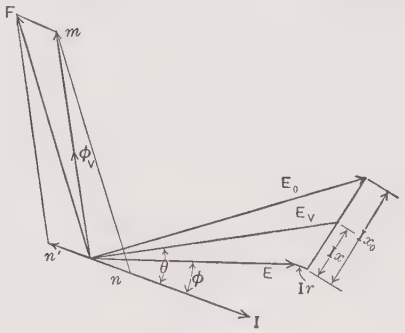


FIG. 96.—Vector diagram for regulation with inductive load by m.m.f. method.

The flux required to produce E_v will lead E_v by 90°, as ϕ_v and the m.m.f. to produce ϕ_v will be in phase with ϕ_v as m. m is taken from a no-load magnetization curve for the value of voltage corresponding to E_v, and represents the magnetomotive force required in the field to produce the actual flux.

The armature reaction m.m.f. is in time, phase, and position with I, and may be represented by n. This is a reaction m.m.f. and must be overcome before any flux can be produced. Let n' be the required impressed m.m.f. in the field capable of overcoming the reaction n; then n' will be roughly n times the leakage factor.

In order to produce the desired flux, the total magnetomotive force in the field must be of such magnitude as to be able to overcome n and leave available m to produce flux. As n' and m are not in the same phase, hence not in the same direction, but differ

by an angle equal to $90 - \theta$, it is obvious that the required gross m.m.f. of a field represented by F is a resultant of m and n' and is numerically equal to:

$$F = \sqrt{m^2 + n^2 + 2mn \cos (90 - \theta)}$$

$$= \sqrt{m^2 + n^2 + 2mn \sin \theta},$$

in which θ is the angle between the virtual voltage and the current and the sine of θ is equal to:

$$\sin \theta = \frac{E \sin \phi + Ix}{E_v}.$$

The above analysis determines F, the magnetomotive force in the field, in ampere-turns per pole, which is required to overcome all of the internal reactions in the machine under load I, and still give the voltage E at the terminals of the machine to the load. If now the field excitation were held at a value equal to F and the load were removed, we should then have open-circuit or no-load conditions and the terminal voltage would rise to a value E_0 which would be found on the no-load magnetization curve for an excitation F, and which would be at right angles to F in time and space phase; this value is represented by the line E_0 in the diagram. This is known as the nominal generated electromotive force of the machine and its relation to E, the terminal voltage with load, is what determines the regulation of the machine.

If the magnetic parts of the machine are not saturated the numerical value of E_0 will be such that it will come graphically at the intersection of Ix prolonged and E_0. Then the vector between the real generated electromotive force and E_0, represented on the diagram by Ix_0, is known as the synchronous reactance drop in the machine, as the upper part is really the voltage effect caused by the armature reaction. It will be noted that this voltage is at right angles to n, the m.m.f. which produces it. To be more specific, we say that the m.m.f. n' produces a certain flux in the same phase with it and that this flux produces the voltage at right angles to the current, while the armature m.m.f., n, produces an equal flux in the opposite direction which produces an electromotive force just opposite and equal to the previous one.

From the above calculation the field excitation required for a given terminal voltage for the load current I, and load power

factor cos ϕ may be determined, which is of great practical importance. Further, by ascertaining from the magnetization curve the voltage corresponding to F, and making the comparison with the terminal voltage, the regulation may be determined, but this is no longer of great practical importance.

If the load is anti-inductive, that is, if the current I leads the terminal voltage E, it will be found that the values of E_0 and F become smaller and for certain anti-inductive power factors E_0 and F may be less than for no-load conditions. In this case E lies below I and E sin ϕ becomes a negative quantity. When the

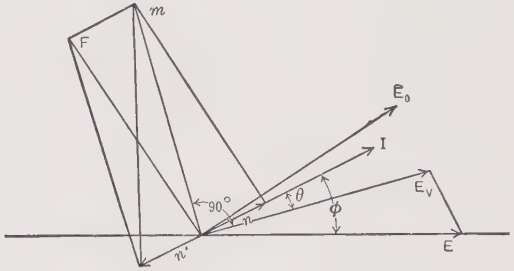


Fig. 97.—Vector diagram for regulation with anti-inductive load by m.m.f. method.

negative value of E sin ϕ becomes greater than Ix then sin θ and θ become negative and the term $2mn$ cos $(90 - \theta)$ becomes negative (see Fig. 97).

Electromotive Force Method

Let E = terminal voltage per phase at the given load;
r = alternating-current resistance per phase in ohms;
Ix_0 = volts per phase on magnetization curve corresponding to the field excitation required to send the assumed value of current through the armature on synchronous impedance test;
I = the current per phase;
cos ϕ = the power factor for the assumed load.

Then the voltage per phase E_0 at no load corresponding to the

correct excitation for the assumed load is the vector sum of E, Ir and Ix_0, or

$$E_0 = \sqrt{(E \cos \phi + Ir)^2 + (E \sin \phi + Ix_0)^2}.$$

The regulation is the ratio of voltages as before and the required field excitation, F, is taken from the no-load magnetization curve for the point corresponding to E_0 on the magnetization curve. This is much the simplest of all methods and is quite generally used for qualitative results, but is not very accurate and generally makes the regulation appear poorer than it really is; in other words, it is a pessimistic method.

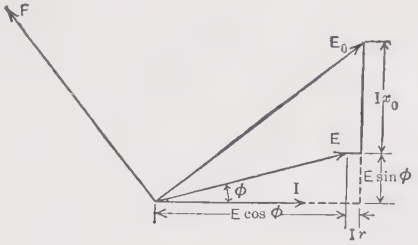


FIG. 98.—Vector diagram for regulation with inductive load by voltage method.

Field Winding.—The field winding is laid out for the value of excitation, F, in ampere-turns per pole for some specified load condition, such as rated kv-a. and 80 per cent inductive power factor. It is a wise precaution to design the field winding so that this value of excitation may be obtained at some exciter voltage less than maximum, thus at 100 volts if a 125-volt exciter is to be used, in order to have a margin of safety.

The proper cross-section of field conductor depends upon the required excitation, F, the assigned value of exciter voltage and the mean length of a field turn, and is entirely independent of the allowable loss in the field. The latter quantity is governed entirely by the number of turns used.

The mean length of turn may be obtained from a drawing of the pole piece, but is fairly accurately approximated by assuming that, since the minimum length of turn is $2(b + f)$, the average or mean length approximates m.l.t. $= 2(a + f)$ inches.

Let p = number of poles;
E_x = assigned voltage for excitation F;

.01 = specific resistance at 75° C.;
q = the cross-section in square inches of the desired field conductor.

Then

$$q = \frac{.01 \times (\text{m.l.t.}) \times F \times p}{E_x \times 12{,}000}.$$

The number of turns is set by the current it is proposed to use, and the value of this current is determined in several ways.

1. It may be decided to operate the field at 1000 to 1500 amp. per square inch density. In this case the field current would be $1000q$ or $1500q$.

2. It may be decided that the individual field coils can radiate a certain number of watts. In this case the suitable field current is this wattage multiplied by the number of poles and divided by the assigned value of E_x.

3. From considerations of the efficiency (see table, page 183) of the machine as a whole, the power to be lost in the field may be limited to a certain value. In this case the appropriate field current is found by dividing this power in watts by E_x.

Let I_f be the value of the field current found by any one of these methods, then the required number of turns on each pole or spool is $t = F \div I_f$. In small machines ordinary double cotton-covered round wire is used, and in some special cases square wire. In large machines requiring a field conductor having a cross-section of 0.10 sq. in. or greater, it is customary to use flat copper strip of dimensions 1 in. by 0.10 in. or greater. This is wound on edge with a strip of paper fiber .005 in. thick between turns. The field spool is then laid out to fit around the pole core of dimensions $b \times f$ and with a height equal to c, which, however, may be readjusted at this stage in the design if necessary. This may be necessary in order to make room for sufficient turns to keep the loss in the field winding within reasonable limits.

The resistance of the entire field winding at 75° C. will be:

$$R_f = \frac{.01 \times \text{m.l.t.} \times t \times p}{12{,}000q},$$

and the total loss in the field is $I_f^2 R_f$ in which I_f is the field current for the specified load assumed.

The outside perimeter is now calculated by estimating the depth of winding and spool δ. Then the cylindrical radiating surface of one spool will be

$$A = 2c(f + b + 4\delta).$$

The watts per square inch on the field coils will be

$$w_f = \frac{I_f^2 R_f}{pA}.$$

The rise in temperature of the field coils of revolving fields with strip windings will be about as in the table.

TABLE XXIX

Peripheral Speed, Feet per Minute	Temperature Rise, ° C. per Watt per Square Inch
1,000	55
2,500	40
5,000	25
10,000	15

On wire-wound coils the rise will be greater

Losses

Coreloss.—The coreloss, consisting of hysteresis and eddy-current losses may be treated in the manner given for d.c. machines, page 103. It is usual to construct the armature of sheets of silicon steel of a thickness of .014 in., having a value of η of from .001 to .002 and a value of ϵ of from .00005 to .0001.

$$\text{Total loss in core} = V_c C_h f + V_c C_e f^2.$$

$$\text{Total loss in teeth} = V_t C_h f + V_t C_e f^2,$$

where V_c, V_t = volume in cubic inches of core and teeth respectively;

C_h = constants from hysteresis loss curve for the respective densities;

C_e = constants from the eddy-loss curve for the proper thickness and respective densities;

f = frequency.

The magnetic densities used should be those corresponding to $(E + IR)$ the **real generated voltage** as mentioned on page 176. These densities vary with the load; but since the IR drop is so very small (about 1 per cent), and the coreloss cannot be calculated as closely as that, it is reasonable to calculate the coreloss for densities corresponding to the rated voltage and consider it constant.

The total coreloss may be somewhat greater (10 to 25 per cent) than the value given by these calculations, if the air gap is short or the slots are wide.

Friction and windage depend in magnitude upon the details of the physical or mechanical construction, the amount of induced ventilation, and the speed. It is impossible to give a method of predetermining these quantities which may be applied to various designs and makes. Each manufacturer has an empirical formula for each line of machines.

Table XXX, page 183, gives usual values which range from 2.5 per cent in a 100 kv-a. machine down to 0.5 per cent in a 10,000 kv-a. machine. Some machines are designed with devices to produce a great deal of ventilation, which increase the friction loss.

Excitation Loss.—If the generator is to be operated at a constant terminal voltage for all loads it is necessary to calculate the excitation for several loads, determine the field current for each load $I_f = F \div t$, and plot the Field-Compounding curve with field current as ordinates and armature currents as abscissae. This will start at a definite field current for no load and curve upward with increasing load. Then the $I_f^2 R_f$ for any load may be calculated. In a separately excited machine the losses in the field rheostat are not chargeable against the machine.

Load Loss.—When a current flows in the armature conductors a local flux is set up which causes eddy currents in those conductors, if they are not well subdivided, and in the surrounding iron, as well as hysteresis in the surrounding iron. This loss is kept within reasonable limits by using stranded conductors in the armature, minimizing the leakage flux and keeping the end connections from coming into close proximity to masses of metal.

It is almost impossible to calculate this loss and very difficult to measure it. The A. I. E. E. Standards recognize the possibility of values up to 1 per cent of the rating. In predeterminations from the design of a machine it is taken into account by

assuming the armature copper losses to be increased by from 15 to 50 per cent. This can be done by assuming the armature resistance to be increased by a given percentage, as the load loss varies as the square of the current in the same way as the armature copper loss.

Armature Copper Loss.—This is calculated for a temperature of 75° C., and the d.c. resistance per phase is increased by a coefficient (1.15) to give the effective resistance in order to allow for the load loss. Let R = effective resistance of armature per phase, n = the number of phases, and I = current per phase in armature; then the total loss in the armature winding is nI^2R.

Efficiency.—The efficiency is found for any assumed output by calculating the excitation and armature loss corresponding to that output and adding to these the coreloss and friction, which are fairly constant irrespective of the load. To these four losses is added the output, to obtain the input.

For accurate results in machines having a high efficiency, it is best to use the formula.

$$\text{Efficiency} = 1 - \frac{\text{all losses}}{\text{Input}}.$$

The efficiency is a maximum for that load at which the armature loss is equal to the sum of the other three losses.

Usual values for the efficiency at full load and the division of the losses at full load for various sizes of generators are given in the following table. These values are merely indications and

TABLE XXX

EFFICIENCY AND LOSSES—USUAL VALUES

Salient Pole A.C. Generators

Rating, Kv-a.	Efficiency, Per Cent	Friction, Per Cent	Excitation, Per Cent	Core, Per Cent	Armature,* Per Cent
100	91	2.5	2.5	2.4	1.6
500	93.7	1.4	1.5	2.2	1.2
1,000	95	0.9	0.9	2.1	1.0
2,000	96	0.6	0.7	1.8	0.9
3,000	96.3	0.6	0.6	1.7	0.8
5,000	97	0.55	0.4	1.6	0.5
10,000	97.2	0.5	0.35	1.5	0.45

* Copper and load loss.

vary with the frequency, voltage, speed, power factor, etc. In general, a 60-cycle or a low-voltage machine will be likely to have a higher efficiency than a 25-cycle or a high-voltage machine of the same power.

Heating.—The armature of the usual a.c. generator is stationary and is cooled by air blown through it by the revolving field whose projecting pole pieces act as fan blades. The armature has two sources of heat in it, the coreloss and the I^2R in the windings. To facilitate cooling, the armature core is subdivided by ventilating ducts $\frac{1}{2}$ in. wide, spaced every 2 or 2.5 inches in the core and so arranged that the air flows radially outward from the air gap to the outside frame, incidentally passing the coils as they pass across the air ducts. The end connections of the windings project outward from the core and are also cooled by air from the revolving field, which in some cases has fan blades attached in such a way as to direct air against the end connections.

In the case of designs in which difficulty is expected from heating, the armature end connections are flared up radially as well as projecting axially from the core. This gives more room so that they will not be in such a compact mass and air may flow through the spaces between coils. It is obvious that the velocity of the air and its cooling effect are proportional to the peripheral velocity of the field structure which acts as a centrifugal fan. It is not possible to make an analysis of the cooling of a machine in which the empirical constants can be used indiscriminately for all types and makes, but the following method is strictly rational and the constants can be adjusted with experience to apply to various types. The principal variation is in the effect of the field structure in inducing the ventilation.

This method for calculating the rise in temperature of the surface of the core and the rise in the average temperature of the copper is the same as that given for d.c. machines (q.v.).

It is first assumed that the heat loss dissipated by the surface of the armature core is:

$$P_1 = \text{coreloss} + nI^2R\left(\frac{2L}{l}\right),$$

where I = armature current per phase;
R = effective resistance of armature per phase;
n = number of phases;

L = total length of armature core;
l = mean length of an armature turn.

The effective radiating surface of the armature core is:

$$A_1 = \pi D_1 L + \frac{\pi}{4}(D_1^2 - D^2)(2 + d);$$

D_1 = outside diameter of armature (Fig. 89);
D = inside diameter of armature;
d = number of air ducts.

The rise in temperature of the surface of the armature core above the temperature of the surrounding air (ambient temperature) is:

$$T_1 = \frac{P_1}{A_1} t,$$

where t varies from 30 for narrow machines ($L < 10$ in.) and high-speed machines ($V > 4000$) to 40 for long machines or slow-speed machines.

The permeance of the path of the heat energy from the copper through the slot insulation to the iron of the core is:

$$M = \frac{U_s L s}{k_1 d_1},$$

where U_s = perimeter of winding in the slot

$= 2(u + w)$;
s = total number of slots;
d_1 = thickness of slot insulation distance copper to iron, inch;
k_1 = specific heat resistance of material of insulation in degrees per watt per square inch of section and one inch length.

k_1 is 160 to 300 for cotton and linen impregnated with shellac and 250 to 350 for composite mica insulation. (See d.c. machines.)

The permeance of the path of the heat from the copper in the end connections through the insulation by conduction, and to the surrounding air by radiation and convection, is:

$$N = \frac{U_c l_c Z}{k_2 d_2 + k_3},$$

where U_c = perimeter of a coil in end connection
$= u + 2w$;
l_c = one-half the length of end connections of a coil
$= 0.5\, l - L$;
Z = total number of coil sides or twice the number of individual coils;
k_2 = same as k_1 if same material;
d_2 = thickness of insulation on end connections;
$k_3 = \dfrac{170}{1 + .00025V}$;
V = peripheral velocity of field, feet per minute.

Then the rise in temperature of the copper above the ambient temperature is:

$$T_c = \frac{P_2 + MT_1}{M + N},$$

where P_2 = the total copper loss in the armature, nI^2R;
T_c = the increase in the average temperature of the copper, as would be indicated by resistance measurements.

EXAMPLE OF DESIGN OF A SALIENT POLE ALTERNATING CURRENT GENERATOR

Item

1. Required: a three-phase, 5000 kv-a., 60-cycle generator for 150 r.p.m. to give 4000 volts. Capable of operating on an inductive load of 80% p.f. when giving 4000 kw.
2. For 60 cycles at 150 r.p.m. there must be 48 poles.
3. Assume a peripheral speed: v = 5000 f.p.m.

$$D = \frac{12 \times 5000}{\pi \times 150} = 127.5''.$$

Modify to 120″ for round numbers.
Then V = 4700 f.p.m.

4. Let $\rho = .65$ $\sigma = 600$ $B = 40{,}000.$ $k_1 \times k_2 = .86$ for preliminary values.

$$L = \frac{5000 \times 144 \times 10^9}{.65 \times 600 \times 40{,}000 \times 4700 \times 120 \times .86} = 95.$$

5. From table we should have 2 slots per pole per phase or $2 \times 3 \times 48 = 288$ slots. Each should be about 2.5″ deep. Minimum air gap should be 0.3125″.
6. Preliminary flux per pole.

$$\phi = \frac{\pi \times 120 \times 95 \times .65 \times 40{,}000}{48} = 19.5 \times 10^6.$$

7. Assume a Y-connection then, per phase

$$E = 2310. \qquad I = 720.$$

Approximate turns in series per phase:

$$S = \frac{2310 \times 10^8}{4.44 \times 60 \times .86 \times 19.5 \times 10^6} = 51.$$

8. Effective conductors per slot:

$$c = \frac{51 \times 2 \times 3}{288} = 1.06 \qquad \text{Say } 1.$$

9. Size of effective conductor: $U = 1500$.

$$q = \frac{720}{1500} = 0.48 \text{ sq. in.}$$

Copper per slot $= .48 \times 1 = 0.48$ sq. in.

10. Size of slots:

Pitch of slots at face $= \dfrac{\pi \times 120}{288} = 1.305''$.

Make slot width $= .65$.

Width of copper in slot $= .65 - .35 = .30''$ approximate.

Depth of copper in slot $= .48 \div .30 = 1.60''$.

After several trials choose strands of $(.15 \times .10)$ bare and $(.158 \times .108)$ with cotton insulation.

For convenience in making up coils make 4 paths in multiple per phase, hence 4 actual conductors per slot each consisting of 8 strands, 2 wide by 4 deep.

32 strands per slot divided into 4 conductors.

Layout of slot:

Width $= .158 \times 2 + .334 = .65''$.

Depth $= .108 \times 16 + 1.12 = 2.85''$.

11. Armature Winding:

288 coils each having two turns and a pitch of 5 slots instead of 6. 96 coils per phase in four parallel paths. 12 poles or 24 coils in series per path. $24 \times 2 = 48$ turns in series per phase.

Area of effective conductor $= .015 \times 8 \times 4 = .48$ sq. in.

12. Final value of flux per pole.

Assuming fractional pitch of $\frac{5}{6}$ $k_2 = .96$.

$$\phi = \frac{2310 \times 10^8}{4.44 \times 60 \times 48 \times .96 \times .96} = 20 \times 10^6.$$

13. Magnetic Circuit:

Armature Core: Total length = 95″.

Assume 32 air ducts of 0.5″ each.

Effective length = $(95 - 32 \times .5) \times .9 = 71$″.

Desire $B = 50{,}000$.

$$\text{then } h = \frac{20 \times 10^6}{2 \times 71 \times 50{,}000} = 2.82\text{″ trial.}$$

Outside diameter stator,

$$D_1 = 120 + 2 \times 2.85 + 2 \times 2.82 = 131.34, \text{ say } 132\text{″.}$$

$$\text{Final } h = \frac{132 - 125.7}{2} = 3.15.$$

Armature Teeth:

$$\text{Pole arc} = \frac{\pi \times 120}{48} \times .65 = 5.1, \text{ say } 5\text{″.}$$

$$\text{Teeth beneath pole} = \frac{5 + .625}{1.305} = 4.3.$$

Equivalent width of tooth,

$$t_0 = \frac{\pi \times (120 + 1.9)}{288} - .65 = .68\text{″.}$$

Air Gap: From table make minimum gap = .3125″, average = .39 and maximum = .625.

Make length of pole 94″, 1 inch less than armature core.

Pole Core: Pole made up of .025″ laminations stacked to 94″ giving an effective length of $94 \times .95 = 90$″. From table assume a leakage coefficient of $v = 1.35$.

Assume $B = 100{,}000$ in pole core,

$$\text{then } b = \frac{20 \times 10^6 \times 1.35}{90 \times 100{,}000} = 3\text{″.}$$

Assume radial length of core $c = 5$″ and total length of pole and shoe = 6″.

Field Yoke:

Flux per path = 13.5×10^6.

Desirable density, $B = 70{,}000$.

Make length of yoke, e longer than length of pole for seat of spools, say, 100″.

$$\text{then } d = \frac{13.5 \times 10^6}{100 \times 70{,}000} = 1.93'', \text{ say, } 2''.$$

May need more for mechanical strength.

14. Leakage Coefficient:

$$P_0 = \frac{5 \times 94}{1.5 \times .31} = 1010$$

$$P_1 = \frac{2 \times 5 \times 94}{4.5} = 209$$

$$l_1 = \frac{\pi \times 114}{48} - 3 = 4.5''$$

$$P_2 = \frac{2 \times 3 \times 5}{4.5 + 1.5} = 5$$

$$P_3 = \frac{4 \times .5 \times 94}{2.85} = 66$$

$$P_4 = \frac{4 \times .31 \times 94}{2.85 + 2.5} = 22$$

$$1312$$

$$v = \frac{1312}{1010} = 1.30 \text{ final.}$$

15. Excitation for rated voltage, no load.

1	2	3	4		5	6	7	8
Part	Mat.	$\phi \times 10^{-6}$	Dimensions	Area	$B \times 10^{-3}$	H	L	
F. Y.	C. S.	13	100×2	200	65	17	3.5	60
P. C.	L. S.	26	90×3	270	97	55	5	275
G.	.....	20	94×5	470	42.5	13,300	.39	5200
A. T.	L. S.	20	71×4.3×.68	206	97	55	2.85	158
A. C.	L. S.	10	71×3.15	224	45	6	4.3	26
Total amp.-turns per pole								5719

Item

16. Armature Resistance and Loss.

$$\text{Mean length of turn} = 2(95 + 1) + \frac{10 \times 120}{48} \times \frac{5}{6} = 213''.$$

$$\text{At } 75^\circ \text{ C. res.} = \frac{.01 \times 213 \times 48}{.12 \times 4 \times 12{,}000} = .0178 \text{ ohm.}$$

Effective res. $= .0178 \times 1.15 = .0204.$
Armature $I^2R = \overline{720}^2 \times .02 \times 3 = 31{,}700$ watts.
Coil surface $= 2(1 + .65) \times 213 \times 288 = 202{,}000$ sq. in.

$$\text{Watts per square inch} = \frac{31{,}700}{202{,}000} = 0.157, \text{ low.}$$

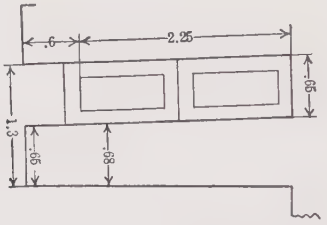


FIG. 99.—Cross-section of slot and coils showing dimensions for machine calculated as example.

17. Leakage Reactance: See Fig. 99.

$$P_s = 3.2 \times 95\left(\frac{2.25}{1.95} + \frac{.6}{.65}\right) = \qquad 630$$

$$P_t = \left(2.35 \log \frac{\pi \times 1.3}{2 \times .65} + .35\right) \times 95 = \qquad 158$$

$$l_c = \frac{213}{2} - 95 = 11.5''.$$

$$U_c = 2(1.125 + 2 \times .65) = 4.85''$$

$$P_c = 1.17 \times \frac{2}{2} \times 11.5 \log \frac{23}{4.85} = \qquad 9$$

$$\Sigma P = \qquad 797$$

$$x = 2\pi 60 \times 48 \times 2 \times 1 \times 797 \times .96 \times .96 \times 10^{-8} = 0.265 \text{ ohm}$$

$$\frac{720 \times .265}{2310} \times 100 = 8.25\%.$$

18. Armature Reaction.

$$n = \frac{2.12 \times 48 \times 720 \times .96 \times .96}{48} = 1400.$$

$$n' = 1400 \times 1.3 = 1820.$$

19. Synchronous Impedance.

Excitation for Ix (= 190 volts) = 450 amp.-turns.
n' = 1820
Synch. Imp. in. 2270 amp.-turns.

From a saturation curve (not shown) the voltage per phase corresponding to 2270 amp. turns is 950.

$$\text{Synchronous reactance} = \frac{950}{720} = 1.32 \text{ ohm.}$$

20. Excitation for 80% p.f.

$$E_v^2 = (2310 \times .8 + 720 \times .02)^2 + (2310 \times .6 + 720 \times .265)^2$$

$$E_v = 2430. \qquad \sin \theta = \frac{1570}{2430} = .645.$$

From a saturation curve (not shown) the excitation for 2430 volts is $m = 6300$.

$F^2 = (6300)^2 + (1820)^2 + 2 \times 6300 \times 1820 \times .645$

$F = 7100$ amp.-turns per pole to give 4000 volts to a load of 5000 kv-a. at a p.f. of 80% inductive.

21. Design field to be able to give 7100 amp.-turns with 200 volts of exciter and with reasonable heating. Minimum length of turn = 2(94 + 3) = 194″. Estimate mean length of turn

$$= 2(94 + 5) = 198''.$$

$$q = \frac{.01 \times 198 \times 7100 \times 48}{200 \times 12{,}000} = 0.28 \text{ sq. in.}$$

Assume 0.8% or 40 kw. loss in field when at 80% p.f.

$$I_F = \frac{40{,}000}{200} = 200. \quad \text{Turns per pole} = \frac{7100}{200} = 35.5.$$

A 5″ spool will allow 36 turns .14 thick.
A 6″ spool will allow 36 turns .16 thick.
Make each turn 1.8 wide by .16 thick = .29 sq. in.
Height of winding = (.16 + .005) × 36 = 5.94.
Must allow room for collars. Spool = 6.5″.
This is 1.5″ more than figured earlier.
Mean length turn = 2(94 + 3 + 4) = 202.″

$$R_F = \frac{.01 \times 202 \times 36 \times 48}{.29 \times 12{,}000} = 1.0 \text{ ohm at } 75^\circ \text{ C.}$$

$$\text{Surface} = 2(94 + 3 + 8) \times 6 \times 48 = 60{,}500 \text{ sq. in.}$$

$$\text{Rise in temp.} = 25 \times \frac{40{,}000}{60{,}500} = 17^\circ \text{ C.} \qquad \text{(low)}$$

22. Coreloss: Vol. core = 92,500 cu. in. Teeth = 41,000 cu. in.

Core hyst. $= .00525 \times .0015 \times 92{,}500 \times \overline{45}^{1.6} \times 60$	$=20{,}000$
eddy $= .254 \times .00007 \times 92{,}500 \times (.014 \times 45)^2 \times \overline{60}^2$	$= 2{,}400$
Teeth hyst. $= .00525 \times .0015 \times 41{,}000 \times \overline{100}^{1.6} \times 60$	$=31{,}000$
eddy $= .254 \times .00007 \times 41{,}000 \times (.014 \times 100)^2 \times \overline{60}^2$	$= 5{,}200$
(See Chapter I.)	
	58,600

23. Efficiency: Assume friction at 0.6% or 30 kw.

Friction	30
Excitation, 100% p.f.	33
Coreloss	60
Armature I^2R	32
All losses	155
Output	5000
Input	5155

Per cent efficiency. 97 for non-inductive load.

24. Heating.

$$P_1 = 60 + 32\,\frac{190}{213} = 88.5 \text{ kw.}$$

$$A_1 = 39{,}500 + 82{,}000 = 121{,}500 \text{ sq. in.}$$

$$T_1 = 37 \times \frac{88{,}500}{121{,}500} = 27^\circ \text{ C.}$$

$$U_s = 5.8 \qquad d_1 = .17 \qquad k_1 = 200.$$

$$M = \frac{5.8 \times 95 \times 288}{.17 \times 200} = 4650.$$

$$U_c = 3.55 \qquad l_c = 11.5 \qquad d_2 = .12 \qquad k_3 = 78.$$

$$N = \frac{3.55 \times 11.5 \times 576}{24 + 78} = 230.$$

$$T_c = \frac{32{,}000 + 4650 \times 27}{4650 + 230} = 32^\circ \text{ C.}$$

Examples of Design of Salient Pole Alternating Current Generators

Column 1 describes a small, laboratory, 60-cycle, 5 kv-a. generator, part of a motor-generator set.

Columns 2 and 3 describe similar machines of larger sizes.

Column 4 describes a waterwheel-driven generator, slow speed, medium capacity.

Column 5 gives the author's calculations based upon incomplete data published in the Electric Journal of April, 1924, for the generators for the Muscle Shoals Plant.

Column 6 gives the author's calculations based upon fairly complete data published in the Electrical World of Oct. 19, 1929, of a new generator for the Niagara Falls Power Co.

Column 7 gives the author's calculations based upon incomplete data given in the A.I.E.E. Journal of June, 1922, for one of the new generators for the Queenston Plant at Niagara Falls.

Column 8 gives the author's calculations based upon incomplete data given in the Electric Journal of August, 1922, for another make of generator for the Queenston Plant.

TABLE XXXI.—SALIENT-POLE A.C. GENERATORS

Mechanical Dimensions and Data

	1	2	3	4	5	6	7	8
Poles	4	4	6	54	72	20	16	16
Rating, kv-a	5	7.5	15	2500	25,000	32,500	45,000	49,500
R.p.m.	1800	1800	1200	133	100	150	187.5	187.5
Voltage, terminals	110	110	220	2300	12,000	12,000	12,000	13,200
Frequency	60	60	60	60	60	25	25	25
Armature diameter, face	8.06	10	10	170	284	198	216	210
Armature diameter, back	11.63	14.5	22	180	304	228	250	.256
Armature, total core length	4.25	6.5	5	20	48	65	68	76
Armature, air ducts	0	0	0	8 − .5	23 − .5	30 − $\frac{3}{8}$	25 − $\frac{3}{8}$	36 − $\frac{3}{8}$
Slots, number	48	72	72	648	459	300	288	312
Slots, dimensions	1.15×.35	1.25×.25	1.18×.27	2.13×.4	$3\frac{1}{4}\times\frac{5}{8}$	$4\frac{1}{4}\times\frac{3}{4}$	7×1	5×.75
Conductors, size	No. 11	No. 16	No. 15	$\frac{3}{4}\times\frac{3}{16}$	.130″ sq .in.	.176″ sq. in.	24 (.16×.09)	.82×.33
Conductors per slot	16	32	44	2	6	6	6	2
Conductors in multiple	2	8	2	2	4	4	4	2
Connection	Y	Δ	Δ	Y	Y	Y	Y	Y
Pitch	$\frac{13}{12}$	$\frac{11}{18}$	$\frac{9}{12}$	$\frac{5}{12}$	$\frac{6}{6.5}$	$\frac{12}{15}$	$\frac{15}{18}$	$\frac{15}{18}$
Air gap, min.	.09	.094	.094	.375	0.5	.5	.5	.5
Air gap, avg.	.115	.094	.125	.47	0.625	.625	.625	.625
Pole arc	3.31	4.7	3.25	6	8.25	21	28	28
Pole length	3.75	6.5	5	19.5	48	65	67	75
Pole core width	1.5	2.25	1.6	4	6	15	24	26
Pole radial length	2	1.75	3	7	16	12.5	20	17
Spool, turns	1850	632	185	33.5	15	42.5	70	80
Spool, size cond.	No. 23	*d*.042	(.057×.081)	1.5×.16	1.5×.25	2.25×.20	1.25×.5	2×.25
Yoke, dimensions	.75×5.5	1.25×6.5	1×5	4×24	8×52	10×72	10×67	14×75
Exciter voltage	115	125	125	125	250	250	250	250

SALIENT-POLE A.C. GENERATORS

Electrical and Magnetic Data

	1	2	3	4	5	6	7	8
Rating, kv-a.	5	7.5	15	2500	25,000	32,500	45,000	49,500
Volts per phase	63.5	110	128	1350	7000	7000	7000	7600
Amperes per phase	26.2	22.6	23	628	1200	1560	3160	2180
S turns per phase	64	48	132	162	114	75	72	52
ϕ flux per pole 10^6	.39	1.04	.68	5.6	24	87	96	139
ρ pole arc per cent	.52	.60	.65	.61	.67	.68	.66	.67
σ Ampere conductors	400	206	580	760	1840	1140	1370	1530
V F.p.m.	3770	4710	3140	5900	7400	7700	10,600	10,300
B Gap, kilolines	31	35	42	48	60	65	62	62
n Armature ampere turns	855	475	990	3600	3860	11,900	18,500	14,900
v Leakage coefficient	1.3	1.15	1.15	1.24	1.2	1.15	1.2	1.2
U Armature amperes, square inches.	2040	1410	4500	2200	2300	2210	1560	1600
Slot factor	.245	.21	.35	.33	.38	.29	.30	.28
X Leakage reaction, per cent	12.5	6	18	8	8.5	8.3	11	5.2
Synch. imp., $a-t$	1050	515	1300	4880	5600	12,800	23,000	19,000
Excitation, no load $a-t$	2050	1500	1300	7200	15,000	13,700	14,000	16,000
Excitation, rated load	2550	1825	1930	8800	16,300	20,600	25,000	22,000
R_A Armature per phase, 75°	.125	.071	.41	.14	.026	.024	.012	.019
R_F Field resistance	89	30	4	.84	.42	.28	.34	.47
Friction, per cent	3.5	3	2.7	.54	1	.50	.75	.....
Coreloss, per cent	2.5	2.6	1.5	1.66	1	.75	.86	.90
Excitation, per cent	2.7	2	2.6	.77	.26	.21	.10	.08
Armature I^2R, per cent.	5.2	1.4	3.9	.90	.44	.54	.54	.53
Load loss, per cent	1	1	.....	.20	.....	.....	.....	.....
Efficiency at rating, per cent	85	90	89	96	97.3	98	97.75	.....
Rise temperature windings	33	32	40	45				

CHAPTER IV

THE TURBINE-DRIVEN ALTERNATING CURRENT GENERATOR

In turbine-driven generators the steam turbine is usually built by the same company that builds the generator, and the two machines are practically one unit (Fig. 67). The steam turbine has been developed to such an extent that for a given capacity in power it weighs less than a reciprocating engine, occupies much less space, costs less, and is more economical of steam and fuel. Good economy in steam turbines, however, is obtained only with high angular velocities. This has made it difficult to apply the turbine to useful purposes. The electric generator has shown itself to be the most suitable device to absorb the power of the turbine and make it available in subdivided form and at convenient speeds for general application. However, it required many years of experience to develop an electric generator which would operate successfully at the high angular velocities suitable for direct connection to the steam turbine. The principal difficulty was the design of a construction that would withstand the enormous stresses in the revolving member, which result from the centrifugal force. Thus the design of the machine as a whole is largely a question of the type and construction of the rotor which is the field.

Normal Weights and Speeds.—Most turbo-generators have much more copper on the fields and less on the armature than do slow-speed machines, although the total amount of copper is not far different in the two classes. The turbo-generator has much less magnetic iron on account of its high speed. It is quite normal in machines of 5000 kv-a. capacity to operate at 3600 r.p.m. and in machines of 20,000 kv-a., at 1800 r.p.m. To do this involves the use of peripheral speeds of from 15,000 to 27,000 ft. per minute. At these high speeds the amount of material per kilowatt is much reduced. Increasing the speed ten-fold reduces the weight of material to about one-quarter. This low weight is

somewhat offset by the higher cost of construction necessary to withstand the enormous centrifugal forces. Most of the machines, even the large sizes, have two or four poles, or at most six poles. This involves the use of a pole pitch of from 30 to 40 ins. in 60-cycle machines, and 60 to 90 in 25-cycle machines. Since such large powers are concentrated in such small bulk, a great deal of energy in the form of losses must be dissipated in a small space. Thus

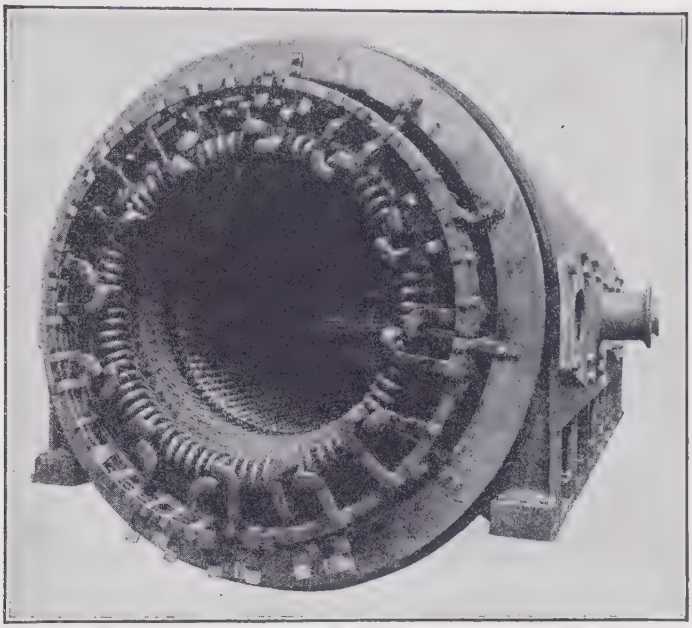


FIG. 100.—Stator or stationary armature of a turbo-generator showing windings.

special means of ventilation must be provided, such as numerous air ducts, fan blades on the rotor or a separate blower and a supply of clean, cool air.

Construction of Rotor.—The design of a turbo-generator differs from that of the salient-pole type described above, because the revolving field of the turbo-generator consists of groups of coils placed in slots and distributed concentrically about the pole centers or cores, which resemble large teeth. The center of a pole occupies about one-third of the pole pitch and has no slots

(see Fig. 102). The distributed field winding gives a peaked or trapezoid shape to the flux wave in the air gap, which is made to approach a sine form as nearly as practicable, as shown in Fig. 103, a developed cross-section of one pole. Thus the pole arc is the same as the pole pitch, but the maximum flux density is approximately 1.50 times the average.

The air gap is usually very large in order to provide a path for the large volume of ventilating air required. On account of the

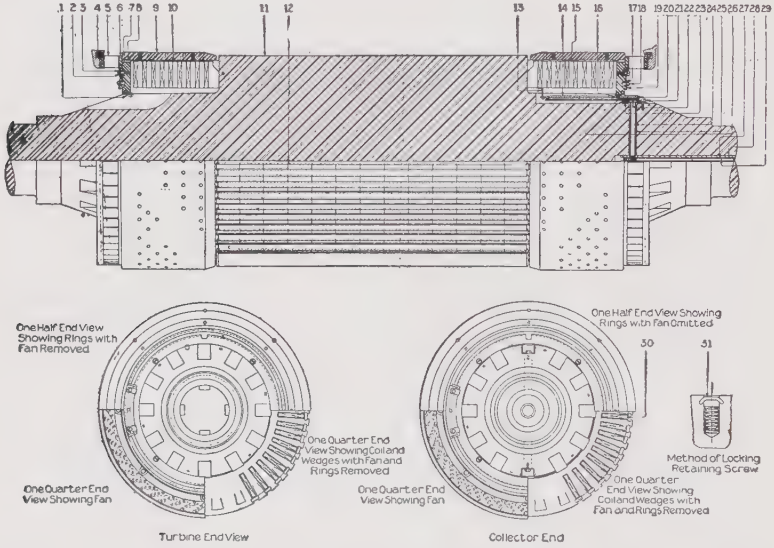

1 2 3 4 5 6 7 8 9 10
11 12
13
14 15 16
17 18 19 20 21 22 23 24 25 26 27 28 29
One Half End View Showing Rings with Fan Removed
One Quarter End View Showing Fan
One Quarter End View Showing Coil and Wedges with Fan and Rings Removed
Turbine End View
One Half End View Showing Rings with Fan Omitted
30
31
Method of Locking Retaining Screw
One Quarter End View Showing Fan
One Quarter End View Showing Coil and Wedges with Fan and Rings Removed
Collector End

FIG. 101.—Cross-section of the rotor or revolving field of a turbo-generator showing slots, windings and retaining rings.

high peripheral speeds that are customary, the diameter per pole is high:

The centrifugal force in the revolving member is from 1000 to 1500 pounds for every pound of material near the periphery. A method for determining the centrifugal force in every part is given in "Electric Machine Design," by Gray, and in "High-speed Dynamo-electric Machinery," by Hobart and Ellis.

The rotor must be of substantial and rigid construction, so that the critical speed of vibration is above the normal speed of operation. There are several methods of construction, among which the following are prominent:

a. A solid steel forging turned to shape with radial slots containing a distributed field winding. The shaft is in one piece with the field core (see Fig. 101).

b. A built-up structure consisting of steel disks about 3 ins. thick, held between the two end-plates by through-bolts, the radial or parallel slots being milled in the assembled structure.

c. Steel laminations with radial slots, assembled on a forged-steel shaft.

d. Laminated pole pieces attached to a spider by keys. (These

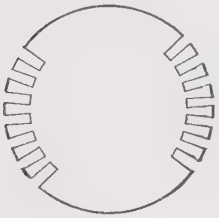


FIG. 102.— Cross-section of Rotor showing arrangement of slots and pole centers for a two-pole machine.

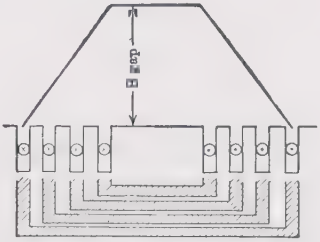


FIG. 103. — Distribution of field windings and of the flux in the air gap.

are no longer being built although there are a large number in service.)

In these revolving fields the length is about the same as the diameter, and in large machines the length is greater than the diameter. The air gap is from 1 to 2 ins. A uniform air gap is necessary to prevent noise and strains.

Construction of Stator.—Since the pole pitch is large, a large number of slots per pole is used (12 to 24) and this gives a low armature self-inductance. Owing to the long pole pitch the end connections are long and are subjected to considerable mechanical forces, as a result of the leakage flux. They must, therefore, be held securely in place by non-magnetic supports. For armature reaction in ampere-turns per pole, values of from 8000 to 20,000 are common, and the field winding is usually of a capacity of three times the armature ampere-turns. The regulation is poor (20 to 30 per cent). This defect is readily overcome by the use of automatic voltage regulators. The current density in the copper (1500 amperes per square inch) is lower than in slow-speed

machines, while the **constant ampere conductors per inch periphery** has high values, viz., 600 to 1500.

Suitable Speed and Number of Poles.—For 25-cycle machines, two poles and 1500 r.p.m. are used for all sizes, including 35,000 kv-a. For 60 cycles, two-poles and 3600 r.p.m. are used up to 6000 kv-a.; four poles and 1800 up to 50,000 kv-a.; and six poles for larger sizes. Table XXXII gives representative values of dimensional constants.

TABLE XXXII

General Dimensions and Design Constants of Turbo-Generators

	1	2	3	4	5	6	7
Frequency	25	25	25	60	60	60	60
Poles	2	2	2	2	2	4	6
Kv-a.	2,000	5,000	20,000	2,000	5,000	20,000	30,000
R.p.m.	1,500	1,500	1,500	3,600	3,600	1,800	1,200
Diameter of rotor, in.	38	38	61	23	25	50	75
Length of rotor, in.	35	56	58	46	60	75	94
Stator of slots	48	48	72	30	36	72	108
Depth of slots, in.	4.5	5	7	3	4	6	8
Peripheral velocity, ft. per min.	15,000	15,000	24,000	22,000	24,000	24,000	24,000
σ, amp.-cond. per in.	600	900	1,300	600	900	1,300	1,500
n, armature reaction ampere turns	11,000	16,000	37,000	6,600	11,000	16,000	15,000
Flux per pole megalines	58	100	175	40	60	83	112
Average gap density, kilolines per sq. in.	28	30	32	24	26	28	30
Air gap, length, in.	1	1.25	2.5	0.75	1	1.5	1.5
Rotor slots	36	36	48	24	24	48	72
Depth of slots, in.	5	5.5	7	3.5	4.5	6	8
Weight of generator, lbs.	50,000	100,000	300,000	34,000	62,000	250,000	430,000

Diameter of rotor is set by whatever value is assumed for the peripheral speed (V), which is usually between 20,000 and 24,000 ft. per minute. The dimensions of the rotor are chosen first as it is usually the controlling feature.

$$D = \frac{12V}{\text{R.p.m.} \times \pi}.$$

Length of Rotor.—From the general output equation this dimension may be found by assuming reasonable values for the arbitrary constants as follows:

$$P_0 = \frac{\rho\sigma BVDLk_1k_2}{144 \times 10^9}$$

where P_0 = output in kv-a;

ρ = unity in most turbo-generators;

σ = the ampere conductors of the armature per inch periphery of the field.

It must be figured for the same diameter as used for V, if the equation is to check. σ has values as in table XXXII.

B, the average magnetic density in the air gap, varies from 25,000 in 60-cycle machines to 32,000 in 25-cycle machines. Maximum density is 1.50 times the average.

V, the peripheral speed at the surface of the rotor, has values as already given.

D, the diameter of the rotor is given above.

L, the length of the field, is to be found.

k_1, distribution constant, .96 for three-phase and .91 for two-phase.

k_2, the pitch constant, assumed for preliminary purposes as .9 for 60 cycles and .8 for 25 cycles.

Number of Turns in Series per Phase in the Stator.—S is found for preliminary purposes by:

$$S = \frac{\pi D\sigma}{2nI},$$

where n = number of phases;

I = rated current per phase.

Number of slots and conductors per slot are determined by trial from the desired number of turns in series; then:

$$cs = 2nS,$$

where c = the effective conductors per slot;

and

s = the total number of armature slots.

With a set value of the product, cs, values of c = 0.5, 1, 2, 4, and any other even number are tried until a combination is found which will give a number of slots approximating those in the table and giving approximately the product, cs, assumed.

s should be divisible by the number of poles and the number of

phases in order to make a conventional winding. It may now be necessary to readjust the value of S to a practicable value which will be called the final value.

Size of Armature Conductor.—For preliminary purposes, a current density of 1500 amp. per square inch may be assumed, which, divided into the current per phase gives the cross-section of one **effective** conductor in square inches. By **effective conductor** is meant the conductor that carries the whole current per phase. On account of the large currents in large machines, it is frequently advisable to divide the winding into multiple paths, two paths in multiple for instance; then each conductor carries half the phase current and is called half an effective conductor. To reduce eddy currents and facilitate bending, the conductors are usually made up of from 4 to 30 strands of rectangular wire laid parallel and insulated from each other. This makes it possible to build up a conductor of convenient shape, but necessarily reduces the slot factor.

Size of armature slots is determined, first by the space necessary for conductors and insulation, and, second, by the depth which is best suited to the diameter of armature. The slot factor is low for machines of this size for three reasons: (1) high voltage is customary; (2) a considerable part of the slot is left vacant to serve as a passage for ventilating air; (3) the winding is placed deep in the slot to give sufficient reactance to limit the instantaneous short-circuit current.

A typical slot would be 6 ins. deep by 1 in. wide with the winding and wedge occupying only 5 ins. of the depth. Customary slot factors are given in Table XXXIII, usual depths in Table

TABLE XXXIII

Slot Factor and Slot Insulation

Machine Voltage	Slot Factor	Insulation, Inches	
		Depth	Width
2,000	0.35	1.25	0.17
4,000	0.35	1.50	0.24
6,000	0.33	1.625	0.31
10,000	0.30	2.00	0.45
13,000	0.27	2.25	0.55

XXXII and space allowed for insulation and air in Table XXXIII. The dimensions given for insulation include all the insulation on both sides of the coil, that is, an allowance of 0.30 in. means 0.15 in. on each side. The area of the slot is equal to the cross-section of the copper divided by the slot factor, and the width of slot is equal to the cross-section divided by a suitable depth as judged from values in Table XXXII.

ARMATURE COIL AND SLOT INSULATION FOR A 13,000-VOLT TURBO-GENERATOR

Stranded cable with two coil-sides in open slots, four turns per coil, each turn consisting of 12 strands laid 3 wide and 4 deep. Each strand to be (.12 × .20 in.) when bare. Double cotton-covered (.128 × .208).

Depth		Width	
Each strand, d.c.c.	.128	Each strand, d.c.c.	.208
Four deep	4		3
	.512	Three wide	.624
025 mica paper on each turn.	.050	Turn insulation	.050
Dimensions over each turn...	.562		
	4		
4 turns stacked	2.248		
13 layers mica tape ½ lap	.325	13 layers mica tape	.325
asbestos	.040	asbestos	.040
corona protection	.035	corona protection	.035
Dimensions over each coil...	2.648		1.074
	2		
Dimensions over 2 coils	5.296		
Wedge	.312		
Clearance	.892	Clearance	.006
Depth of slot	6.500	Width of slot	1.080

$$\text{Slot factor} = \frac{(.12 \times .20) \times 96}{6.5 \times 1.08} = 0.328.$$

(This insulation is more conservative than that of Table XXXIII.)

The armature winding is usually a double layer barrel with as many coils as slots, and one, two, or three turns per coil. The

coils usually have a pitch two-thirds that of a pole in order to shorten the end connections, save copper, and space, and reduce the copper loss. It is of the lap-winding type shown in Fig. 82. There may be as many circuits in multiple in each phase as there are poles, in order to reduce the amount of current per circuit. Each turn is usually given two twists so that it lies near the top of the slot on one side and near the bottom on the other side.

The **total useful flux** in the armature is found as given on page 160, and has very high values, as shown in Table XXXII. This is because the output per pole is so great. The leakage coefficient runs quite uniformly from 1.1 to 1.25 and is low because of the large pole pitch and distributed field winding.

Leakage Factor.—Usual values of the leakage coefficient in turbo-generators are:

TABLE XXXIV

Leakage Coefficient

	25 Cycles	60 Cycles
2 pole	1.05	1.10
4 pole	1.10	1.15
6 pole	1.15	1.20

The **magnetic circuit** is adjusted so that the magnetic densities are about as given in Table XXXV. In the stator and rotor teeth and in the air gap, the density is not uniform, so that the maximum density (in the center of a field pole) is about 1.50 times the average density (see "Induction Motors, Polyphase"). For a preliminary design this ratio may be assumed.

TABLE XXXV

Usual Maximum Magnetic Densities

Part	25 Cycles	60 Cycles
Armature core	90,000	80,000
Armature teeth	100,000	85,000
Air gap	50,000	45,000
Rotor teeth	100,000	100,000
Rotor core	90,000	90,000

Rotor Slots and Construction.—The rotor teeth in the final analysis must be very carefully studied to determine if there is sufficient material at the root to withstand the centrifugal force due to the masses of the tooth and the material in one slot. For the preliminary layout a number approximating those given in Table XXXII is selected, say 36 for a two-pole machine, and these are spaced as if there were 54 slots, but two groups of 9 each are omitted at symmetrically placed positions, thus forming the pole centers. This will give 18 slots per pole, in which will be placed 9 coils per pole, each of a different pitch and all concentric with the pole center.

For each diameter of rotor there is a certain number and depth of slots which gives the best proportion between amount of copper and strength of tooth and this may be judged from Table XXXII. The width of a slot is made approximately 0.5 to 0.66 of the pitch on the circle at the bottom of the slots.

The **length of air gap** is determined primarily by its function of carrying air for ventilation of the interior, the air going into the air gap at both ends and then passing radially out through the air ducts. The method of calculating this is given under "Heating and Ventilation." Usual values for the length of air gap are given in Table XXXII.

The reluctance of the path of the flux crossing the air gap is usually so great that about 80 per cent of the total ampere turns at no load are required for this purpose. For magnetic reasons the air gap is usually taken of such a length that the total ampere-turns per pole at no load are about equal to the armature reaction per pole.

No-load Magnetization Curve.—This is calculated as for induction motors (q.v.) on account of the distributed field winding and the peak wave of flux. The average densities are figured on a basis of full-pitch pole arc and all stator teeth per pole carrying flux. The maximum density in gap, stator teeth, and rotor teeth is obtained by multiplying the average density by 1.50 for the usual shape. With saturation in the teeth (over 110,000) or with specially wide pole centers, this constant becomes 1.45 or 1.4 respectively.

On account of the long air gap (long compared with slot openings and duct widths) the effect of slots in contracting the flux in the gap is not marked, so that the density in the air gap

may be taken the same as at the surface of the rotor. Investigation shows that the increase in density in the air gap due to the slots is only from 2 to 4 per cent and this is balanced by figuring the density at the surface of the rotor instead of at the mean gap diameter. The section of rotor teeth is figured at a point one-third the distance from the bottom of the slots, and the total section includes the one large center tooth and several smaller teeth.

The average length of the path of the magnetic flux in the two cores is considerably less than one-half the pole pitch on account of the distributed flux. It is usually about 50 to 60 magnetic degrees.

The materials used are generally silicon steel for the stator and forged steel for the rotor. The tendency of the flux to pass through the slots instead of the teeth is not so great in these machines as in other types because, in the rotor, the flux can spread out with saturation, and in the stator, the densities are not very high. A magnetization curve between no-load volts between terminals and ampere-turns per pole is usually plotted up to a voltage 120 per cent of rated voltage.

TABLE XXXVI

1	2	3	4	5	6	7	8	9
Part	Material	Flux	Cross-section	$B_{avg.}$	$B_{max.}$	H	L	Amp.-t
F. Yoke		$\frac{v\phi}{2}$	$d \times L$		$\frac{v\phi}{2dL}$		i	
F. Teeth		$v\Phi$	See text		$1.5B_{av}$		n'	
A. Gap		Φ	$\frac{\pi DL}{\text{poles}}$		$1.5B_{av}$		g	
Arm. T		Φ	Tt_0l_1		$1.5B_{av}$		n	
Arm. C		$\frac{\phi}{2}$	$h \times l_1$		$\frac{\phi}{2hl_1}$		k	

Total Amp.-turns per Pole = Sum.

Symbols for Turbo-Generators

d = radial depth of rotor core.
D = diameter of rotor at gap.
D_1 = diameter of armature at back.
D_2 = diameter of armature at gap.
g = radial length of air gap. (Uniform).
h = radial depth of armature core.
i = length of flux path in rotor core.
k = length of flux path in armature core.
L = total length of core, stator and rotor the same.
l_1 = effective length of iron in armature core.
n = depth of armature slots.
n' = depth of rotor slots.
z = pitch of armature slots at face.
t_0 = effective width of a tooth, rotor or stator.
T = stator teeth per pole carrying flux.
w = width of armature slots.
w' = width of rotor slots.
R_2 = radius of circle through effective section of rotor teeth.
a_2 = pole arc through effective section of rotor teeth.

Explanation of Magnetic Calculations (Table XXXVI)

Column 1 gives the name of each part of the magnetic circuit. Only the length of the circuit supplied by the field coils of one pole is calculated, as the rest of the circuit is identical. Column 2 gives the material of each part.

Column 3 gives the value of the flux that passes through each part of the circuit.

Column 4 gives the cross-section, in square inches, of each part of the circuit, the symbols referring to the dimensions taken from a drawing similar to Fig. 104.

Column 5 gives the average magnetic density, in maxwells per square inch, in the field teeth, gap and armature teeth. Owing to the distributed m.m.f. of the several field coils, the flux wave in the gap and teeth is peaked in shape and it is difficult to calculate the maximum directly. The average density is therefore calculated as a basis for determining the maximum by means of a ratio or form factor.

Column 6 gives the maximum density in each part, as the

maximum is the density that governs the m.m.f. required. In the case of the field yoke and armature core, this occurs midway between pole centers, as at d and h in the drawing, and the value is simply the respective fluxes of Column 3 divided by the area of Column 4. In the case of the field teeth, gap and armature teeth, the flux wave has a peaked shape whose maximum is usually 1.5 times the average where the pole center is one-third of the pole pitch, and may vary from 1.4 times, where the wave is flat-topped because of a wide pole center or saturation in the armature teeth, to 1.57 for a very peaked wave approaching a true sine shape.

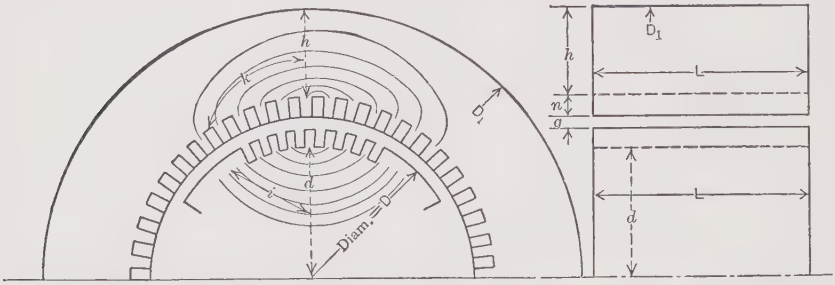


FIG. 104.—Dimensional drawing of a turbo-generator showing cross-section and length of magnetic path.

Column 7 gives the ampere turns per inch for the maximum densities taken from saturation curves for the materials used in the respective parts. For the air gap the ampere-turns per inch is always equal to $B \times .313$.

Column 8 gives the length of the separate paths whose reluctance is overcome by the m.m.f. of one pole. In the case of the teeth and gap, the length is definite and obvious. In the case of the field and armature cores the length of path is a mean length and must be approximated empirically or by judgment. Each magnetic line has a different length and the density is different at all parts, hence an equivalent length is sought. An empirical method is to take the length of an arc of 50 to 60 electrical degrees on the outer circumference of each core.

Column 9 gives the ampere-turns required for each part and is obtained by multiplying the respective values of m.m.f. of Column 7 by the lengths of Column 8.

The sum of all the values in Column 9 gives the required

number of ampere-turns per pole for the useful flux and voltage assumed. The tabulation is then repeated for two or three other values of voltage up to 120 per cent of rated voltage.

Field Yoke.—The field yoke takes one-half the flux each way. This flux filters in from the several teeth and all passes through the section under the neutral tooth, thus giving maximum density here. The material is usually a solid steel forging.

Field or Rotor Teeth.—These are part of the same piece as the core, left by milling out the slots. The slots usually occupy about two-thirds of the pitch of the pole. That is, if there are 8 slots per pole the center tooth will occupy the space meant for 4 slots and 5 teeth. If there are 32 slots on the rotor perimeter they will be cut with such a pitch that there could have been 48 slots but 4 slots are omitted at each pole center to form the big center tooth. This is a usual proportion but not a universal one.

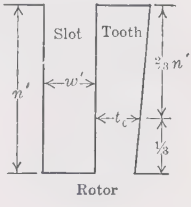


FIG. 105. — Diagram of slot and tooth in Rotor showing dimensions.

The effective width of one tooth is the width at a point one-third the distance from the smallest section (bottom of the slot) towards the largest section. This is at a radius $R_2 = \left(\frac{D}{2} - \frac{2n'}{3}\right)$. If there were no slots at all the pole arc of metal at this radius would be $a_2 = \frac{2\pi R_2}{\text{poles}}$. But from this arc there has been cut out an amount of metal represented by the slots per pole. The cross-section of the remaining metal is therefore a_2 diminished by the aggregate width of the slots or $a_2 - \left(w' \times \frac{\text{slots}}{\text{poles}}\right)$.

As the tooth midway between pole centers is at a neutral point and is not subjected to any m.m.f., it carries no flux and its cross-section should be still further subtracted from the total section for greater accuracy.

Thus the exact cross-section of the rotor teeth is:

$$\left[a_2 - \frac{w' \times \text{slots}}{\text{poles}} - t_0\right] \times L.$$

Air Gap.—The air gap is of uniform length all around in order to avoid noise and strain at the high peripheral speeds used. The

gap is so long compared to the slot openings and duct widths that the effect of these is negligible in contracting the flux. The density in the air gap is figured at the surface of the rotor instead of at the center of the gap, to balance such constriction as may be due to armature ducts and slots.

Armature Teeth.—All the teeth per pole carry flux; therefore T equals total slots divided by poles. However, at a given instant all the slots opposite a pole do not have the same density, but as the rotor revolves each tooth goes through the same cycle. The effective length of the iron in the core is greater than in other types of machines because the laminations are packed together under a greater pressure, primarily to reduce vibration. The stacking factor is usually .95 and the effective length l_1 = (L − air ducts) × .95. The material is usually silicon steel.

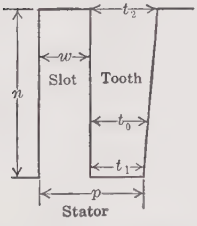


FIG. 106.— Diagram of slot and tooth in Stator showing dimensions.

Armature Core.—The total length of the armature core is usually the same as that of the field. The effective length of iron is the same as that of the teeth and, as the core and teeth are in one piece, it is of the same material. The core usually consists of 2-in. lengths of punchings alternating with $\frac{1}{2}$-in. air ducts. There may be as many as one air duct for every 2 ins. of gross length of core. The magnetic density figured is the maximum, which occurs along the section h. The mean length of path is estimated. The shortest line has a length equal to the width of one slot, and the longest line goes from a tooth opposite the pole center out to the outer perimeter and along it to h. A reasonable equivalent length would be about one-quarter to one-third of the maximum length. This allows for varying density as well as varying lengths.

Armature Resistance.—The armature resistance is figured at this point to be sure that the heating of the armature winding will not be excessive. The resistance is figured as for salient-pole machines, except that the straight portions of the coils extend 1 to 2 ins. beyond the core, making x equal to 2 or 4 and adding 4 to 8 ins. to the mean length of turn. It is customary to calculate the resistance of the armature at 100° C. at which temperature the resistance of 1000 ft. of copper of 1 sq. in. cross-section is .0107 ohm. The total I^2R loss in the armature should

be from 0.2 per cent to 0.5 per cent in moderate-sized machines, and lower in large machines. (See Table XXXVII.)

The I^2R loss in watts per square inch of coil surface ranges around 0.5 and may be higher in low-voltage machines or machines with unusually good ventilation (see page 216).

Armature Leakage Reactance.—This is calculated the same as for salient-pole machines. Usual values are 8 to 15 per cent for 25-cycle machines and 10 to 20 per cent for 60-cycle machines.

Armature Reaction.—This is computed in the same manner as for salient-pole machines, but has much higher values, on account of the large output per pole and long pole pitch. Usual values are from 8,000 to 25,000 as in the table.

Effectiveness of the Field.—Since some of the coils of the field winding span only a small part of the pole pitch (one-third) this winding is not as effective in overcoming the m.m.f. of the armature as if all coils spanned the whole pole. If the shortest coil had a pitch of one-third and the longest coil had full pitch, then the average pitch would be two-thirds and the effectiveness of the winding would be 0.866, the same as the pitch constant for any winding with a two-thirds pitch. But in these field windings the longest coil cannot have a pitch of 100 per cent because then the outer coils of adjacent poles would have to lie in one and the same slot. Consequently, the longest coil spans something less than the 100 per cent and the average pitch is less than two-thirds, varying from 58 per cent in four-pole machines to 61 per cent in two-pole machines. The pitch constant or effectiveness of the field therefore varies from .78 to .83.

The exact value is obtained by finding the average value of the pitch constants of all the coils. Thus, referring to the four-pole field shown in Fig. 104, there are 8 slots per pole or 32 in all. There is space for 4 slots in each pole center, therefore there is space for 48 slots of this size on the perimeter of the field and each slot and tooth would occupy $\frac{360}{48} = 7.5$ mechanical degrees or 15 electrical degrees (360 electrical degrees per pair of poles). Assume the usual construction, in which the 2 innermost slots are spaced 5 slots apart or 75 electrical degrees center to center, then the next two would be spaced $75 + 2 \times 15 = 105$, etc. (See Fig. 107 for two-pole equivalent.)

The calculation of the average pitch factor would be:

	Span, Electrical Degrees	Half Span		Factor
Slots 4+5	75	37.5	sin 37.5 = .61	.610
3+6	105	52.5	sin 52.5 = .79	.790
2+7	135	67.5	sin 67.5 = .925	.925
1+8	165	82.5	sin 82.5 = .991	.991
			Sum	3.316
			Average = Sum ÷ 4 =	.829
			Effectiveness or Pitch Factor =	.829

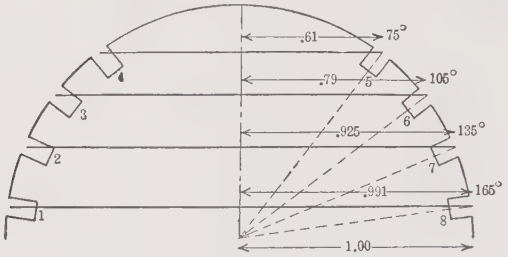


FIG. 107.—Diagram of arcs and chords spanned by the field coils to be used in calculating the average pitch factor or "Effectiveness of Field."

If the 2 innermost slots had been spaced just 60 electrical degrees center to center, then in the 120-degree arcs which are slotted there would be the space for 7 times the slot pitch and the slot pitch would be $\frac{120}{7} = 17.2$ degrees. On this basis the calculation would be:

	Electrical Degrees	Half Span	
Slots 4+5	60	sin 30 =	.500
3+6	94.4	sin 47.2	.725
2+7	128.8	sin 64.4	.900
1+8	163.2	sin 81.6	.985
			4|3.110
		Effectiveness or Pitch of factor =	.78

Excitation.—The excitation in ampere-turns per pole for any specified loading is calculated in a manner quite similar to that used in the design of salient-pole machines (q.v.) except that the component of the field m.m.f., n', required to overcome the armature reaction, is greater because of the lesser effectiveness of this particular type of field winding.

Thus, $$n' = \frac{n \times \text{leakage factor}}{\text{effectiveness}},$$

where n is the armature reaction for the load under consideration. The rest of the procedure for the determination of the synchronous reactance, regulation, and excitation are determined as for salient-pole machines; and the calculated results usually check more closely with test results in this type of machine than in the salient-pole type, because the more uniform reluctance of the field throughout the pitch of the pole causes the different magnetic forces to retain the same relative values no matter what the phase differences between them.

Thus the excitation, F, for any specified load, is determined, and particularly for the load which requires the greatest excitation in order that the field winding may be designed to meet this condition without overheating.

Regulation.—From the **no-load excitation curve,** the voltage corresponding to the computed value of F, and the relation between this no-load voltage and the assumed terminal voltage at rated load, the regulation is calculated.

The regulation of turbo-generators is usually from 20 to 30 per cent at rated load and unity power factor. As voltage regulators are always used, this poor inherent regulation is of no practical importance; in fact, it is an advantage in the case of large power stations as it places a limit on the maximum steady current which a machine will give in case of a short-circuit and limits the strain on the switches and circuit breakers.

Field Winding.—The size of field conductor and the number of turns per pole are computed as for salient-pole machines, except that the mean length of a field turn is not so obvious, as each of the several coils has a different length of turn. The average pitch is computed by taking the average of the angular spans of all the coils. Thus in the first example given on page 212, the average span is:

$$(75 + 105 + 135 + 165) \div 4 = 120 \text{ degrees},$$

and in the second case:

$$(60 + 94.4 + 128.8 + 163.2) \div 4 = 112 \text{ degrees}.$$

The average pitch is then 0.67 and 0.62 respectively.

The mean length of turn is:

$$2(L + x) + \frac{10D_3}{\text{poles}}k,$$

where x is excess of the straight portion of a coil over the length of the core, D_3 is the pitch diameter of the field coils (diameter to a point at about the middle of the slot), and k is the average pitch just found.

The cross-section of the conductor is given by the same relations as in the salient-pole type, and the number of turns is governed by the loss which the coils can dissipate or which can be allowed from the point of view of the efficiency. In this type of machine the heating is usually the limiting condition.

The current density is usually between 1500 and 2000 amperes per square inch. This gives a first approximation of the field current and number of turns. This number of turns is divided equally among the chosen number of coils, and the shape of the conductors chosen to fit in the slot, thus .85 × .125-in. leaving about .075 to .10 in. clearance for insulation on each side of the slot, requiring in this case a slot 1 in. wide. The number of conductors per slot is the same as the number of turns per coil. About 1.25 in. is left unoccupied at the top of the slot for the wedge and for ventilation. A cross-section of a typical field slot is given in Fig. 108.

There is usually about .005 in. insulation between turns and it is customary to use asbestos in the insulation of the field coils.

In cases where it is very difficult to obtain sufficient space for the winding without encroaching upon the width of tooth required for mechanical strength, the slot is made in two steps with a narrow portion at the bottom as shown in Fig. 109. The bottom portion contains a few conductors of the same cross-section as the upper portion, but of necessity these are thicker and narrower.

From a preliminary layout of the coils the resistance of all coils of all poles in series is calculated, and the I^2R for excitation at the limiting load conditions. Then the total surface of all coils is calculated from the perimeter or circumference of one coil (2 × depth + 2 × width), thus:

$$\text{Surface} = \text{perimeter} \times \text{m.l.t.} \times \text{no. of coils.}$$

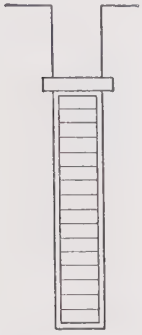


Fig. 108.—Diagram of a rotor slot showing arrangement of field conductors, wedge and space for ventilation.

Fig. 109.—Special form of rotor slot for difficult conditions.

The average watts per square inch (total I^2R divided by total surface) should run from 0.5 to 1.0 watt per square inch, although higher values are sometimes used in special cases (intermittent loads or continuous loads with exceptional ventilation). One watt per square inch would give a rise of about 100° C. The ultimate temperature of the field coils with asbestos insulation may be 125° C.

Efficiency and Losses.—The losses are friction, excitation, coreloss, armature I^2R, and load loss. The load loss is usually combined with the armature I^2R by multiplying the value of that loss by a constant from 1.15 (25 cycles) to 1.35 (60 cycles). The principal friction loss is that due to windage, is a function of ventilation, and cannot be expressed in any formula that is broadly applicable. The excitation is generally different for every load, as the field is adjusted to hold a constant terminal voltage with varying load. The coreloss is calculated as for salient-pole machines, the maximum densities being used and the loss per cubic inch per cycle at the respective densities being taken from

standard curves for the appropriate material, which is usually silicon steel ($\eta = .001$ and $\epsilon = .00007$):

Usual efficiencies are given in the attached table for the rated load at unity power factor:

TABLE XXXVII

Kv-a.	Efficiency, Per Cent	Friction, Per Cent	Excitation, Per Cent	Coreloss Per Cent	Armature, Per Cent
2,000	96	1.50	0.70	1.20	0.60
5,000	97	1.20	0.45	1.00	0.35
10,000	97.5	1.00	0.35	0.90	0.25
20,000	98	.80	0.25	0.77	0.18
30,000	98.2	.70	0.20	0.75	0.15

Heating and Ventilation.—In turbo-generators the ventilation is a most important feature, as, on account of the large powers concentrated in a small volume, temperatures would become excessive if the body of the machine were not traversed by numerous ventilating ducts, by which the air may carry away the heat from a point as near as possible to that at which it is generated. There are two methods of providing the ventilation, known as self-ventilation and separate ventilation. In the self-induced ventilation the air is impelled by centrifugal fans forming part of the revolving field, and is forced into the air gap from both ends and then flows radially outward through the armature. In the case of separate ventilation, an entirely independent blower is used, forcing air through suitable piping to, and through, the machine. In some cases the air leaving the machine is sent through coolers consisting of pipes of cold water and then returned to the blower and circulated continuously through the machine. This has the advantage of assuring a supply of clean air and avoiding the danger of clogging up the machine by sending in dust and dirt from the outside air.

There must be provided: (*a*) sufficient air to carry off the heat generated with but a reasonable rise in temperature of the machine and the air; (*b*) ample duct capacity to prevent too high velocity of the air in the ducts; (*c*) proper spacing of the ducts so that there is no part very far from an air duct; (*d*) proper precautions to

prevent the air from carrying dirt and moisture into the machine.

In general, it is desirable to supply 100 cu. ft. of air per minute for each kilowatt lost in the machine, including the loss due to windage, and it is desirable to keep the maximum velocity of the air as it passes through the ducts of the machine and the air gap to between 5000 and 6000 ft. per minute, although greater velocities are sometimes found in practice. These figures determine the space in the air gap and in the tops of the stator and rotor slots required for ventilation. The total air required for the machine is divided into two equal parts, one coming in at each end of the machine. About 20 per cent of each part passes radially through the end windings and the remaining 80 per cent goes into the air-gap space. After entering the air gap axially at either end, the air passes out radially through the ventilating ducts step by step.

Thus the air velocity in the gap decreases towards the center of the machine, and where the two streams meet at the center the velocity is zero.

In some machines there are axial ducts in the stator and rotor cores, back of the slots, and this reduces the amount which must enter the air gap. In such machines it is often the custom to use only two or three radial ducts, but these are made 3 ins. wide instead of the $\frac{1}{2}$ in. ducts more generally used (see Fig. 111).

This is called **axial** ventilation as contrasted with **radial.** Generally the ventilation of every machine partakes of both methods and the classification depends upon which principle predominates.

The method of calculating the temperature drop in the machine is the same as that given for salient-pole generators (q.v.) but some of the constants are different because of the different conditions. Thus in the composite insulation used on the coils the drop in temperature by conduction is in the neighborbood of 160° C. per watt per square inch per inch thickness of insulation, while the surface of the coils and the cores shows a rise of 10 to 12° per watt per square inch above the circulating air for air velocities of from 5000 to 6000 ft. per minute. The air itself shows a rise of about 18° when carrying off one kilowatt of loss for every 100 cu. ft. of air per minute.

In a well-designed machine operating at the conventional temperature of 125° at the hottest spot, in an ambient temperature

of 40° C. according to the limits of the standards, we should have the following probable distribution of temperatures:

Hottest spot of the copper	125° C.
Average temperature of the copper by resistance	120° C.
Temperature at the surface of the coils and core measurable by thermometer	105° C.
Temperature of the air leaving machine	60° C.
Temperature of the air entering machine	40° C.

Thus we should have in the formula for temperature drop the constant for the heat resistivity of the insulation, $k_1 = 160$,

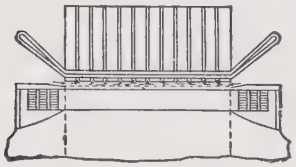


FIG. 110.— Arrangement of the ventilating ducts for the system of Radial Ventilation.

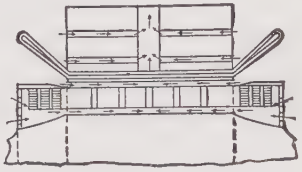


FIG. 111.— Arrangement of the ventilating ducts for the system of Axial Ventilation.

and the constant for the drop due to convection $k_3 = 10$ to 12 for air velocities of from 5000 to 6000 ft. per minute. However, in order that these constants may apply, the special subdivision of the body of the machine by air ducts must be very carefully and intelligently laid out, since, if the heat has to travel any great distance from its source of generation to a cooling surface, the drop due to conduction will be exceeded, and if there is any obstruction of the air in the ducts the velocity will be decreased and the constant for surface convection will be accordingly increased.

Sample Calculation for the Design of a Turbo-Generator.

1. Required a four-pole, 10,000 kv-a. 60-cycle generator for 6600 volts. The r.p.m. must be 1800.

2. Assume a peripheral velocity $V = 21{,}000$. Then outside diameter of rotor $D = 45''$.

3. Assume $\rho = 1$; $\sigma = 1200$; $B = 30{,}000$; $k_1 = .96$; $k_2 = .90$.

$$\text{Then } L = \frac{10{,}000 \times 144 \times 10^9}{1 \times 1200 \times 30{,}000 \times 21{,}000 \times 45 \times .96 \times .90} = 49.$$

Say, 50 ins.

4. Assume a Y connection, then $E = 3820$; $I = 875$.

$$S = \frac{\pi \times 50 \times 1200}{2 \times 3 \times 875} = 32 \text{ (trial)}.$$

5. $cs = 2 \times 3 \times S$ $\quad cs = 2 \times 3 \times 32 = 186$.
186 slots with one effective conductor.
93 slots with two effective conductors.
62 slots with three effective conductors.

6. From Table, try 72 slots with 3 effective conductors in each.
For a conventional barrel winding with two coil-sides per slot use 6 real conductors per slot and two circuits in multiple per phase. Requires 72 coils having 3 turns each.

7. Desirable cross-section of conductor. 438 amperes in each path, $U = 1500$.

$$q = \frac{438}{1500} = 0.29 \text{ sq. in.}$$

Copper per slot $= .29 \times 6 = 1.74$ sq. in.

8. From Table, asume a gap of 1.25 ins. Inside diameter of armature $= 45. + 2 \times 1.25 = 47.5$ ins.

9. Slot pitch $= \dfrac{47.5 \times \pi}{72} = 2.07$ ins. Make slot width 40 per cent of pitch.

$$w = 2.07 \times .4 = .83 \text{ in.}$$

10. Slot insulation on sides $= .31$. Width of copper in slot $b = w - i = .83 - .31 = .52$. Depth of copper $= 1.74 \div .52 = 3.40$.

11. Make conductors three strands wide $= \dfrac{.52}{3} = .17$. Say strands are $.16 \times .09$, a convenient size. Strands per conductor $= \dfrac{.29}{.16 \times .09} = 20.2$. Say $21 = 3 \times 7$ as it must be divisible by 3. Dimensions of each strand over cotton $= .168 \times .098$.

12. Final layout of slot (see text).
Slot width $= .168 \times 3 + .326 = .504 + .326 = .83$.
Slot depth $= .098 \times 7 \times 6 + 1.625 = 4.12 + 1.625 = 5.75$.
Each slot is $5.75 \times .83$ and has 6 conductors in it, and each conductor consists of 3×7 strands of $(.16 \times .09)$ rectangular wire. Each coil-side contains 3 conductors. 18 slots per pole assume $\frac{1}{1}\frac{5}{8}$ pitch of coils.

13. Turns in series per phase: $S = \dfrac{72 \times 6}{3 \times 2 \times 2} = 36$. (final)

14. Flux per pole: $k_1 = .96 \quad k_2 = .965$.

$$\phi = \frac{3820 \times 10^8}{4.44 \times 60 \times 36 \times .96 \times .965} = 43. \times 10^6.$$

15. Assume a leakage coefficient $v = 1.15$.

16. $E = 3820$ $E = 4200$.

		$\phi 10^6$	Dimensions	Area	Avg. B	Max B	H	L	Amp.-t	Max B	Amp.-t
					$\times 10^3$	10^3					
A.C.	$S_i.S.$	22	36×7.5	270	82	82	17	16	272	90	560
A.T.	$S_i.S.$	43	36×1.41×18	910	47	71	9	5.75	52	78	75
G	....	43	50×35.3	1765	24.4	36.6	11450	1.25	14300	40.3	15750
R T	F.S.	50	50×16.3	815	61	91.5	72	5.5	395	100	575
R C	F.S.	25	50×17	850	29.4	29.4	11	8	88	33	96
									15107		17056

Armature core: Desired density = 85,000.
Effective length: assume 24 air ducts of $\frac{1}{2}$ in. each

$$l = [50 - 24 \times .5] \times .95 = 36.$$

Desired area $= 22 \times 10^6 \div 85{,}000 = 258$.
Desired radial dimension, $h = 258 \div 36 = 7.2$.
Outside diameter $= 47.5 + 2 \times 5.75 + 2 \times 7.2 = 73.4$.
Call it 74, then $h = 7.5$.

Armature teeth: diameter at equivalent section $\frac{1}{3}$ distance from gap

$$= 47.5 + \frac{2 \times 5.75}{3} = 51.33.$$

Pitch at this diameter $= \pi \times 51.33 \div 72 = 2.24$ ins.
Width of equivalent section of tooth $= 2.24 - .83 = 1.41$.
18 slots per pole.

Air gap: Pole arc $= \pi \times 45 \div 4 = 35.3 \qquad L = 50$.

Rotor teeth: From table make rough assumption of 48 slots 5.5 ins. deep by 1.10 wide as outside limits. If these dimensions are exceeded later a recalculation of excitation is necessary.

Diameter at equivalent section $= 45 - \frac{4 \times 5.5}{3} = 37.7$.

$a_2 = \pi \times 37.7 \div 4 = 29.5$.
section of metal left $= 29.5 - 12 \times 1.1 = 16.3$.

Rotor core: Diameter at bottom of slots $= 45 - 2 \times 5.5 = 34$,
radius $= 17$.

$$\text{Length of path in armature core} = \frac{\pi \times 74}{8} \times \frac{50}{90} = 16.$$

$$\text{Length of path in rotor core} = \frac{\pi \times 34}{8} \times \frac{50}{90} = 8.$$

17. Armature resistance:

$$\text{M.l.t.} = 2 \times (50 + 4) + \frac{10 \times 47.5}{4} \times \frac{15}{18} = 208''.$$

$$R_A = \frac{.0107 \times 208 \times 36}{.0144 \times 21 \times 2 \times 12{,}000} = 0.011$$

and $.011 \times 1.15 = .0126$ effective.

$$\frac{875 \times .0126}{3820} = .0029 \text{ or } 0.29 \text{ per cent } IR \text{ and } I^2R.$$

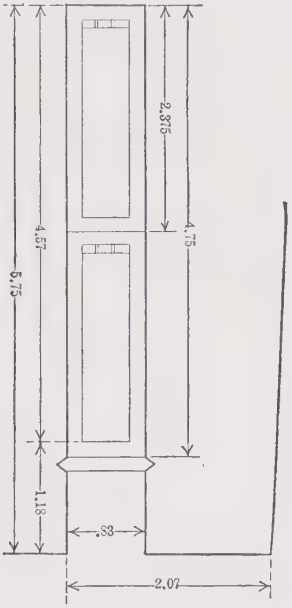


FIG. 112.—Diagram of the stator slot of the example calculated showing necessary dimensions.

Heating: From sketch, Fig. 112, we get:

perimeter of coil $= 2 \times (2.375 + .83) = 6.41.$

Surface of all coils $= 6.41 \times 208 \times 72 = 96{,}000$ sq. in.

$$\text{Watts per sq. in.} = \frac{29{,}000}{96{,}000} = 0.30, \text{ very conservative.}$$

18. Armature Leakage Reactance.

Dimensions from sketch, Fig. 112.

$u = 4.57 \qquad w = .83 \qquad p = 1.18 \qquad q = .83$

$z = 2.07 \quad l_c = 104 - 50 = 54. \quad U_c = 4.57 + 2 \times 6 \times .83 = 14.57$

$$P_s = 3.2 \times 50 \times \left(\frac{4.57}{2.49} + \frac{1.18}{.83}\right) = 520$$

$$P_t = \left(2.35 \times \log \frac{\pi \times 2.07}{1.66} + 2.1\right) \times 50 = 176$$

$$P_c = 1.17 \times 3 \times 54 \log \frac{2 \times 54}{14.57} = 165$$

$$\Sigma P = 861$$

$$x = 377 \times 4 \times 6 \times 9 \times 861 \times .96 \times .965 \times 10^{-8} = 0.645 \text{ ohm.}$$

$$\frac{Ix}{E} = \frac{875 \times .65}{3820} = 0.149 \text{ say } 15 \text{ per cent.}$$

19. Armature reactive m.m.f.

$$n = \frac{2.12 \times 36 \times 875 \times .96 \times .965}{4} = 15{,}500. \text{ amp.-turns.}$$

20. Assume effectiveness of field = 0.83.

$$n' = \frac{15{,}500 \times 1.15}{.83} = 21{,}500.$$

21. Synchronous impedance in amp.-turns.

From magnetization curve, the amp.-turns

for $Ix = 550$ volts = 2,200

for armature reaction = 21,500

Synchronous impedance = 23,700 $a - t$

22. Excitation for full non-inductive load.

$E^2 = (3820 + 10)^2 + (550)^2 \qquad E = 3835.$

Excitation m for 3835 is 15,200. $\qquad \sin \theta = \frac{550}{3835} = .143.$

$F^2 = (15{,}200)^2 + (21{,}500)^2 + 2 \times 15{,}200 \times 21{,}500 \times .143.$

$F = 27{,}100.$

Excitation for full kv-a. at 80 per cent inductive load.

$E^2 = (3040 + 10)^2 + (2280 + 550)^2 \qquad E = 4200V$

$m = 17{,}000. \qquad \sin \theta = \frac{2830}{4200} = .69$

$F = 31{,}500.$

23. Field. 48 slots, 24 coils, 6 coils per pole.

$$\text{M.l.t.} = 2 \times 54 + \frac{10 \times 40}{4} \times \frac{2}{3} = 175 \text{ ins.}$$

Assume we wish 27,100 amp.-turns per pole with an exciter voltage of 175; then the correct cross-section of conductor:

$$q = \frac{.0115 \times 175 \times 27{,}100 \times 4}{175 \times 12{,}000} = 0.103.$$

for non-inductive load we assume a loss in field of 0.28 per cent or 28 kw. at 175 volts for $F = 27{,}100$.

Then $I_F = 28{,}000 \div 175 = 160$.

Turns per pole $= 27{,}100 \div 160 = 170$.

Use 174 to subdivide into 6 coils.

Turns per coil = conductors per slot $= 174 \div 6 = 29$.

Assume a slot 1 in. wide; then width of conductor $= 1 - .15 = .85$

Thickness of conductor $= .103 \div .85 = .121$.

Say, conductor $= .85 \times .12 = .102$ sq. in.

Conductor + insulation $= .12 + .005 = .125$.

Depth of slot $= .125 \times 29 + 1.5 = 5.125$.

24. $R_F = \dfrac{.0115 \times 175 \times 174 \times 4}{.102 \times 12{,}000} = 1.14$ (final)

I_F for non-inductive load $= 27{,}100 \div 174 = 156$.

Loss $= \overline{156}^2 \times 1.14 = 27{,}800$ watts.

Surface $= 2(3.62 + 1) \times 175 \times 24 = 38{,}700$ sq. in.

Watts per square inch $= .72$.

For an excitation of 31,500 $a - t$.

$I_F = 182$, watts 37,500.

Watts per square inch $= .975$ high for 80 per cent inductive p.f.

25. Coreloss:

Volume of armature core $= 60{,}000$ cu. in.

Vol. of armature teeth $= 21{,}900$ cu. in.

Using silicon steel of $\eta = .002$ and ϵ of .00007.

In core	$B = 82{,}000$	$C_H = .0127$	$C_E = .0000235$		
In teeth	$B = 71{,}000$	$C_H = .00955$	$C_E = .0000176$		
Armature Core	$H = 60{,}000 \times .0127 \times 60$			$= 46{,}000$	
	$E = 60{,}000 \times .0000235 \times \overline{60}^2$			$= 5{,}100$	51,100
Armature Teeth	$H = 21{,}900 \times .00955 \times 60$			$= 12{,}600$	
	$E = 21{,}900 \times .0000176 \times \overline{60}^2$			$= 1{,}390$	13,990
					65,090

26. Efficiency: Assume windage and friction are 1 per cent or 100 kw.

	100 kw.	100 kw.
Friction	27.8	37.5
Excitation	65	65
Coreloss	29	29
Arm. I^2R		
All losses	221.8	231.5
Output	10,000	8000
Input	10,221.8	8231.5
Losses / Input	.0218	.028
Efficiency	.9782 for p.f. = 1.	.972 for p.f. = .8

27. Heating and ventilation: 222 kw. loss. At 100 cu. ft. air per minute per kw. loss we require 22,200 cu. ft. air per minute. This is 11,100 entering at each end.

80 per cent entering gap and slots		= 8900
Section of gap	= 220 sq. in.	
Openings in stator slots	= 60	
Openings in rotor slots	= 48	
	328 sq. on.	

328 sq. in. = 2.28 sq. ft.

$$\text{Maximum velocity of air in gap} = \frac{8900}{2.28} = 3900 \text{ ft. per minute.}$$

28. Weight:

Vol. of steel in armature core and teeth	= 81,900 cu. in.
Wt. of steel in armature core and teeth	= 23,000 lbs.
Vol. of rotor, gross, including slot	= 79,000 cu. in.
Wt. of rotor, gross, including slot	= 22,200 lbs.
Wt. of copper, in armature	= 1,450 lbs.
Wt. of copper in field	= 4,000
Total weight of steel	= 45,200
Total weight of copper	= 5,450
Weight of active material	50,650 lbs.

Examples of Design of Turbine Driven Alternating Current Generators

Column 1 gives data on a moderate-size, 25-cycle turbo-generator.

Column 2 gives the author's calculations based upon data published in the Proc. A.I.E.E. of June, 1920, for a large 25-cycle turbo-generator installed in Chicago.

Column 3 gives data on a moderate-size, 60-cycle turbo-generator for comparison with the 25-cycle machine of similar capacity given in column 1.

Column 4 gives the author's calculations based upon data published in the Proc. A.I.E.E. of June, 1920, for a large, 60-cycle turbo-generator installed in Chicago.

Column 5 gives data on a large six-pole, 60-cycle turbo-generator installed in Philadelphia and fully described in the N.E.L.A. Current News of April, 1916.

TABLE XXXVIII

A.C. Turbo-Generators

Mechanical Dimensions and Data

	1	2	3	4	5
Poles	2	2	4	4	6
Rating, kv-a	6,250	30,000	6,250	30,000	35,000
R.p.m.	1,500	1,500	1,800	1,800	1,200
Volts between terminals	6,600	9,000	4,000	12,000	13,200
Frequency	25	25	60	60	60
Armature diameter, face	36	56	40	52	68
Armature diameter, back	74	105	64	82	98
Armature, total length	50	112	52	126	125
Armature, air ducts	20–$\frac{1}{2}$	52–$\frac{1}{2}$	24–$\frac{1}{2}$	62–$\frac{1}{2}$	63–$\frac{1}{2}$
Slots, number	54	72	72	72	90
Slots, dimensions	$6\frac{3}{4}\times1.02$	$6\frac{3}{8}\times1.1$	$3\frac{1}{16}\times\frac{3}{8}$	7×1	$7\frac{1}{8}\times1\frac{1}{8}$
Conductor, size	12 (.2×.12)	27 (.17×.11)	39 (.16×.09)	36 (.15×.09)	36 (.15×.09)
Conductors per slot	6	2	2	4	4
Conductors in multiple	1	4	1	2	2
Winding pitch	$\frac{13}{27}$	$\frac{25}{36}$	$\frac{19}{38}$	$\frac{13}{18}$	$\frac{13}{18}$
Winding connection	Y	Y	Y	Y	Y
Air gap radial	1.5	2.75	.812	1.5	1.625
Rotor diameter, face	33	50.5	$38\frac{3}{8}$	49	64.75
Rotor, total length	52	112	52	126	125
Slots, number	28	44	48	40	72
Slots, dimensions	$6\frac{3}{4}\times1.05$	$6\frac{3}{4}\times1.05$	$4\frac{1}{4}\times.78$	7×1.05	$6\frac{3}{4}\times1.05$
Slots, pitch, inches	2.47″	2.42	1.89	2.56	1.88
Conductor, size	$\frac{7}{8}\times.21$	$\frac{7}{8}\times.29$	$\frac{5}{8}\times.19$	$\frac{7}{8}\times.28$	$\frac{7}{8}\times.34$
Conductors per slot	20	18	15	19	13
Field turns per pole	140	198	90	95	78
Exciter voltage	125	250	125	230	250

Electrical and Magnetic Data

Rating, kv-a	6,250	30,000	6,250	30,000	30,000
Volts per phase	3,820	5,200	2,310	7,000	7,620
Ampere per phase	550	1,925	902	1,410	1,530
S	54	24	24	24	30
ϕ	82.5	276	48.4	117.5	115
σ	1,590	1,740	1,033	1,520	1,350
V	13,000	19,800	18,000	22,600	20,200
B	29.5	31.4	30.5	24	27
n	3,300	34,500	8,300	17,000	12,900
v	1.1	1.1	1.13	1.15	1.2
U	1,520	1,920	1,800	1,650	1,575
Slot factor	.25	.276	.436	.28	.235
X, per cent	16	9	18	15	12
Synch. imp. amp-turns	7,200	42,000	11,400	25,000	19,700
Excitation, no-load	24,600	45,000	15,000	22,000	25,000
Excitation, rated load	38,000	66,000	17,640	35,000	31,000
R_a	.036	.0085	.006	.008	.0096
R_F	1.02	.57	.46	.47	.46
Friction, per cent	.80	1.00	1.6	1.00	1.30
Coreloss, per cent	.90	.74	1.0	.67	1.00
Excitation, per cent	1.20	.21	.44	.23	.21
Armature I^2R, per cent	.50	.32	.26	.20	.19
Load loss, per cent					
Efficiency at rating	96.6	97.7	96.7	96.7	97.3
Rise in temperature winding					

CHAPTER V

The Transformer

General Description.—The electrical transformer, commonly called the static transformer, is a piece of stationary apparatus used to transform alternating-current energy at one voltage to some other higher or lower voltage. The single-phase transformer consists of two electrical circuits, usually of a large number of turns, interlinked with a common magnetic circuit of iron. Since the power is approximately the same in both windings, the currents in the two windings are inversely proportional to the volt-

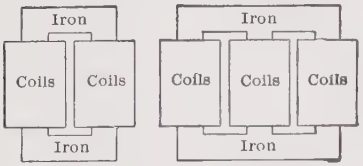


Fig. 113.—Core-type Transformers.

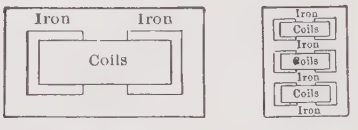


Fig. 114.—Shell-type Transformer.

ages in the windings. The polyphase transformer is essentially two or more single-phase transformers made into a single piece of apparatus, but so designed that at least a part of the magnetic circuit is common to all the phases. A three-phase transformer has three high-tension and three low-tension windings arranged on a single iron core; see Figs. 113 and 114.

Terminology.—The winding by which the energy enters the transformer is logically the **primary** and the one by which the energy leaves the transformer is called the **secondary.** Since either winding of the transformer may be connected to the source of energy, these terms are not definite unless the manner of connection is also stated. When referring to the transformer as a separate piece of apparatus, the terms **high-tension** winding and **low-tension** winding are used to distinguish the two windings, the high-tension winding being the one with the greater number

of turns. When the high-tension winding is connected to the source of supply it is the primary, and the transformer is said to be used as a **step-down** transformer; when the low-tension winding is connected to the source of supply, the high-tension winding is the secondary, and the transformer is said to be used as a **step-up** transformer.

Classification.—Transformers may be classified according to their operating characteristics, to their construction, or to the method of cooling.

Constant-potential and Constant-current Transformers.—Transformers may be either constant-potential, such as are intended to give an approximately constant potential on the secondary side, or constant-current, intended to give an approximately constant current on the secondary. Both types are intended to operate on a supply circuit of a constant potential.

Series transformers are connected in series with the main circuit and receive a variable voltage and current in the primary. The secondary circuit is closed through a path of low impedance and thus the secondary current will be proportional by the ratio of turns to the load current flowing in the primary or supply circuit. They are usually used to supply low-reading ammeters and wattmeters from circuits carrying very heavy currents.

Auto-transformers or compensators, sometimes called single-circuit transformers, consist of one electric circuit interlinked with the magnetic circuit and a tap brought off from some part of the winding. The voltage between this tap and either terminal of the electric circuit will be a fraction of the total voltage and thus a fractional voltage may be secured from this piece of apparatus. It is customary to proportion the windings on each side of the tap in accordance with the current to be carried. Auto-transformers are generally used where the ratio of voltages is quite near to unity, as in this case they can be constructed with much less copper than the regular transformer.

Potential regulators are a form of transformer in which the voltage of one member may be varied from zero to a fixed maximum either by changing the direction of the magnetic flux or by changing the phase of the electromotive force of the secondary with respect to the electromotive force of the primary.

Core and Shell Types.—Two methods of arranging the electric and magnetic circuits are in use, the corresponding construction

being designated as the **core type** or the "**shell type.**" At present a large number of lighting transformers of small and medium capacity are constructed in a manner which is a composite of the core and shell types.

Core Type (Fig. 113).—The single-phase core-type transformer consists of a single magnetic circuit interlinked with two electric

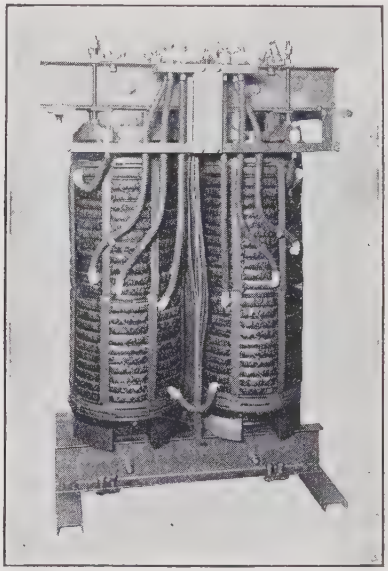


FIG. 115.—Core-type Transformer. Assembled core and coils.

circuits, each consisting of a group of coils as shown in Fig. 113. The three-phase core-type transformer is also shown in Fig. 113.

Shell Type (Fig. 114).—In the shell type of transformer each electric circuit is interlinked with two magnetic circuits having a common path inside the coils but branching outside of the coils as shown in Fig. 114.

Distributed Core Type (Fig. 120) is a compromise between shell and core type and is much used for small lighting transformers because it economizes in iron and distributes the heat. It consists of the two electric circuits wound concentrically about one central core which divides at the top and bottom into four radiating magnetic circuits surrounding the coils on four sides. It is shown in Fig. 120 in plan view.

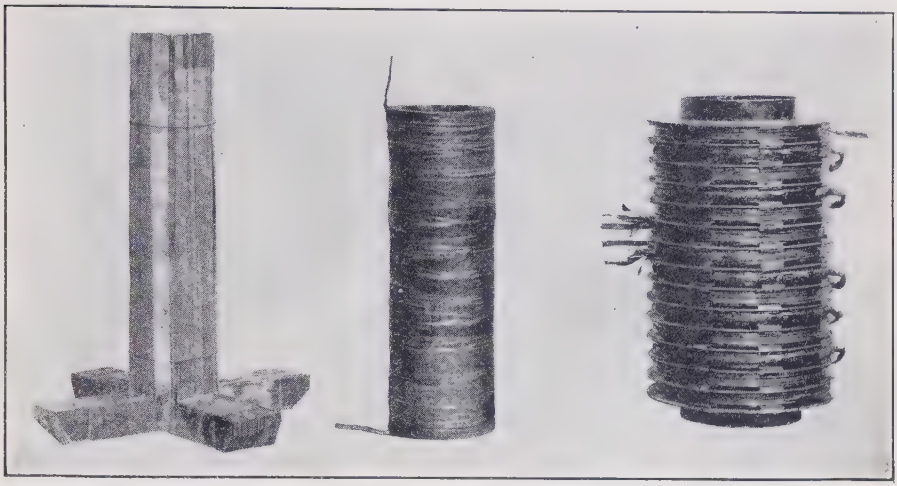


FIG. 116.—Core-type Transformer. Bare core, low tension and high tension coils.

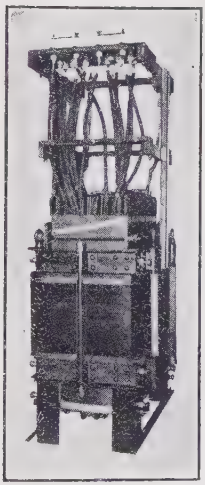


FIG. 117.—Shell-type Transformer, Assembled core and coils.

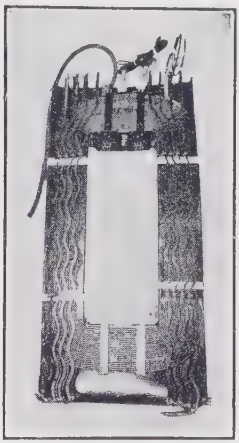


FIG. 118.—Flat coil for shell-type showing spaces to form air ventilating ducts.

Comparison of Shell and Core Type.—The core type of construction is best adapted for high-voltage low-capacity transformers, and the shell type for low-voltage high-capacity transformers. This arises from the fact that the most economical disposition of material in the core type demands a large number of turns and small cross-section of iron, while in the shell type a large cross-section of iron and small number of turns may be used to advantage. In the shell type the coils are usually wound in flat "pan-cakes," this type of construction being particularly well suited to the use of heavy copper ribbon or straps. In the core type the coils usually consist of two or more spools, long in comparison with their diameter. The use of the core type is increasing both in number and size of transformers as it is found possible to arrange the coils so they will better withstand the mechanical strain due to short circuits.

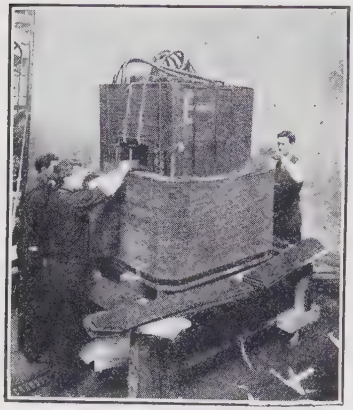


FIG. 119.—Building up the core of a shell-type around the formed coils.

Classification According to Method of Cooling.—Transformers may be subdivided into classes in accordance with the method used for dissipating the heat due to their internal losses. As a transformer is a very compact piece of apparatus, the problem of carrying away the heat is very important and various ingenious means have been devised for the purpose.

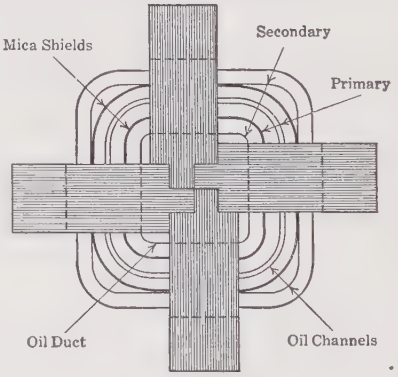


FIG. 120.—Distributed core-type. Plan view showing core and coils.

Naturally Cooled Transformers.—As the name implies, this type of transformer has no special means of cooling but relies upon the ordinary circulation of the air. It is only

used in transformers of very small sizes, such as those intended to supply meters.

Oil-cooled Transformers.—In this type of transformer the core and windings are submerged completely in oil in a tank and the windings and core are subdivided by ducts in order that the oil may circulate and carry off the heat from the internal parts. The heat is carried to the surface of the tank which contains the transformer and from there carried off by the surrounding air. The tank is specially designed to provide large air-cooling surfaces, e.g., provided with deep corrugations, or with projecting vanes or tubes.

FIG. 121.—View of assembled core and coils of distributed core-type.

Air-blast transformers are designed with special passages through which a current of air is forced by means of a blower, the heat being carried off to the atmosphere in this manner.

Water-cooled transformers consist of a construction similar to that of the oil-cooled type, and in addition a coil of pipe carrying running water is submerged in the oil.

Forced-oil Transformers.—Transformers artificially cooled by circulation of oil are used when the size is too great for the self-cooling oil type and no cooling water is available. The oil circulates through external coils or tanks which give a greater cooling surface.

Methods of Rating.—The "Standardization Rules" of the A. I. E. E. specify that the continuous rating of a transformer is that load in kilovolt-amperes which when applied continuously will cause a rise in temperature of 50° C. as measured by a thermometer, or 55° as measured by resistance of the windings. The iron of transformers takes a long time to reach a constant temperature, that of oil-cooled transformers in particular requiring

from 10 to 12 hours. Thus a transformer will stand a considerable overload for a short time without overheating, the principal danger being excessive mechanical stresses.

Transformer Principles.—The essential features of a transformer consist of a primary winding having a number of turns interlinked with a magnetic circuit, and a secondary winding also interlinked with the same magnetic circuit, as shown in Fig. 123.

FIG. 122.—Tanks for oil-cooled transformers.

Simple Theory, Neglecting Leakage Reactance.—In the following discussion the assumption will first be made that all the flux links both primary and secondary windings; the effect of the leakage flux will be discussed later.

In Fig. 123, S_1 is the primary winding, M the magnetic circuit (of iron) and S_2 the secondary winding. If an alternating current is caused to flow in S_1 it will set up a flux in M which at any instant is proportional to the current i_1. Thus:

$$\phi = \frac{4\pi S_1 i_1}{10R},$$

where S_1 is the number of turns, i_1 the instantaneous value of the current and R the magnetic reluctance of the path in M.

Thus ϕ alternates with i_1 and since it interlinks with S_2 it will induce a voltage in S_2 at any instant equal to:

$$e_2 = -S_2 \frac{d\phi}{dt} 10^{-8} \text{ volts},$$

in which S_2 is the number of turns. The negative sign means that any current due to e_2 will tend to diminish ϕ.

No-load Conditions.—When there is no current in the secondary, the vector relations are as shown in Fig. 124. Let E_1 be the voltage impressed upon S_1, then the maximum value of the alternating flux will be:

$$\phi = \frac{10^8 E_1}{4.44 f S_1},$$

where f is the frequency of alternation of E_1. Strictly, the numerator is $10^8(E_1 - r_1 I_{00})$, where I_{00} is the exciting current

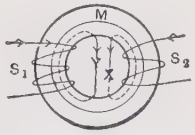


Fig. 123.—Elementary Transformer.

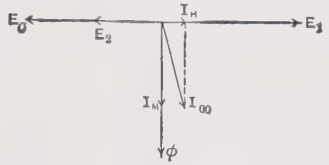


Fig. 124.—Vector diagram for transformer for No-load conditions.

and r_1 the resistance of the primary, but the term $r_1 I_{00}$ is practically negligible. This flux will lag 90° behind E_1 or 90° ahead of the counter-electromotive force E_c which it induces in S_1. The true magnetizing current I_M will be in phase with the flux and the hysteresis component of current I_H will be in phase with E_1. Hence the total no-load current I_{00} will assume some phase a little less than 90° behind E_1.

The flux, ϕ, will induce in the secondary turns, S_2, an e.m.f. 90° behind the flux, hence 180° behind E_1 or in phase with the primary counter e.m.f. The effective value of this e.m.f. is:

$$E_2 = 4.44 f S_2 \phi 10^{-8}.$$

Load on Secondary.—When the secondary is closed through a non-inductive external circuit a current will flow. This current will tend to demagnetize the iron, that is, to reduce the flux and hence the counter e.m.f. of the primary. This action, however, allows the current in the primary to increase until the secondary m.m.f. is balanced and there is left an excess m.m.f. in the primary to give sufficient flux to induce the counter e.m.f. E_1'. The vector relations are as shown in Fig. 125, again neglecting the leakage flux.

Leakage Reactance.—Due to the fact that a part of the lines of induction set up by the currents in the two windings pass through the air space (X in Fig. 123), the m.m.f.'s set up by the secondary current and the load current in the primary (the current I_1' in Fig. 125) cannot neutralize each other, since the leakage fluxes established by the two currents are in the same direction, and not in opposition as are the fluxes in the iron. The primary leakage flux is in phase with the total primary current and the secondary leakage flux is in phase with the secondary current; these leakage fluxes are therefore not in phase with the useful flux (i.e., the flux which links both primary and secondary). The result is that the leakage fluxes cause a decrease in the secondary voltage and also a shifting of its phase with respect to the primary voltage.

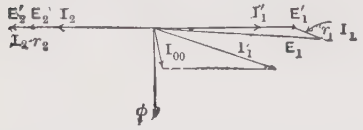


FIG. 125.—Vector diagram for full-load, no leakage reactance.

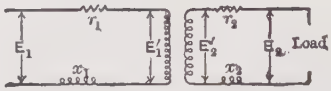


FIG. 126.—Diagram of complete circuits of a transformer.

Since the two leakage fluxes are in phase with the total currents in the primary and secondary respectively, the voltages induced by the alternation of these fluxes are in quadrature with the currents. The quotient of the voltage induced in the primary by the alternating primary leakage flux divided by the primary current is called the **primary leakage reactance,** and the quotient of the voltage induced in the secondary by the alternating secondary leakage flux divided by the secondary current is called the **secondary leakage reactance.** These reactances are practically constants, since the major portion of the leakage path is in the air.

Complete Theory of Transformer.—The leakage reactance in ohms is divided arbitrarily into two parts, primary and secondary, each expressed in terms of the number of turns in each winding. This proportioning may be easily made in the calculation of the reactance from the drawings (see page 254); but if the values are to be taken from a test the allotment is merely made by assigning an equal percentage of voltage drop in the two members:

$$\frac{I_1' x_1}{E_1} = \frac{I_2 x_2}{E_2}.$$

Hence if X represents the total reactance from test in terms of the primary, then $x_1 = \frac{X}{2}$ and $x_2 = \frac{x_1}{u^2}$.

Figure 126 shows diagrammatically the transformer circuits, and Fig. 127 the complete vector diagram.

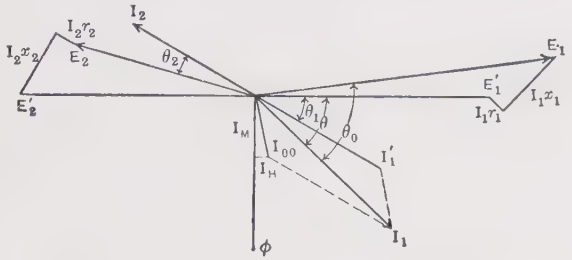


FIG. 127.—Complete vector diagram for a transformer.

Transformer Symbols

E_2 = secondary terminal e.m.f.;
I_2 = secondary current lagging E_2 by angle θ_2;
$\cos \theta_2$ = power factor of the load;
P_2 = watts output of secondary = $E_2 I_2 \cos \theta_2$;
$I_2 r_2$ = secondary resistance drop;
$I_2 x_2$ = secondary reactance drop;
E_2' = induced e.m.f. in secondary (hypothetical);
E_1' = counter e.m.f. of primary = uE_2';
u = ratio of transformation = $\frac{S_1}{S_2}$;
ϕ = mutual or useful flux at 90° to E_2';
$I_1' = \frac{I_2}{u}$ = load component of primary current;
I_{00} = primary exciting or "no load" current;
I_H = active component of I_{00} = coreloss ÷ E_1';
I_M = reactive component of I_{00} in phase with ϕ;
θ_1 = phase angle between E_1' and I_1';
I_1 = total resultant primary current;
θ = phase angle between E_1' and I_1;
$I_1 r_1$ = primary resistance drop;
$I_1 x_1$ = primary reactance drop;
E_1 = required impressed e.m.f. on primary;

$\cos \theta_0$ = power factor of primary input;
P_1 = watts input to primary;
η = overall efficiency of transformer.

Complete Analysis of Transformer

$$E_2' = \sqrt{(E_2 \cos \theta_2 + I_2 r_2)^2 + (E_2 \sin \theta_2 + I_2 x_2)^2};$$
$$E_1' = uE_2';$$
$$I_1' = I_2 \div u;$$
$$\sin \theta_1 = (E_2 \sin \theta_2 + I_2 x_2) \div E_2';$$
$$I_1 = \sqrt{(I_1' \cos \theta_1 + I_H)^2 + (I_1' \sin \theta_1 + I_M)^2};$$
$$\sin \theta = (I_1' \sin \theta_1 + I_M) \div I_1;$$
$$E_1 = \sqrt{(E_1' \cos \theta + I_1 r_1)^2 + (E_1' \sin \theta + I_1 x_1)^2};$$
$$\cos \theta_0 = (E_1' \cos \theta + I_1 r_1) \div E_1;$$
$$P_1 = (E_1' \cos \theta + I_1 r_1) \times I_1;$$
$$\eta = P_2 \div P_1;$$
$$\text{Per cent reg.} = \frac{E_1 - uE_2}{uE_2} 100.$$

Equivalent Circuit of Transformer.—In practice it is not possible to measure separately the primary and secondary reactances. They must be measured together and treated as one quantity, called the total reactance, X, of the transformer, which may be expressed in terms of either the primary turns or the secondary turns.

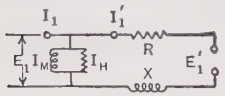


FIG. 128.—Equivalent circuits of transformer.

A simple approximate method which is sufficiently accurate for all practical purposes is based on the equivalent circuits of Fig. 128. This method is used in calculating the characteristics of a transformer from constants obtained from tests. The results are slightly optimistic.

Let E_2, I_2, $\cos \theta_2$, u, I_1', I_M, E_1, I_1, $\cos \theta_0$, η, P_1 and P_2 be as in the previous case.

Also let $E_1' = uE_2$;
R = total resistance in terms of primary
$= r_1 + u^2 r_2$;
X = total reactance in terms of primary
$= x_1 + u^2 r_2$;
I_H = coreloss $\div E_1'$;
θ_1 = angle between E_1 and I_1';

then
$$E_1 = \sqrt{(E_1' \cos \theta_2 + I_1'R)^2 + (E_1' \sin \theta_2 + I_1'X)^2};$$
$$\sin \theta_1 = \frac{E_1' \sin \theta_2 + I_1'X}{E_1};$$
$$1 = \sqrt{(I_1' \cos \theta_1 + I_H)^2 + (I_1' \sin \theta_1 + I_M)^2};$$
$$\cos \theta_0 = \frac{I_1' \cos \theta_1 + I_H}{I_1};$$
$$P_1 = E_1 I_1 \cos \theta_0 = P_2 + (I_1')^2 R + E_1' I_H;$$
$$\eta = P_2 \div P_1.$$
$$\text{Per cent regulation} = \frac{E_1 - E_1'}{E_1'} 100.$$

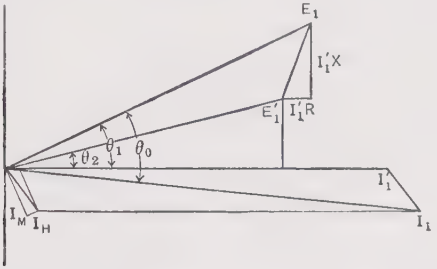


FIG. 129.—Vector diagram for the equivalent circuits.

Short-cut Empirical Method.—This process, while not strictly rational, gives very good results, slightly pessimistic, and reduces the amount of labor to a minimum. It is recommended for all cases where exact logic is not essential.

$$I_1 = \sqrt{(I_1' \cos \theta_2 + I_H)^2 + (I_1' \sin \theta_2 + I_M)^2};$$
$$E_1 = \sqrt{(E_1' \cos \theta_2 + I_1 R)^2 + (E_1' \sin \theta_2 + I_1 X)^2};$$
$$P_1 = E_2 I_2 \cos \theta_2 + I_1^2 R + E_1 I_H;$$
$$\cos \theta_0 = \frac{P_1}{E_1 I_1};$$
$$\eta = \frac{P_2}{P_1}.$$
$$\text{Per cent regulation} = \frac{E_1 - E_1'}{E_1'} 100.$$

Constant-current transformers are used for supplying current to arc lamps connected in a series circuit which requires from 4 to 10 amperes and about 2000 volts. The principle of operation for this type of transformer is based on the existence of the leakage or useless flux which causes a repelling force between the primary

and secondary windings. One winding is arranged so that it may move with respect to the other. As the current in the windings increases, the repelling force increases and the windings move apart. This allows more flux to leak or pass between the windings and less to thread the secondary. Therefore the voltage induced in the secondary decreases as the current tends to increase. A counterweight is attached to the movable winding to regulate the amount of movement. The current in the primary remains fairly constant, but the power factor decreases with decreasing load on the secondary, so that the power in the primary is approximately proportional to the output of the secondary.

Series transformers operate under conditions similar to those in a constant-potential transformer with short-circuited secondary. Under operating conditions the flux is small, as the only voltage to be induced in the secondary is that required to overcome the impedance drop in that winding and the instrument which it supplies. The only counter e.m.f. in the primary is that due to impedance drop. The series transformer must be operated with the secondary, short-circuited by a low impedance. If the secondary circuit is open there is no counter m.m.f. to balance the primary turns, which, being in series with the main line, carry a current irrespective of the secondary circuit. The primary ampere-turns would then set up a magnetic flux of considerable magnitude. As the transformer is not designed for this condition, the density would become very high in the magnetic circuit of small cross-section, and the transformer is likely to burn up from the heat due to coreloss.

DESIGN

The methods of design for single-phase and polyphase transformers are similar, the chief difference being that of the magnetic circuit. In the design of polyphase transformers each leg is treated as an independent single-phase transformer. The number of turns is adjusted to the voltage per phase, and the cross-section of the conductors to the current per phase. All legs are wound alike and the phases are connected up as described on pages 135–137.

The alternating-current transformer is the most efficient piece of electrical apparatus. Its efficiency at full load is usually better

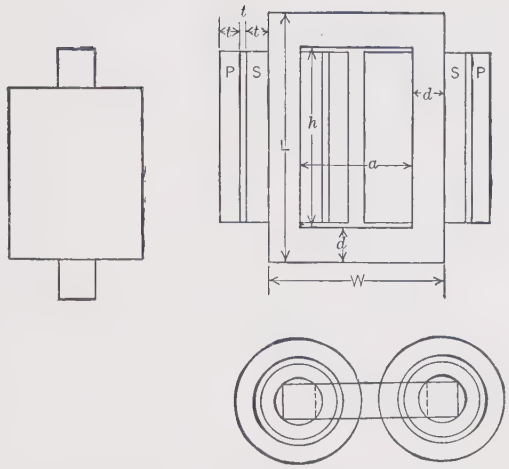


FIG. 130.—Plan and elevations showing dimensions of a core-type transformer.

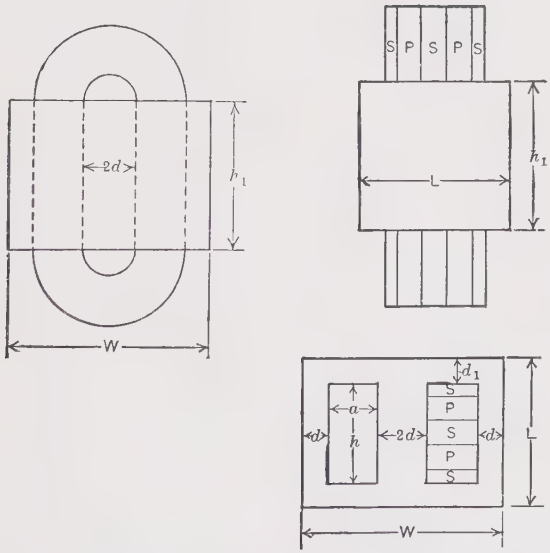


FIG. 131.—Plan and elevations showing dimensions of a shell-type transformer.

than 95 per cent and frequently better than 98 per cent. For this reason it is very small in volume and light in weight for a given output, compared with other pieces of electrical apparatus, and hence the chief difficulty in design is the necessity of providing sufficient cooling surface and a proper means to carry off the heat developed.

The difference in mechanical construction between the core and shell type is that the winding of the core type usually consists of two or more long spools (in which the length is considerably greater than the diameter) surrounding each leg of the transformer. The shell type consists usually of large flat "pan-cake" coils, laid alongside of each other on the central core with proper separating devices to permit ventilation. See Figs. 130 and 131.

The following discussion applies alike to both core and shell types except where specific mention of one or the other is made:

Preliminary Calculation of Main Dimensions

Let E_1 = primary voltage, effective value;
I_1 = primary full-load current, effective value;
E_2 = secondary voltage, effective value;
e = volts per turn;
I_2 = secondary full-load current, effective value;
f = frequency in cycles per second;
S_1 = number of primary turns in series;
S_2 = number of secondary turns in series;
ϕ = maximum value of flux;
C = an arbitrary design-constant;
B = maximum value of magnetic flux density in lines per square inch;
A = cross-section of core in square inches;
f_1 = space factor of winding space;
D = total section of copper in square inches;
U = current density in amperes per square inch;
σ = ampere conductors per inch length of core.

Determination of Flux, ϕ, and Number of Primary Turns, S_1.—. In order to estimate a desirable value of the flux to be used, there is employed a factor, C, which is defined by the relation:

$$C = \frac{\phi}{S_1 I_1} = \frac{\phi}{S_2 I_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The proper value of this factor, C, depends upon the type, capacity, and voltage of the transformer, and the relative weights of copper and iron.[1] Usual values are given in the accompanying table:

TABLE XXXIX

VALUES OF C

Form of Transformer	Voltage	Core-type Value of C	Shell-type Value of C
Natural-draft	0– 6,000	55–70	500–700
	6,000 up	70–75	500–700
Oil-cooled	0–10,000	75–100	
	10,000 up	100–150	
Air-blast or water-cooled	0–10,000	110–160	600–1000
	10,000 up	160–240	600–1000

The relation between the effective value of the voltage and the maximum value of the flux is:

$$E_1 = 4.44 f S_1 \phi \times 10^{-8};$$
$$E_2 = 4.44 f S_2 \phi \times 10^{-8}.$$
$$\phi = C S_2 I_2.$$

Inserting in equations (1) or (2) the proper values of the primary or secondary voltages respectively, the frequency, f, the constant, C, and the primary or secondary current at rated load respectively, the equation may be solved to give a value for S_1 or S_2:

$$S_2^2 = \frac{E_2 \times 10^8}{4.44 f C I_2}.$$

In the preliminary estimation the respective currents may be taken as rated volt-amperes divided by either primary or secondary voltage, as the efficiency or power factor may be ignored at this stage of the design.

It is more convenient to start by determining an approximate value for S_2, the secondary or low-voltage turns in series. Then

[1] A discussion of this factor C will be found in Arnold's "Transformers" and in S. P. Thompson's "Dynamo-electric Machinery," Vol. II.

by multiplying this number by the ratio of the voltages, the primary turns, S_1, may be obtained. This is more convenient because S_2, being the smaller number, is less susceptible of variations than the primary.

Volts per Turn, e.—A more convenient criterion of design is the value of the volts per turn which, though an empirical constant, is directly dependent upon the more general and rational constant, C. The desirable value of the volts per turn depends upon the size and frequency of the transformer, the relative amount of copper and iron to be used, and the method of insulation. A high voltage per turn means relatively more iron and less copper. A shell type of transformer naturally has much iron and therefore may have a greater value of volts per turn. Usual values of the volts per turn are given in Table XL.

TABLE XL

VOLTS PER TURN

Kv-a. Rating	CORE-TYPE				SHELL-TYPE	
	25 Cycles		60 Cycles		25 Cycles	60 Cycles
	Low	High	Low	High	Average	Average
1	.26	.40	.40	.60	.75	1.15
5	.65	.78	1.00	1.2	1.7	2.55
10	.95	1.60	1.5	2.5	2.3	3.65
20	1.10	2.0	1.7	3.0	3.3	5.10
25	1.15	2.2	1.8	3.4	3.7	5.70
50	1.8	3.4	2.8	5.3	5.4	8.30
75	2.6	4	4.0	6.3	6.7	10.30
100	3.2	4.7	5	7.2	7.5	11.60
500	7.1	8.5	11	13	17	26
1000	9.7	12	15	18	23	36

Number of Secondary Turns, S_2.—The number of turns in series in the secondary should be $S_2 = \frac{E_2}{e}$. It will considerably simplify the mechanical arrangement of the coils if S_2 is divisible by 4. So for a value of S_2 is taken that number, divisible by 4, which is the nearest approximation to S_2 as above.

With this value of S_2 accepted as fixed, the final values of ϕ and S_1 are calculated thus:

$$\phi = \frac{E_2 \times 10^8}{4.44 f S_2},$$

$$S_1 = \frac{E_1}{E_2} S_2.$$

Cross-section of Core, A.—The core section is determined by the magnetic density which it is desirable to use, which in turn is determined by the heating due to the coreloss. It is found that in average practice a loss of one watt per pound of iron can be dissipated without excessive rise in temperature. In very well cooled transformers this value may be exceeded. The densities customarily employed in different types of transformers are given in Table XLI. These values are approximate only, since the quality of the iron used varies and these densities are for silicon-steel sheets having a thickness of 14 mils, or No. 29 gauge:

$$A = \frac{\phi}{B}.$$

TABLE XLI

TRANSFORMER DENSITIES

Method of Cooling	B, Maxwells, Square Inch		U, Current Density, Amperes, Square Inch	
	25 Cycles	60 Cycles	From	To
Self-cooled	90,000	75,000 90,000	1600	2000
Forced-air	95,000 105,000	95,000 100,000	2000	2500
Water-cooled	95,000 100,000	90,000 100,000	2500	3000
Forced-oil	95,000 100,000	90,000 100,000	2500	3000

It should also be noted that transformers for distribution purposes, which operate many hours a day with a very light load, should have a lower coreloss in order to have a high **all-day efficiency.**

Design of Core Type

Assuming arbitrarily from the table a value for the magnetic density, the net cross-section of the core (central core in the distributed core type) is:

$$A = \frac{\phi}{B}.$$

This section may be assumed to be a square whose side is d. Since the core is made up of laminations whose effective length one way, due to stacking, is $0.9d$, the value of d is:

$$d = \sqrt{\frac{A}{.9}}.$$

Sometimes the core is in the form of a Maltese cross in order to make less lost space between the core and the circular coils. This applies particularly to large transformers. In this case the major dimension is:

$$d = \sqrt{\frac{4A}{3 \times .9}}.$$

In case the core is more than 3 ins. thick it should have ducts in it from $\frac{1}{4}$ in. for oil to $\frac{1}{2}$ in. for air. In this case the cross-section is laid out by trial so that after allowing for the ducts the required net section of magnetic material is left.

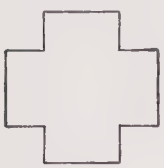


Fig. 132.—Cruciform cross section of core for core-type.

The outer limbs of the distributed-core type must have an aggregate cross-section equal to that of the central core. In the four-branch type the outer cores are $\left(d \text{ by } \frac{d}{4}\right)$.

Dimensions of Winding Space.— (Fig. 130). The area or cross-section of the **window** depends upon the amount of copper, insulation and ventilation space. The first can be determined accurately by assuming a conventional value for the current density. The window contains $S_1I_1 + S_2I_2$

ampere-turns or ampere conductors. The cross-section of copper is then:

$$D = \frac{S_1 I_1 + S_2 I_2}{U} = \frac{2 S_2 I_2}{U};$$

where U is the assumed current density as given in Table XLI.

The space for insulation and ventilation is allowed for by a space factor (f_1) which gives the ratio of the cross-section of copper (D) to the gross cross-section of the window. Usual values of the space factor (f_1) are given in the Table XLII.

TABLE XLII

Space Factor in Transformers (f_1.)

Size of Transformer Kv-a.	Core-type			Shell-type		
	Up to 2200 Volts	11,000 Volts	33,000 Volts	Up to 2200 Volts	11,000 Volts	33,000 Volts
0–50	0.22	.20	0.15	0.35		
50–1000	0.33	.30	0.25	0.40	0.33	0.18
Above 1000	0.38	.35	0.33	0.45	0.36	0.21

The cross-section of window ($h \times a$) is then $ha = \frac{D}{f_1}$ sq. in.

Height of Core (h).—This depends upon the ampere conductors per inch length of coil, which in turn governs the heating.

$$\sigma = \frac{S_1 I_1 + S_2 I_2}{2h} \quad \text{and} \quad h = \frac{S_2 I_2}{\sigma}.$$

In the distributed core type it is:

$$h = \frac{2 S_2 I_2}{\sigma}.$$

For a maximum rise in temperature of 50° C. σ should have values as in the accompanying table.

TABLE XLIII

Form of Transformer	σ
Natural-cooled, instrument	500–750
Oil-cooled or air-blast	750–1250
Water-cooled	1250–2000
Forced-oil cooled	1250–2000

From this a reasonable value can be found for h and in turn for the other factor a.

Details of Winding.—The cross-section of each individual turn depends upon whether there will be multiple circuits or not. It is convenient to lay out each winding for the highest voltage for which it may be used. Thus if the low-tension winding is to be used either for 110 or 220 volts the winding is laid out with two equal parts in series for 220 volts. Then it is easy to connect these two parts in multiple for 110 volts. In the same way the high-tension winding may be easily designed for 2200 and 1100 volts. Choosing the current for rated load of each winding respectively on this basis, we have:

The cross-section of a low-tension conductor should be $\frac{I_2}{U}$

where U is the chosen current density in amperes per square inch. Theoretically, for minimum loss, the copper density should be the same in low-tension and high-tension members; but to make a good mechanical arrangement of coils and to allow for the greater space required for insulation in the high-tension member, the current density here is usually 10 to 20 per cent greater than in the low-tension conductor. The low-potential member is usually placed next to the iron core.

In the core type there will be one high-tension coil on each leg and one or two low-tension coils on each leg. In the latter case one low-tension coil is inside the high-tension and one outside, forming a sandwich arrangement to reduce the leakage reactance. This particularly applies to the distributed core type.

After from $\frac{1}{8}$ to $\frac{1}{4}$ in. has been allowed for the insulating and protecting collars at both the top and bottom, the remaining space (of h) is taken up by adjacent turns of wire or strips of the required cross-section, each covered with double cotton covering. Some-

times square wire is used for economy of space. As many turns are placed in a layer as the available space will permit. Dividing the available height by the overall thickness of a conductor and its insulation gives the turns per layer. Dividing the required turns per coil by the turns per layer gives the number of layers. If the last layer should have only a few turns it may be desirable to readjust the height, h, to get an equal number of turns in each layer.

In the case of the high-tension coils, care must be taken that the voltage per layer does not exceed 200 volts as this would make 400 volts between the first turn of the first layer and the last turn of the second layer, which two conductors lie right next each other.

If the voltage per layer works out to be greater than 200 volts it is necessary to subdivide that particular coil horizontally into two or more coils, endeavoring to keep the volts per coil to less than 5000.

Thus if the coil on each leg has 6000 volts and has 16 layers, the volts per layer would be 375, which is excessive. But if this coil is divided into two coils each having half the height, half the turns per layer, and the same 16 layers, then the volts per layer would be 188, or within the limits. See Fig. 133.

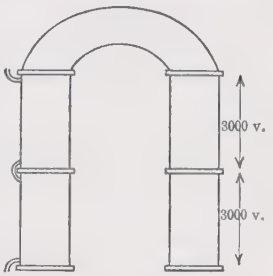


Fig. 133.—Method of subdividing high tension coils to reduce voltage strain between layers.

Between the windings and the core a layer of fiber or press board of about 40 mils is placed. Between layers there are about 10 mills of paper; and between primary and secondary, sheets of micanite of about 40 mils and usually an oil duct of .25 in. or an air duct of .375 in.

As no part of the winding should be more than $\frac{1}{2}$ in. from a cooling surface, the oil ducts should be about 1 in. apart.

The thickness of a coil is figured by adding to the thickness of the several layers the 10 mils between layers and the 40 mils on the two sides, and from this the overall physical dimensions of the coils are determined. The coils are laid out concentrically with each other and with the oil ducts and coil, and the several diameters determined, making possible a drawing as in Fig. 130 and a final determination of the values for h and a.

Insulation of Windings.—The insulation on each turn usually consists of double cotton covering, later impregnated with a suitable varnish or asphaltic compound by immersing the whole transformer in a vat of hot compound in a vacuum. If copper strap is used, then mica paper may be used upon the turns. Between layers there is placed at least 10 mils of special paper projecting beyond the first and last turn of each layer. The coils are wound on a spool of fiber or micanite, and when finished usually have two or three thicknesses of micanite on the outside.

Transformers for very high voltages have the end turns of the high-tension winding given special extra insulation between turns and between layers for about 75 ft. from each terminal. This is to protect the windings from the transients or surges caused by switching and discharges in the circuit. These high local potentials result from very sudden changes in the value of the current through the high inductance of the transformer winding and from the distributed capacity of the transformer between the high-tension winding and the core, frame, and the other winding.

Design of Shell Type (Fig. 131)

In the shell type the cross-section of the central core is:

$$A = \frac{\phi}{B} \qquad A = .9(h_1 \times 2d.)$$

The shape of the central core is a rectangle whose length h_1 is from two to four times the width, $2d$, as in Fig. 131. The greater the dimension h_1 in comparison with $2d$ the less the amount of iron required, the shorter the magnetic path and the length of idle copper, and the more economical the design. But if $2d$ is too small the magnetic structure becomes mechanically weak and flimsy. The outer limbs of the magnetic circuit have one-half the cross-section of the central core as each carries one-half the flux. Their dimensions are $(h_1 \times d)$.

Window.—There are two windows, each containing $(S_1I_1 + S_2I_2)$ ampere conductors. The cross-section of the copper in each window is

$$D = \frac{2S_2I_2}{U} \text{ sq. in.}$$

The cross-section of each window will then be:

$$ha = \frac{D}{f_1}.$$

It is general practice to make the width of window, a, about the same dimension or a little greater than the width of the central core, $2d$. That is, a = from $.75 \times 2d$ to $1.25 \times 2d$. The dimension a having been chosen arbitrarily, the value of h is found by dividing the desired cross-section by a.

In the shell type of transformer the windings are subdivided into a large number of coils of the flat, or **pan-cake,** type, each coil being only from $\frac{1}{2}$ to 2 ins. high or thick. There may be an equal number of low and high tension coils, which is usual, or the number of the low-tension coils may be one greater than the number of high-tension coils. This latter arrangement reduces the leakage reactance (q.v.)

As shell-type transformers are usually cooled by air there is a duct between coils. This duct is made as narrow as it can possibly be and still give good cooling effect, because the wider the duct the greater is the leakage flux and reactance.

If the low-tension current is quite large (hundreds of amperes), the coils are connected in multiple and the number of coils is adjusted so that the current per coil can be easily handled and will require a convenient size of conductor.

The low-tension coils usually consist of copper ribbon or strap wound flat, with a strip of insulation wound with it, as ribbon and protective paper are commonly wound in a roll. If the current per coil were to be 300 then it would be reasonable to select a strip 1 in. wide and 0.10 or 0.125 in. thick and wind this around the form that fits around the central leg ($2d \times h_1$) until the radial dimension of the coil was a little less than a. Or, the net allowable space in the direction of a may be divided by the desired turns per coil and the quotient will be the proper thickness of copper plus insulation between turns. The latter should be 8 or 10 mils.

The high-tension coils are wound with wire, the number of turns per layer being governed by the size, so that the thickness of coil between air ducts shall not be so great that the temperature differences will be excessive.

To keep the leakage reactance low an arrangement is sometimes used in which there is a half-coil of the low potential winding (half the current capacity) at each end of the winding space, as in Fig. 136. In exceptional cases, to reduce the leakage reactance still further, all coils may be divided into halves, with a ventilating duct between the halves of one coil, and only insulation between a primary half and a secondary half, as in Fig. 137.

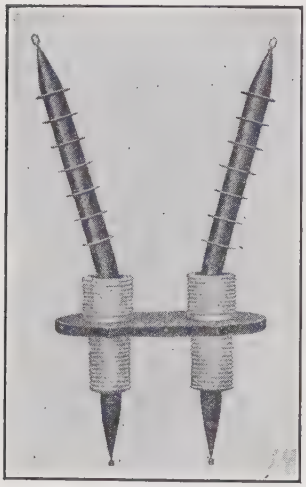


Fig. 134.—View of the high tension leads and bushings of oil-filled type.

The terminal from the inside turn of each coil has to be brought out along the side of the coil in the duct space, but firmly attached to its coil and well insulated.

Terminal Bushings.—At the point where the high-potential leads pass through the case there is a very great dielectric stress which must be taken care of by the use of a proper kind of insulation and a proper disposition of the insulation, to prevent a concentration of the dielectric flux at a few points. For voltages below 40,000 a porcelain bushing is customarily used.

For higher voltage it is necessary to supply a large creepage distance and to have the surface submerged in oil to prevent corona effect. This is accomplished in one form of bushing, known as the **condenser type,** by surrounding the terminal with layers of insulation and putting sheets of tinfoil between the layers. This arrangement is equivalent to a series of condensers. By properly proportioning the area of the successive layers of tinfoil the drop in potential across the insulation is kept uniform. The whole terminal is enclosed in an oil-filled casing. Another form of bushing, known as the **oil-filled type,** consists of a long cylinder of composition insulation which surrounds the lead and is filled with oil. The cylinder is divided into compartments to keep the oil properly distributed, and disks or collars project outward from the outside to increase the creepage distance.

Adjustment of Core Dimensions.—After the final details of the windings are arranged, the cross-section and length of core are finally settled.

Predetermination of the Performance

From the previous calculations, a drawing to scale of the transformer with its core and windings may be made. The next step is to calculate its performance, i.e., to predetermine its magnetizing current, coreloss, resistance, reactance, efficiency, regulation, and heating.

Magnetizing Current, I_M.—The final flux density will probably differ by a few per cent from the value assumed earlier in the calculation. The cross-section of core is usually proportioned to give the same magnetic density in all parts, as this condition gives minimum coreloss for a given weight of iron.

The mean length of magnetic path in the iron is measured on the drawing, or calculated. From the magnetization curve of a sample of the iron to be used, the number of ampere-turns per inch for the required density is found and multiplied by the length of path for the ampere-turns consumed in the iron. Sometimes it is desired to allow for the minute air gaps at the joints of the punchings. In most cases the layers of punchings are lapped or staggered so that the flux may either cross a butt-joint in the same layer or leak from both sides of a sheet to the two punchings in the neighboring layer. It is generally considered by authorities that with the punchings reasonably compressed the effect of this jumping of the flux is the equivalent of crossing an air gap of 2 mils or .002 in. In some cases three parts of the magnetic path are stamped from one sheet of steel and the fourth part consists of an assembled keeper or a yoke with surfaces machined, to make the air gap as small as possible. In the former construction there would probably be four joints per magnetic path, and in the latter construction two joints per magnetic path. The density in these gaps is assumed the same as in the iron in both cases, and by this means the ampere-turns required for all the air gaps would be:

$$0.313\ B \times .002 \times (\text{number of joints}).$$

The total magnetomotive force required would be the sum of that required for the iron and for the air gaps and hence the magnetiz-

ing current would be obtained by dividing this sum by the square root of 2 times S_1 or the square root of 2 times S_2, depending on whether we wished the value of the magnetizing current in terms of the primary or secondary, thus:

$$I_M = \frac{\Sigma \text{ amp.-turns}}{\sqrt{2}\, S_1}.$$

Coreloss.—The coreloss consists of hysteresis and eddy losses in the steel punchings. These losses have been considerably reduced in recent years by improvements in the manufacture and quality of the steel, the use of silicon steel, and improved methods of annealing.

The coreloss is most commonly found by referring to a curve provided by the manufacturers of the steel, which shows the coreloss per pound or per cubic inch at standard frequencies for various magnetic densities. In this case it is merely necessary to pick the total value of the coreloss per pound (including hysteresis and eddy currents) at the desired frequency and density, and multiply by the cubic inches or pounds as it may be.

In case this particular information on the steel is not given it may be necessary to refer to curves giving the hysteresis loss per cubic inch at one cycle per second for various densities and the eddy-current loss for similar conditions as mentioned in previous sections. In this case we have:

$$\text{Hysteresis loss} = C_1 f V_c \text{ watts};$$

$$\text{Eddy-current loss} = C_2 f^2 V_c \text{ watts},$$

in which C_1 is the value taken for the hysteresis loss at the suitable density at one cycle per second, C_2 is the constant for eddy-current loss, f is the frequency, and V_c is the volume of iron in cubic inches.

Copper Loss.—The mean length of turn of both primary and and secondary windings is obtained from a sketch to scale. The direct-current or ohmic resistance of each member at 75° C. is obtained by substituting the proper values in the formula·

$$R = \frac{.01lS}{12{,}000qm},$$

where l = mean length of turn of primary or secondary respectively, in inches;
S = number of turns in series, S_1 or S_2 respectively;
q = cross-section of conductor in square inches;
m = number of conductors or coils in multiple.

To allow for eddy currents caused by the leakage flux, the above resistance is increased by 15 per cent to give the effective resistance r_1 and r_2. The total copper loss is then $I_1^2 r_1 + I_2^2 r_2$.

Transformer Reactance.—As explained in the section on "Transformer Principles," the leakage of flux between the primary and secondary windings of a transformer causes a component of voltage in each member which is out of phase with the current; if the current lags very much this voltage may have a considerable component opposed to the useful voltage and may cause a loss of voltage or poor regulation. It is, therefore, necessary in designing to estimate the amount of this flux and calculate the voltage it would produce. In practice, empirical formulae are usually used, but a logically deduced formula is desirable as it is more easily adapted to unusual cases.

The phase of the current in the secondary coil is nearly opposed to that in the primary and may be assumed to be exactly opposed without any great error. The result as shown in Fig. 135 is that all these ampere conductors, both primary and secondary, tend to set up a flux in the same direction in the space between the windings and to a lesser degree in the windings themselves. A part of the flux in the intervening space interlinks with the primary turns and another part with the secondary turns. In addition the flux in the windings themselves interlinks with some of the turns. The flux in each part is proportional to the ampere-turns producing it and to the permeance of the path. In this case, since the path is in air, the permeance is the area divided by the length, where the length is the average length of the flux lines.

The leakage flux passes between the windings and one part closes its path in the iron inside the inner coil and the other part in air outside the outer coil (in the core type). The reluctance of the path between the two coils is large compared with that of the return path, because the inside path has a high permeability and the outside path has a large cross-section. The reluctance of the path between the coils is therefore accurately (and easily)

calculated and a constant used to allow for the rest of the path.

In order to calculate the inductance it is necessary to have a cross-section of the windings, showing their thickness, length, and arrangement, as in Figs. 135 and 136.

Two cases must be considered:

(*A*) Where there are as many primary coils as secondary coils, and all coils are full size (usual core type).

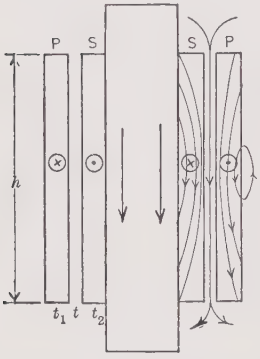


FIG. 135.—Coils and Leakage Flux in the core-type.

(*B*) Where there is a "half" secondary coil at each extremity of the group of windings (the usual arrangement in the shell-type transformers).

Arnold's Method of Calculating Reactance.—Arnold calculates the inductance of a single primary coil and its secondary mate or mates, and multiplies this by the number of primary coils in series or divides by the number in multiple. This gives the total **short-circuit** inductance of the transformer in terms of the primary voltage or turns.

Referring to Figs. 135 and 136 for the core and the shell-type respectively, let the various quantities be represented as follows, all dimensions being in inches:

q = number of primary coils in series;
p = number of primary coils in parallel;
s_1 = number of turns in one primary coil;
s_2 = number of turns in one whole secondary coil;
h = height of coils in cylinder type;
a = depth of coils in flat type or shell type;
t_1 = thickness of a whole primary coil;
t_2 = thickness of a whole secondary coil;
t = distance between coils, copper to copper:
U_1 and U_2 = mean length of primary and secondary turns respectively;
U_m = average of U_1 and U_2;
k = empirical constant, varying from 0.95 with flat coils to 1.06 with cylinder coils (for return path).

Reactance of Core Type (Fig. 135).—In a core-type transformer with cylindrical coils and an equal number of full-sized primary and secondary coils, the permeance of the path of the flux in the duct which interlinks with a primary coil is $\frac{tU_1}{2h}$ and the equivalent permeance of the path in the winding itself is $\frac{t_1 U_1}{3h}$ (see d.c. generators).

The inductance of one primary coil is:

$$L_1 = \frac{3.2ks_1^2 U_1}{h}\left(\frac{t_1}{3} + \frac{t}{2}\right)10^{-8}.$$

The inductance of one secondary coil is:

$$L_2 = \frac{3.2ks_2^2 U_2}{h}\left(\frac{t_2}{3} + \frac{t}{2}\right)10^{-8}.$$

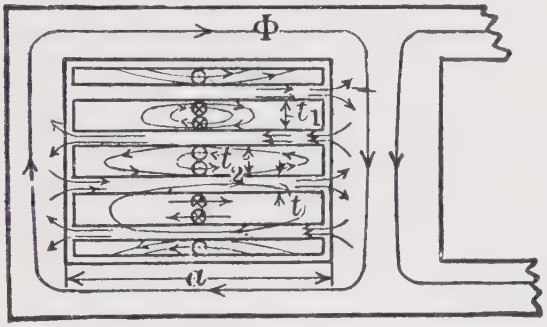


FIG. 136.—Coils and Leakage Flux in the shell-type.

Reducing the latter to terms of the primary turns by multiplying by $\frac{s_1^2}{s_2^2}$, adding together and multiplying by $2\pi f$, the total reactance of the transformer in terms of the primary is:

$$X = 2\pi f \frac{3.2qks_1^2 U_m}{ph}\left(\frac{t_1 + t_2}{3} + t\right)10^{-8}.$$

Reactance of Shell Type.—If a shell-type transformer has an equal number of primary and secondary coils, the distribution of the leakage flux between any pair of coils follows the same laws as the leakage flux in the core type, and the formula for the

reactance is the same except for the change in symbols representing certain dimensions thus:

$$X = 2\pi f \frac{3.2qks_1^2U_m}{pa}\left(\frac{t_1 + t_2}{3} + t\right)10^{-8},$$

total ohms, in terms of primary.

If, however, there is a half secondary coil at each end, then the magnetic relations are the same as if there were really twice as many primary coils, each having half as many turns and half the thickness as in reality, so that the inductance is changed by having $\left(\frac{s_1}{2}\right)$ instead of s_1, $\frac{t_1}{2}$ instead of t_1 and $\frac{t_2}{2}$ instead of t_2. On this basis the reactance of this special arrangement becomes:

$$X = 2\pi f \frac{3.2qks_1^2U_m}{2pa}\left(\frac{t_1 + t_2}{6} + t\right)10^{-8},$$

in ohms in terms of primary.

If all the coils are divided into halves and the ducts placed between the two halves of one member, then t becomes the thickness of insulation between primary and secondary, which, being small, makes the reactance much lower. The ducts then become a dead or inactive space.

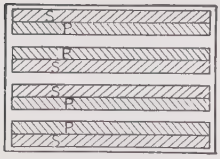


FIG. 137.—Sub-divided coils in the shell-type for low leakage reactance.

Efficiency and Regulation.—The efficiency and regulation may be calculated by any of the three methods given on pages 235–237.

Cooling of Transformers.—It is necessary to keep the temperature of the various parts of a transformer within such limits that the materials of which it is constructed do not become damaged and deteriorate too rapidly. When subjected to too high a temperature the insulating materials disintegrate and lose their mechanical and dielectric strength, and the iron deteriorates in its magnetic qualities so that the coreloss becomes greater for a given density. This so-called aging of the iron may cause an increase of as much as 50 per cent in the loss in the iron. The effect varies with the character of the iron and is practically negligible in silicon steel.

Maximum Allowable Temperatures.—The maximum temperature at which the various materials of a transformer should be operated are given in the table:

TABLE XLIV

Material	Temperature C.
Iron	95–100
Cotton	85
Paper	95
Mica or asbestos	115

The maximum allowable temperature rise measured by resistance and checked by thermometer, is 55° C.

Means of Dissipating Heat.—The problem is to provide a path of low thermal resistance by which the heat energy may pass to the surrounding air. To accomplish this it is necessary, first, to provide sufficient surface in the subdivided parts of the coils and core to transfer the heat to the cooling agent, air or oil, without too great a difference in temperature; second, to subdivide the transformer so that no part of the iron is more than 1 in. and no part of the copper more than $\frac{1}{2}$ in., from a cooling surface; third, to provide a sufficient quantity of the cooling agent, air, oil or water, to carry away the heat at the same rate as it is being generated; and fourth, to provide sufficient surface on the containing case or tank to transfer the heat from the internal oil to the external air without too great a difference in temperature.

Calculation of Exposed Surface of Transformers.—The practical method of estimating the rise in temperature in a transformer consists of calculating the drop in temperature in the successive media through which the heat must pass to be dissipated. The first step is to calculate the exposed or superficial surface of the core, core ducts, coils and coil ducts. Calling A_i the gross surface of the iron in square inches and A_c the gross surface of the coils in square inches, we endeavor to meet the following normal relations:

For a rise in temperature of 55° above the surrounding air the allowable watts loss per square inch should be within the following limits:

TABLE XLV

ALLOWABLE WATTS PER SQUARE INCH

	Coil Surface	Iron Core Surface
Oil-immersed self-cooled	0.40	1.5
Air-blast	0.75	1.0
Forced-oil cooled	1.00	1.75
Oil-immersed water-cooled	.85	1.75

In the case of the coils the rate is found by dividing the sum of the I^2r losses by the total exposed surfaces of all the coils A_c.

In the case of the core the rate is found by dividing the total coreloss by the total exposed surface of the iron core, A_i.

Naturally air-cooled transformers (instrument transformers) should have from 4 to 6 sq. in. for each watt loss.

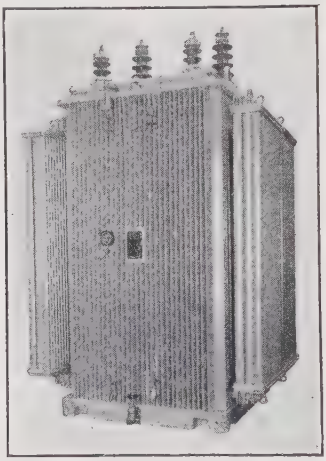


FIG. 138.—Large oil-cooled transformer showing corrugated tank to increase cooling surface.

Oil-immersed Self-cooled Transformers.—The transformer is submerged in oil in a tank so that the level of the top of the oil is at least 3 ins. above the top of the transformer proper (in large sizes 6 to 10 ins.). The oil ducts should be $\frac{1}{4}$ to $\frac{3}{8}$ in. wide and arranged so that the flow is vertically upward through the transformer and down along the periphery of the tank.

The quantity of oil required is from 5 to 10 lbs. of oil per kilovolt-ampere of rating, or 250 to 500 lbs. per kilowatt of loss in the transformer, depending upon the potential of the high-tension winding, thus:

TABLE XLVI

POUNDS OF OIL PER KV-A. RATING

Potential, Volts	Oil	
	Small, Lbs.	Large, Lbs.
2,200	8	6
6,600	10	8
11,000	12	10
33,000	14	11

The tank surface (not including top and bottom) should be from 3 to 6 sq. ins. per watt loss of the transformer if smooth, and 6 to 10 sq. ins. of radiating surface if the tank is corrugated, as the air does not circulate as rapidly in the grooves as over the surface of a smooth tank. A greater specific surface is required in large transformers because there is a greater difference between maximum and mean temperature of the oil.

For a rise in temperature of 55° C. in the winding and core of the transformer, it is expected that there will be about 5° drop between the inner conductors and the surface of the coils, about 2° between the surface of the coils and the oil, and about 40° to 45° between the maximum temperature of the oil and the surrounding air. The oil circulates through the ducts in the transformer by convection, absorbs heat and rises to the top, then passes outward to the walls of the tank where it gives up its heat and flows down to the bottom of the tank. Thus the oil at the top of the tank is the hottest, the rise being from 1.3 to 1.5 times as great as that at middle height. The difference between the maximum temperature of the oil and the surrounding air is from 130 to 160° per watt per square inch of cooling surface of the tank for smooth tanks. With corrugated tanks this figure becomes 160 to 200° per watt per square inch of gross or total surface of corrugations.

On this basis the formulae for the maximum rise in temperature of the coils and core by thermometer above the ambient temperature would be:

$$T_c = \frac{W_c}{A_c} \times 2 + \frac{W_c + W_i}{A_T} \times k_1,$$

$$T_i = \frac{W_i}{A_i} \times 2 + \frac{W_c + W_i}{A_T} \times k_1,$$

where W_c = total copper loss watts;
W_i = total coreloss;
A_c = cooling surface of coils, square inches;
A_i = cooling surface of core;
A_T = cooling surface of tank (exc. top and bottom);
k_1 = 130 to 160 for smooth tanks;
160 to 200 for corrugated tanks.

(2° per watt per square inch assumed core and coil to oil.) The drop between the oil and the iron of the tank is negligible.

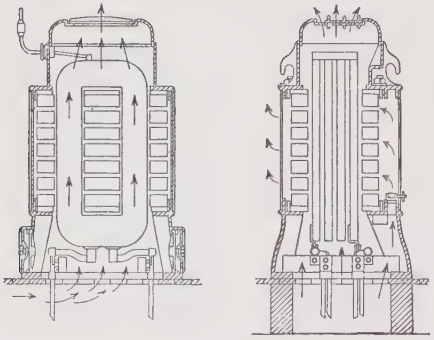


FIG. 139.—Cross-section of an air-cooled transformer showing paths of the air in coils and in iron.

Air-blast Transformers.—For this method of cooling the shell type of construction is almost universally employed. The transformer is usually set over a large air duct in the floor and air is supplied to this duct by a centrifugal blower driven by a motor. The pressure impressed is from $\frac{1}{2}$ to 1 ounce ($\frac{3}{4}$ to $1\frac{1}{2}$ ins. of water level). The supply ducts are proportioned so that the velocity of the air is about 500 ft. per minute. The air enters the transformer at the bottom and is divided into two streams; one passes vertically

through the coils, and the other, after rising at one side, passes horizontally through the ducts in the iron core. There is a damper or valve at the exit of each stream so that the proper amount of air is provided for each part independently. Ducts $\frac{1}{2}$ in. wide are provided every 2 ins. in the iron, and ducts $\frac{3}{8}$ to $\frac{1}{2}$ in. wide are provided every inch in the coils. These cooling surfaces are adjusted so that the core surface operates at about one watt per square inch and the surface of the coils at about 0.75 watt per square inch. A liberal amount of air for a rise of 55° C. is 140 to 150 cu. ft. per minute per kilowatt loss. In the ducts inside the transformer the air may have a velocity of from 1000 to 2000 ft. per minute. The difference in temperature between the surface of the coils and the air at any particular point is:

$$T_s = \frac{32{,}000}{V} w,$$

where w = watts per square inch;

V = velocity of the air, feet per minute.

The air is expected to rise from 15° to 20° C. in its passage through the transformer. This corresponds to 0.75 watt per square inch on the coils and an air velocity of 1500 ft. per minute.

The theoretical quantity of air required is:

$$Q_a = \frac{1.65W}{T_a} \text{ cu. ft. per minute,}$$

where W is the total watts loss and T_a the assigned rise in temperature of the air.

The rise in temperature of the surface of the coils and the iron is given by the relation:

$$T_c = \frac{32{,}000}{V} \frac{W_c}{A_c} + T_a,$$

$$T_i = \frac{32{,}000}{V} \frac{W_i}{A_i} + T_a.$$

Water-cooled Transformers.—The cooling coils are suspended in the tank in the oil near the top, usually above the transformer proper, and carry a continually circulating stream of water. It is

customary to provide $\frac{1}{6}$ gallon of water per minute for each kilowatt of loss and to allow the outgoing water to rise 25° C. above the incoming. There should be one square inch of surface of water coils, between oil and water, for each watt loss. There would be a drop of 20 to 25° C. in the oil, 2 to 3° between oil and water, and 20° to 25° in the water. In all cases the 5° to 15° C. drop between copper and surface of windings is additional and not part of the observable rise of 55° C.

$$Q_w = \frac{.031W}{T_w} = \text{lbs. of water per minute,}$$

where T_w = rise in temperature of water.

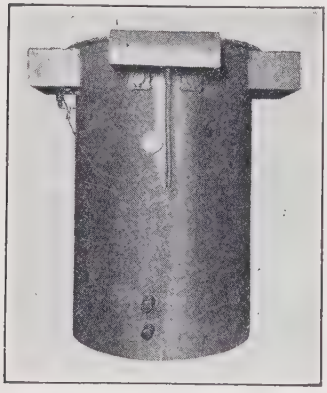


FIG. 140.—Outside view of a large transformer for forced-oil cooling.

Forced-oil Cooled Transformers.—In this type the cooling surface of the tank is supplemented by the surface of cooling coils external to and frequently separate from the tank, through which the oil flows and gives up its heat to the surrounding air. It is figured that these cooling coils will dissipate one watt of loss for every 5 or 6 sq. ins. of surface. A suitable pump is supplied to circulate the oil, and the rate at which the oil should be circulated is determined by the amount of heat each unit of oil will carry for a given difference in temperature. Thus this quantity of oil should be, by theory:

$$Q_0 = \frac{.08W}{T_0} \text{ lbs. per minute,}$$

where Q_0 = lbs. of oil per minute;

W = watts loss to be carried off;

T_0 = rise in temperature of oil in C.

The specific heat of oil is taken as 0.4 of that of water.

EXAMPLE OF TRANSFORMER DESIGN

Required: A 10-kv-a., 60-cycle transformer for either 2200 or 1100 volts on high tension and 220 or 110 volts on low tension.

To be of the oil-immersed self-cooling type for distribution purposes, high all-day efficiency, low coreloss. Choose the core type.

Call the high-tension side the primary, then $I_1 = 4.55$ and $I_2 = 45.5$.

Try $C = 100$, then

$$S_2^2 = \frac{220 \times 10^8}{4.44 \times 60 \times 100 \times 45.5} \text{ and } S_2 = 135.$$

Try volts per turn: $e = 2$, then $S_2 = \frac{220}{2} = 110.$

Choose a compromise value, $S_2 = 120$.

Then $S_1 = 120 \times \frac{2200}{220} = 1200 \qquad u = 10.$

$$\phi = \frac{220 \times 10^8}{4.44 \times 60 \times 120} = 683{,}000.$$

Try $B = 85{,}000$, then $A = \frac{683{,}000}{85{,}000} = 8$ sq. ins.

$$d = \sqrt{\frac{8}{0.9}} = 3 \text{ ins.}$$

Try $U = 1500$, then $D = \frac{2 \times 120 \times 45.5}{1500} = 7.3$ sq. ins.

Assume $f_1 = 0.22$, then $ha = \frac{7.3}{.22} = 33$ sq. ins.

Assume $\sigma = 750$, then $h = \frac{120 \times 45.5}{750} = 7.3$ ins.

$$a = 33 \div 7.3 = 4.5 \text{ tentative.}$$

Low-tension winding: size conductor $= \frac{45.5}{1500} = .0305$ sq. in.

Equivalent to No. 4 B. & S., 41,700 c.m., diameter = 0.204 bare and 0.216 d.c.c.

Turns per layer $= \frac{7.3 - .25}{.216} = 33.$ Say two layers of 30. Sixty turns on each leg.

Thickness of coil $= .04 + .216 + .010 + .216 + .04 = 0.522$ in., say 0.53 (the .04 is the insulation on inner and outer surface).

High-tension winding: size conductor $= \frac{4.55}{1200} = .00305.$

Choose a No. 14 B. & S. 4106 c.m., diameter = .064 bare and .075 d.c.c.

$$\text{Turns per layer} = \frac{7.05}{.075} = 94. \quad \text{Layers} = \frac{600}{94} = 6.4.$$

Say 6 layers of 86 and one of 84, making 600 turns per leg or per spool

$$\text{Volts per layer} = \frac{1100}{7} = 157, \text{ safe.}$$

Thickness of coil = $.075 \times 7 + .01 \times 6 + .04 \times 2 = 0.665$ in., say .67.
Height of low-tension coil = $.216 \times 30 = 6.5$.
Height of high-tension coil = $.075 \times 86 = 6.45$.
Make $h = 6.5 + .25 = 6.75$ final.
Calculate distance center to center of two upright cores.
Diagonal of rectangular core = $3 \times \sqrt{2} = 4.25$.
Radius of inner surface of LT = 2.125 + clearance = 2.15.

Inner radius secondary	2.15
Thickness secondary	.53
Oil duct	.25
Thickness primary	.67
Thickness primary	.67
Oil duct	.25
Thickness secondary	.53
Inner radius	2.15
Center to center of cores	7.20
Say	7.25

$a = 7.25 - 3 = 4.25$ final.

Mean length magnetic path = $2(6.75 + 3 + 4.25 + 3) = 34$ ins.

$$B = \frac{683{,}000}{3 \times 3 \times .9} = 84{,}500. \quad \text{Final.}$$

Requires 20 ampere-turns per inch.

Ampere-turns for iron = 20×34	=	680
Ampere-turns for joints = $84{,}500 \times .313 \times .002 \times 4$	=	212
		892

$$I_M = \text{magnetizing current} = \frac{892}{\sqrt{2} \times 1200} = 0.525.$$

Coreloss: Vol. of iron = $34 \times 8.1 = 275$ cu. ins. 77 lbs.
Coreloss for $B = 84{,}500$ $C_1 = .0065$ $C_2 = .000025$.

Hysteresis = $.0065 \times 60 \times 275$	=	108
Eddy = $.000025 \times 60 \times 60 \times 275$	=	25
Coreloss		133

Resistances:

Low-tension, mean radius of coil = 2.42 ins. Mean length of turn = 15.2.

$$r_2 = \frac{.01 \times 15.2 \times 120}{.0328 \times 1 \times 12{,}000} = .047 \text{ and } .054 \text{ effective.}$$

High-tension winding: mean radius = 3.27 ins.

Mean length of turn = 20.6.

$$r_1 = \frac{.01 \times 20.6 \times 1200}{.0032 \times 12000} = 6.5 \text{ and } 7.5 \text{ effective.}$$

Reactance:

(For 2200-volt connection).

$q = 2$ $\quad p = 1$ $\quad s_1 = 600$ $\quad h = 6.75$ $\quad U_m = 17.9$

$t_1 = .67$ $\quad t_2 = .53$ $\quad t = .25$ $\quad k = 1.$

$$X = 2\pi \times 60 \frac{3.2 \times 2 \times 1 \times \overline{600}^2 \times 17.9}{1 \times 6.75} \left(\frac{.67 + .53}{3} + .25 \right) \times 10^{-8}$$

= 15 ohms. Total reactance in terms of high tension.

$R = r_1 + u^2 r_2 = 7.5 + 5.4 = 12.9$

$$I_M = .525 \qquad I_H = \frac{133}{2200} = .06.$$

Efficiency and Regulation by Short Cut.

$$I_1 = \sqrt{(4.55 + .06)^2 + (.525)^2} = 4.61$$

$$E_1^2 = \sqrt{(2200 + 60)^2 + (69)^2} = 2260$$

$$P_1 = 10{,}000 + 133 + 272 = 10{,}405$$

$$\text{Eff.} = 1 - \frac{405}{10{,}405} = 0.961$$

$$\text{Reg.} = \frac{2260 - 2200}{2200} = 0.027 \text{ or } 2.7 \text{ per cent.}$$

For a load of 10 kw. at 100 per cent power factor.

Heating:

Surface of coils = $4 \times 6.75 \times 15.2 + 4 \times 6.75 \times 20.6 = 970$ sq. ins.

Surface of tank should be = $405 \times 4 = 1620$ sq. ins.

Height of tank should be 20 ins.

Perimeter of tank should be 80 ins.

Tank = 10×30.

Tank cubic contents 6000 cu. ins.
Surface of core $= 12 \times 34 = 408$ sq. ins.
Quantity of oil $= 500 \times 0.405 = 202$ lbs.
One cubic inch of oil weighs .03 lb.
202 lbs. requires 6750 cu. ins.
Volume of core and coils $= 565$ cu. ins.
Tank volume should be $= 6750 + 565 = 7315$ cu. ins.
Tank should be 24 or 25 ins. high.

Surface $= 24 \times 80 = 1920.$

$$T_c = \frac{272}{970} \times 2 + \frac{405}{1920} \times 130 = 28°.$$

$$T_i = \frac{133}{408} \times 2 + \frac{405}{1920} \times 130 = 27.$$

Weight of iron	=	77 lbs.
Weight of copper	$= .0328 \times 15.2 \times 120 \times .32 = 18.5$	
	$.0032 \times 20.6 \times 1200 \times .32 = 25.5$	44
Weight of active material		121 lbs.

$$D = 0324 \times 120 + .0032 \times 1200 = 7.76$$

$$f_1 = \frac{7.76}{6.75 \times 4.25} = .272.$$

$$U = \frac{2 \times 1200 \times 4.55}{7.76} = 1400.$$

TABLE XLVII

Examples of Transformer Design

	1	2	3	4	5
Form of cooling	Oil	Oil	Oil	Oil	Air
Type of core	Distrib.	Core	Distrib.	Distrib.	Shell
Frequency	60	60	60	60	25
Rating, kv-a	5	10	20	50	75
High-tension voltage	2200	2200	2200	6600	2500
High-tension amperage	2.3	4.58	9.16	7.6	30
Low-tension voltage	220	220	220	230	320
Low-tension amperage	23	45.8	91	219	234
Design constant C	205	125	222	98	1550
Volts per turn, e	1.67	1.83	3.45	3.6	11.4
High-tension winding:					
Total series turns	1320	1200	640	1836	219
Coils in series	2	2	2	2	3
Coils in multiple	1	1	1	1	1
Conductor size	No. 16	No. 14	.105×.09	.085×.085	34×.08
Resistance 25°	11	6.5	1.43	5.85	.49
Low-tension winding:					
Total series turns	132	120	64	64	28
Coils in series	2	2	4	4	4
Coils in multiple	1	1	1	2	1
Conductor size	No. 5	No. 4	2(.23×.155)	4(.415×.113)	6(.34×.09)
Resistance 25°	.08	.047	.0204	.0053	.0095
Flux density	58	85	63	57.2	67.5
Core length	4	3	4.6	5.125	8
Core width	3	3	4.6	5.125	21
Window height	6	6.75	9.2	15.8	5.5
Window width	2	4.25	3.1	6	8
Magnetizing current	.03	.52	.432	.125	1.82
Coreloss, watts	40	133	164	900	1520
No-load current, per cent	1.3	12	4.8	1.6	6.9
Coreloss, per cent	.8	1.3	.8	1.7	1.96
HTI^2R, per cent	1.4	1.3	.59	.7	.57
LTI^2R, per cent	1.0	.95	.81	.5	.67
Efficiency at rating	96.8	96.5	97.8	97.1	96.8
Total IX drop, per cent	2	3.1	2.9	4	1.36
Average amperes per square inch, U	1000	1400	1100	1100	1200
Space factor, f	.50	.27	.37	.28	.25

CHAPTER VI

THE INDUCTION MOTOR

An induction or asynchronous motor is an alternating-current motor of approximately constant speed, and resembles in its operating characteristics the direct-current shunt motor.

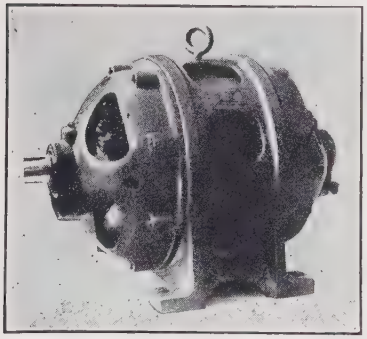


FIG. 141.—Complete Induction Motor.

It consists of an external stationary part, called the **stator,** and an internal revolving part or armature, called the **rotor.** The stator has a distributed polyphase drum winding placed in slots on its internal periphery, very much like the stator of a revolving-field a.c. generator. The rotor has a similar winding in slots on its outer periphery like the armature of a revolving armature a.c. generator.

One member, usually the stator, is connected to the source of electrical supply and is called the primary. The other member, called the secondary, is short-circuited upon itself (or through a low resistance for purposes of starting) and receives its energy from the primary by electro-magnetic induction as in the **transformer.**

The polyphase alternating currents flowing in the primary set up a rotating magnetic field, according to the same laws by which the armature reaction of a polyphase generator sets up a rotating magnetic field. This magnetic field cuts the conductors of the secondary, and generates an alternating voltage in them. As a result of the closed secondary circuit, a current flows. This current reacts on the primary flux, and produces a torque and motion. The secondary is dragged around by the rotating

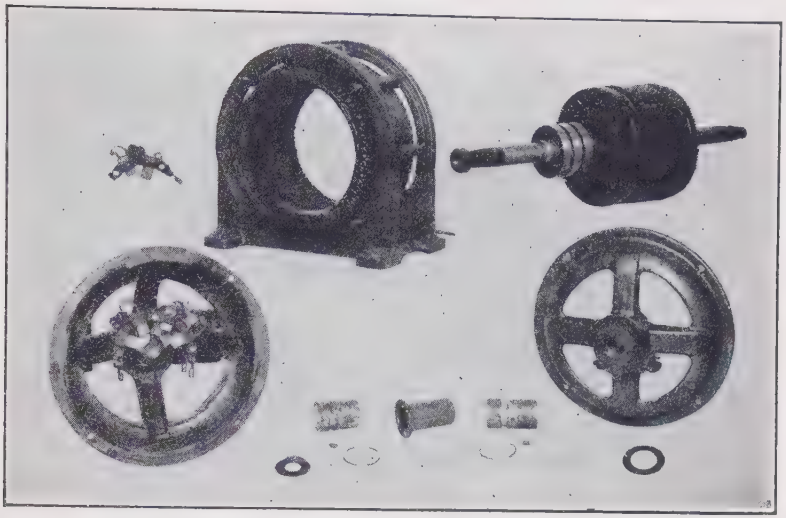


FIG. 142.—Induction Motor taken apart showing parts.

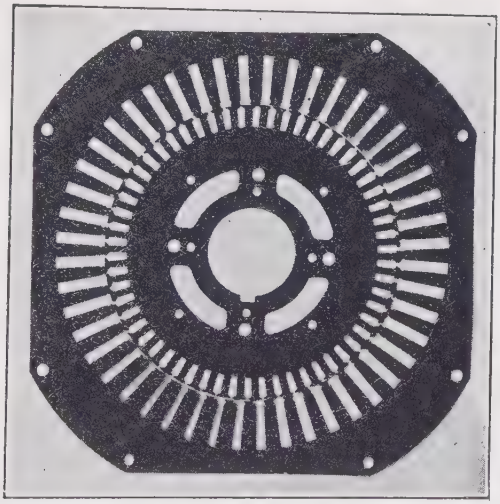


FIG. 143.—Punchings of sheet steel for induction motor. Outside, stator with open slots. Inside, rotor with partly closed slots for a bar winding.

magnetic field and tries to keep up with it. However, the secondary always rotates at a slightly lower speed than the primary flux as there must be a difference in speed in order that a voltage may be generated in the secondary. The voltage generated in the secondary is proportional to the difference between the speeds of the primary flux and the secondary winding. Thus the difference in speeds is what causes the secondary voltage and current. Hence the difference in speeds is roughly proportional to the torque and output of the motor. This is reasonably exact under normal operating conditions near synchronous speed.

The rotating magnetic field of the primary revolves at a speed fixed by the number of poles and the frequency of the currents supplied to the motor. This is called the **synchronous speed** and is:

$$N = \frac{120 \times \text{frequency}}{\text{poles}}.$$

The difference in speed is called the **slip,** and is expressed as a percentage for convenience, thus:

$$s = \frac{N - \text{rotor speed}}{N}.$$

At rated load the slip of small motors is between 6 per cent and 8 per cent, and of large motors between 2 per cent and 3 per cent. It is very nearly the same as the percentage I^2R loss in the secondary, just as the drop in speed of a shunt motor is very closely proportional to the percentage of IR drop in the armature. The lower the copper loss in the secondary, the less is the slip and the better the speed regulation of the motor.

The secondary winding must be arranged for the same number of poles as the primary but may have a different number of phases and a different fractional pitch. The secondary may be definite-wound (standard polyphase winding, Fig. 142) or may be a squirrel-cage winding, Fig. 144. The squirrel cage has one bar per slot and each end of every bar electrically joined to conducting rings at the two ends of the core. With the definite winding it is possible, by means of collector rings, to connect an external resistance into each of the phases of the secondary for the purpose of controlling the starting and speed of the motor. The currents set up in the secondary of an induction motor produce a rotating magneto-

motive force, which travels in the same direction and at the same speed as the primary flux, but at any point the direction or polarity of this secondary m.m.f. is opposed to the polarity of the primary m.m.f. Hence the resultant flux is due to the difference between these two m.m.f.'s and remains practically constant irrespective of the value of the secondary currents, just as the resultant flux in a transformer is practically independent of the secondary current, the primary current increasing and decreasing automatically with the secondary current. The component of primary current which is necessary to produce this *resultant* flux is called the

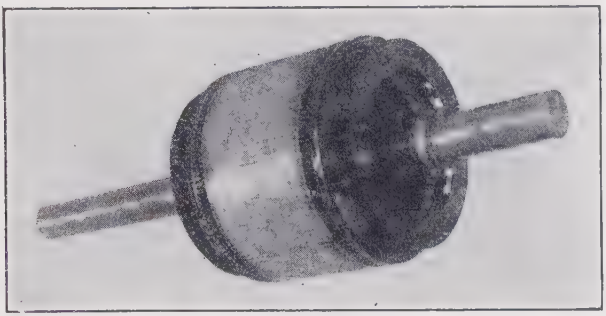


FIG. 144.—Squirrel cage rotor showing narrow slot openings, end ring and fan for ventilation.

magnetizing current and is very nearly equal to the current in the primary when the motor is running without load, in which case the current in the secondary is extremely small.

The distribution of the windings of a polyphase induction motor is exactly the same as that of a polyphase generator, and the magnetizing current flowing in this winding produces a magnetomotive force in the same manner as the load current of a generator produces the armature m.m.f. But the magnetizing current flowing in these windings produces the m.m.f. which sets up the mutual or useful flux in the motor. In a three-phase machine with full-pitch coils, the shape and magnitude of this m.m.f. varies slightly between two limits. When the current in one phase has its maximum instantaneous value the current in the other two phases has an instantaneous value of one-half the maximum, in which case the distribution of these currents along the pole pitch gives a shape of m.m.f. as in Fig. 145*a*, in which the

maximum value is 1.72 times the average. One-twelfth of a cycle, or 30° of time phase later, the current in phase 1 will be 87 per cent of the maximum, that in Phase 2 will be 87 per cent also, and that in Phase 3 will be zero. The distribution of these currents along the pole pitch gives a shape like that shown in Fig. 145*b*, in which the maximum is 1.5 times the average. One-twelfth of a cycle later the current in Phase 1 will be equal to one-half maximum, that in Phase 2, maximum, and that in Phase 3, equal to one-half of maximum. This distribution of currents will produce an m.m.f. curve exactly the same as the first (145*a*) but displaced along the pole pitch or air gap by one-sixth of a cycle

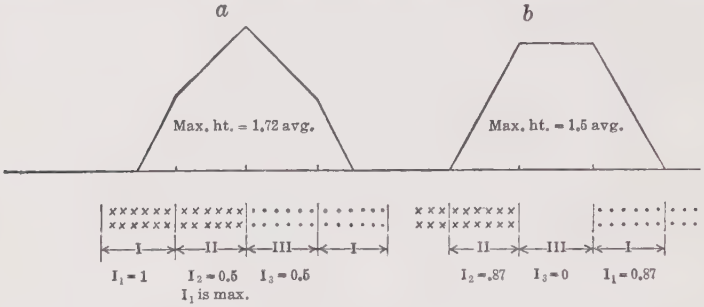


FIG. 145.—Distribution of the m.m.f. of the primary for two limiting conditions.

or 60 electrical degrees. Therefore at all times the shape will vary between these two limits, reaching one or the other of them every twelfth of a cycle. If fractional pitch is used the conditions are slightly more complicated and the resultant m.m.f. is not as great as in the case of full pitch, but the ratio of maximum to average remains about the same.

If the secondary of the motor is open-circuited the flux will try to assume a shape similar to these m.m.f. curves, and would do so if all the reluctance were in the air gap. But, on account of saturation in the teeth at the center of the wave, the peak is somewhat lowered by the reduced permeability of the teeth. However, when the secondary circuits are closed on themselves and the secondary winding is distributed in a considerable number of slots, as is usual, a current will flow in the secondary, which reacts on the primary and causes the flux to assume a smooth shape that approximates a sine wave except that in the case of considerable saturation in the teeth the wave is flattened.

This flattening of the flux wave reduces the maximum magnetic density in the center teeth and thereby the required m.m.f. for a given gross value of flux per pole. This is often found in practice and is the cause of the fact that where the calculations indicate an extreme density in the teeth and high magnetizing current the tests will show a much lower value of magnetizing current because the wave shape has been flattened and the theoretical magnetic densities in the central teeth do not exist.

Voltage.—Motors may be wound for a line voltage up to 13,000 but the great majority of motors, and all small motors, are wound for voltages of 110, 220 or 440, between lines. Other standard voltages are 1100, 2200, 4400, 6600, and 11,000.

Frequency and Speed.—Induction motors may be built for any frequency, but 15 cycles and 133 cycles are the limits for power purposes. The standard frequencies are 25, 50, and 60 cycles. Lower frequencies, such as 25 cycles, are more favorable where frequent overloads or large starting torques are required.

The nominal speed of an induction motor is the synchronous speed, which is practically the speed at which the motor operates without a load. Only a limited number of speeds are possible, and these speeds are definitely set by the frequency of the supply and the number of poles, as given in a preceding paragraph.

The speed at rated load is from 2 to 8 per cent less than the synchronous speed, depending upon the losses in the secondary circuit.

Examples of available speeds are as follows:

25 Cycles		60 Cycles	
Pole	R.p.m.	Pole	R.p.m.
2	1500	2	3600
4	750	4	1800
6	500	6	1200

Rating.—The rating is that output in horse power which the motor will give continuously (unless specifically stated otherwise) without the temperature rise, as measured by thermometers, exceeding 40° C. or 50° C. respectively, depending upon the nature of the insulation employed.

Phase Connections.—Two-phase or quarter-phase motors are usually wound with independent phase windings. Three-phase motors are connected in Y or Δ, depending upon the convenience of the designing engineer.

In a single-phase and two-phase motor, the voltage and current per phase are the same as the voltage between lines and current in line; in a Y-connected three-phase motor, the current per phase is equal to the line current, and the voltage per phase is equal to the line voltage divided by $\sqrt{3}$; in a Δ-connected three-phase motor, the current per phase is equal to the line current divided by $\sqrt{3}$, and the voltage per phase is equal to the line voltage.

Currents Taken by Motors.—

Let P_0 = horse-power output;
I = current in each line;
ϵ = efficiency as a decimal fraction;
$\cos \phi$ = power factor as a decimal fraction;
E = voltage between lines (between one outside wire and the middle wire for three-wire two-phase line).

Then for

$$\text{Two phase: } I = \frac{373 P_0}{\epsilon E \cos \phi};$$

$$\text{Three phase: } I = \frac{431 P_0}{\epsilon E \cos \phi};$$

Usual efficiencies and power factors at full rated load for polyphase motors are as follows:

TABLE XLVIII

Power Factors and Deficiencies

Horse Power	25 Cycles		60 Cycles	
	Efficiency	Power factor	Efficiency	Power factor
1	0.79	0.78	0.78	0.78
5	0.85	0.88	0.82	0.88
20	0.88	0.91	0.84	0.91
50	0.90	0.92	0.87	0.92
100	0.905	0.925	0.89	0.92
200	0.91	0.925	0.905	0.92

DESIGN OF INDUCTION MOTORS

The procedure for the design of a polyphase induction motor is very similar to that for an alternating-current generator, but the constants used are different, as the design must be more conservative with respect to magnetic densities and ampere conductors per inch. The designs for a two-phase and a three-phase motor are very much the same except for the subdivision of the windings, and the same general constants apply; but the design of a single-phase motor is very special and follows a different proportioning. The design of all induction motors is very much influenced by precautions for obtaining a good power factor, which requires a generous use of material, a small air gap, and a careful arrangement of the windings. A high power factor at light load involves a small magnetizing current. A high power factor at overloads requires a low value of leakage flux. A small value of magnetizing current is obtained by using a short air gap and a large value of diameter per pole. A low value of leakage flux is obtained by using a large value of diameter per pole and by subdividing the windings in a large number of slots.

Statement of Problem.—It is usually required to design a motor capable of giving an output of a specified horse power at a given speed when supplied with electric power at a certain voltage, frequency, and number of phases, two-phase or three-phase.

The relation between speed, poles, and frequency is fixed, and only certain definite speeds may be had, as explained in a previous paragraph.

Thus the problem of design starts with a statement of the desired values for: horse power, speed, frequency, voltage, and number of phases.

Symbols, Dimensions in Inches

B = average magnetic density in air gap in maxwells per square inch;
D = inside diameter of stationary member, usually the primary and called the stator;
E = volts per phase, primary;
f = frequency in cycles per second;
g = length of air gap in inches;
I = primary current per phase at rated load;

L = length of stator core between heads (the rotor core has the same length);
L_1 = actual length of iron in core = L − space for ducts;
N = revolutions per minute;
n = number of phases in primary;
p = number of poles;
k = ampere conductors per slot;
S = turns in series per phase, primary;
V = peripheral velocity of rotor, feet per minute;
ϕ = flux per pole, maxwells;
ρ = ratio of width of the face of a stator tooth to the pitch of slots;
σ = ampere conductors per inch periphery, practically the same in primary and secondary.

Diameter at Air Gap.—The upper limit is determined by the permissible peripheral speed, and the lower limit by the permissible magnetizing currents. For normal motors without special mechanical construction, the range is from 4000 to 5000 ft. per minute for definite-wound rotors, and up to 6500 ft. per minute for squirrel-cage rotors. With special attention to mechanical design, peripheral speeds of 10,000 to 11,000 ft. per minute are in use. The peripheral speed having been chosen, the diameter will be:

$$D = \frac{12V}{N} \text{ ins.}$$

Length of Machine.—In the case of induction motors the length that counts, L_1, is the actual length of stacked iron, exclusive of the air ducts. The air gap in induction motors is so small that there is no considerable spreading of the flux into the space opposite the air ducts, particularly as both members have the same number and size of ducts exactly facing each other.

There is usually a $\frac{1}{2}$ in. air duct for every 2 ins. of stacked punchings (14 mils in both members); thus the total length of core, L, is about 25 per cent greater than the actual length of iron L_1.

With this reservation, the general equation used for all machines may be applied with a proper selection of the constants:

$$K\text{v-a.} = \frac{\rho\sigma BVDL_1k_1k_2}{144 \times 10^9}.$$

Kv-a. is the apparent power input obtained by assuming a power factor and efficiency as in Table XLVIII:

$$\text{Kv-a.} = \frac{\text{h.p.} \times 746}{(\text{eff.})(\text{p.f.}) \times 1000}.$$

ρ has values of from 0.4 to 0.6 in machines with open slots and 0.5 to 0.7 with partly closed slots in the stator. The rotor should always have partly closed slots with an opening of from $\frac{1}{16}$ to $\frac{1}{8}$ in.

Representative values of σ are given in Table XLIX.

TABLE XLIX

Data on Induction Motors

Diameter, Inches	Slots	Depth of Slots, Inches	Gap, Radial	σ Ampere-conductors per Inch
10	48–60	1 –1.4	.02–.03	350
20	60–90	1.25–1.8	.03–.04	500
30	72–120	1.5 –2	.04–.05	600
50	168–192	1.8 –2.12	.06	850
100	300–420	3 –4	.10	1350
150	260–450	3.5 –4.5	.15	1500

σ may be higher than shown in a large machine operating at a low voltage, and must be less for a small machine or a high-voltage machine.

B, the average magnetic density in the air gap, should be around 30,000 in 25-cycle motors and around 25,000 in 60-cycle motors.

L_1 is the unknown quantity to be determined.

k_1 and k_2 are the winding and pitch constants respectively. For a first assumption, the product of $k_1 \times k_2$ may be taken as .83 for 25-cycle, and .90 for 60-cycle machines.

Preliminary Value of Turns in Series per Phase.—The chosen values of D and σ impose a value for S, thus:

$$S = \frac{\pi D \sigma}{2 \times n \times I'}$$

where n is the number of phases, and I the primary current per phase.

Number of Slots.—As the number of ampere conductors which it is desirable to put in one slot is limited in order to keep the leakage reactance low, we find a desirable number of slots by the relation:

$$\text{Total slots} = \frac{\pi D\sigma}{k},$$

where k is the proper number of ampere conductors per slot and has the following values:

TABLE L

AMPERE CONDUCTORS PER SLOT

Size of Motor, H.p.	Value of k
Up to 5	Up to 250
5–50	250–300
50–100	350–450
100–200	450–600
Greater	600–800

The number of slots indicated is only an approximation, as the figure chosen finally should in general be divisible by both the number of phases and the number of poles. As discussed under "A.C. Generators," it is possible to make a winding in a number of slots which is prime to the number of poles, but the number of slots must be divisible by the number of phases. (See Table XLIX.)

Conductors per Slot, c.—The number of effective conductors per slot is the number of conductors in series per slot; i.e., if each phase contains two parallel windings each actual wire is counted as half an effective conductor because it carries only half the current. Hence the effective conductors per slot are:

$$c = \frac{2Sn}{\text{total slots}}.$$

c must be a whole number and, to make a conventional winding, should be divisible by 2 for two coil-sides per slot, or by 4 if a

winding of four coil-sides is to be used. The type with two coil-sides per slot is more usual. The actual number of real conductors per slot may be made a multiple of c by using multiple circuits.

Size of Conductor.—As a preliminary calculation, the cross-section of the conductor to be used in the primary is determined by the primary current and an assumed value for the current density in amperes per square inch (U) as given in the table.

$$q_1 = \text{cross-section} = \frac{I}{U} \text{ sq. in.}$$

TABLE LI

Size of Motor	Voltage	Amperes per Square Inch, U
Up to 10 h.p.	Below 220	3000
Up to 10 h.p.	Up to 550	2000
Above 10 h.p.	Below 550	2000
Above 10 h.p.	Up to 2300	1600
Above 10 h.p.	Above 2300	1000
Very large motors	.....	1500

It is not advisable to use round conductors in sizes larger than No. 7 B. & S., and not usual in sizes larger than No. 10. If the conductor needs to be larger than No. 10 it is better to use two or more in parallel, or stranded rectangular cables or rectangular bars. Two wires may be used as twin wires, but in large sizes it is better to have independent circuits in multiple. There may be as many multiple circuits as there are poles on the motor, as the windings of each phase under each pole are entirely independent and may be connected up in series or multiple like so many batteries.

Size of Slots.—The first approximation of the cross-section of a slot is found by dividing the total cross-section of the copper of all the conductors which are to go into one slot by a slot factor as given in Table LII.

The width of the slot, w, is next assumed as a certain percentage of the slot pitch at the air gap diameter, 40 to 60 per cent according to the value assumed for ρ in the earlier calculations.

$$w = \text{pitch} \times (1 - \rho).$$

TABLE LII

SLOT FACTOR

Machine Voltage	Slot Factor	
	Small sizes	Large sizes
500	0.35	0.50
1000	0.30	0.45
2000	0.25	0.40
4000	.....	0.35
6000	.....	0.30

From the desired cross-section of slot and the allowable width, the approximate depth may be determined. This gives approximate slot dimensions.

For a more specific determination of slot dimensions: Let a represent the required cross-section of copper per slot (sq. in.). Let w represent the allowable width of slot and let i be the space required for insulation at the two sides of the slot, as in Table LIII.

Then the allowable width of copper is $b = w - i$.
The necessary depth of copper is then $a \div b = d$.
The total depth of slot will be equal to d plus the space required for depth of insulation in the slot as given in Table LIII.

It remains to lay out the a square inches of copper into a shape $b \times d$ by subdividing it into suitable wires, bars, or strands.

TABLE LIII

SLOT INSULATION

Type of Slot	Voltage	Coil-sides per Slot	Allowance for Depth	Insulation Width
Straight.....	500	2	0.30	.08
Straight.....	2200	2	0.45	.15
Straight.....	6600	2	0.65	.25
Overhung.....	500	4	0.40	.15
Overhung.....	2200	4	0.50	.20

The allowance given for insulation in the table includes the insulation on both sides of the slot. In the case of the depth of the slot, the allowance includes: one thickness of insulation at the bottom of the slot, two thicknesses between the two coils, one thickness at the top of the slot and, in addition, space for the wedge which holds the coils in the slot.

Where it is found awkward to fit the required number of conductors into a slot of reasonable proportions, the actual number of conductors may be varied by choosing to use two, four, or six circuits in multiple, in which case the actual conductors per slot is equal to the number of effective conductors multiplied by the number of multiple circuits ($c \times m$). Or the number of conductors required may be varied by a change in the method of connecting the primary winding, Y to Δ or *vice versa*.

The decision as to whether an induction motor shall have a Δ- or a Y-connected primary winding depends upon such minor details of design as the convenience of arranging the conductors in the slots. For instance, if for 110 volts line and a Y connection with 64 volts per phase, it was found that 7 conductors per slot were required, then a Δ connection would have 110 volts per phase and require 12 conductors per slot. In a two-phase machine there is no such latitude.

The final value of the turns in series per phase will then be:

$$S = \frac{\text{conductors per slot} \times \text{total slots}}{2 \times n \times m} = \frac{c \times \text{slots}}{2n}.$$

n = number of phases;
m = number of multiple circuits;
c = effective conductors per slot.

The winding is then laid out in detail and is the same as that described for a.c. generators. If a high value of diameter per pole has been chosen, then the magnetic density in the air gap may be low enough to warrant the use of fractional-pitch coils. A pitch of $\frac{5}{6}$ in 60-cycle machines and $\frac{2}{3}$ in 25-cycle machines are common values.

But fractional pitch should not be definitely chosen until a calculation of the magnetic circuit shows that the magnetizing current will be reasonable.

The secondary should have more slots than the primary, usually one more slot per pole per phase for a definite-wound

secondary. The total gross cross-section of copper in the secondary should be approximately the same as, or a little less than, in the primary, i.e., if the secondary has 20 per cent more slots there may be from 20 to 30 per cent less copper cross-section in each slot. The secondary is always wound for a low voltage, hence the insulation allowance is depth = 0.40, width = 0.15.

If the secondary is to be squirrel cage, the number of secondary slots should be prime to the number of primary slots and should exceed the number of primary slots by about 50 per cent. This prevents magnetic locking at starting.

With a squirrel-cage secondary the secondary resistance must be high enough to give good starting torque, hence the total cross-section of copper should be from one-half to two-thirds the total cross-section of primary copper (square inches copper per slot $\times$ total slots).

In the squirrel cage the insulation is of no importance, and in some motors the bars are driven into the slots like rivets, without any insulation. The slots are always partly closed with an opening $\frac{1}{16}$ to $\frac{1}{8}$ in., merely to limit the leakage flux and reactance.

If partly closed slots with overhanging teeth are used, the slot may take up from 60 to 80 per cent of the pitch, as then tooth density is the limiting factor instead of air gap density. Slots usually are proportioned to have a depth of from four to six times their width.

The slots of one member, usually the rotor or secondary, should be overhung. It is more convenient to make the secondary slots overhung because the voltage of the secondary may have any odd value. It is therefore permissible to specify arbitrarily four bars per slot, which makes a cheap and convenient winding for overhung slots.

It would be better for both members to have overhanging teeth as this would require a lower excitation; but this would require more labor in the winding, and the cost of this labor in the United States is usually greater than the cost of the extra material required with open slots.

Flux per Pole, Final Value.—The phase connection, and therefore the volts per phase, having been decided, the flux per pole is given by the formula:

$$\phi = \frac{10^8 E}{4.44 f S k_1 k_2},$$

where E is the voltage per phase and k_1 and k_2 are the distribution and pitch constants, pages 140 and 142.

Magnetic Circuit.—A tentative layout of the magnetic circuit is next made in the same manner as described under "A.C. Generators." Usual values for the maximum instantaneous flux densities expressed in maxwells per square inch are given in Table LIV.

Since the wave shape of the flux due to the distributed exciting winding is peaked and approximates the shape of a sine wave, the maximum value of the density in the stator teeth, air gap, and rotor teeth will be 1.5 to 1.57 times the average density, while the densities in the two cores will be as found in the armature core of a d.c. machine.

TABLE LIV

USUAL MAXIMUM FLUX DENSITIES

	25 Cycles	60 Cycles
Stator core	70,000	50,000
Stator teeth	90,000	70,000
Air gap	48,000	40,000
Rotor teeth	80,000	60,000
Rotor core	90,000	80,000

The magnetic circuit is chosen in such a manner that the cross-section of each of the five distinct parts will carry its allotted share of the flux with a maximum density not exceeding the values given in the table. If the constants used in the general formula have been chosen with judgment, the only further adjustment to be made would be the choice of the outside diameter of the stator punching and the inside diameter of the rotor punching, which would determine respectively the cross-section of stator core and rotor core. By means of these calculations a preliminary dimensional sketch to scale may be made, showing the parts of the magnetic circuit in two planes, as shown in Fig. 146.

Air Gap.—The air gap or clearance in induction motors is very small, to avoid excessive magnetizing currents. The radial clearance between stator and rotor has usual values as in Table XLIX. In large machines it is about $\frac{1}{1000}$ of the diameter.

With this preliminary layout, the design proceeds by a calculation of the various constants that determine the performance of the motor.

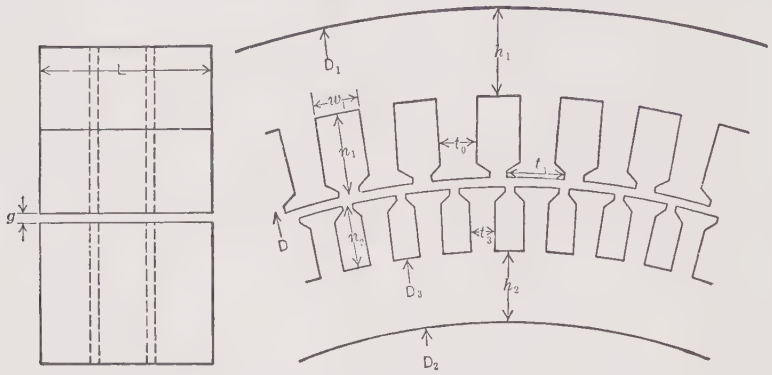


FIG. 146.—Dimensional drawing of the magnetic circuit of an induction motor.

PREDETERMINATION OF PERFORMANCE OF INDUCTION MOTOR FROM ITS DIMENSIONS

Calculation of Exciting Current.—The first step is the calculation of the exciting current. This current is practically constant at all loads and is equal to the no-load current, that is, the current taken by the motor when it is running light. The exciting current has two components, the magnetizing current, which lags 90° behind the impressed voltage, and a component in phase with the impressed voltage, which supplies the coreloss and friction. The magnetizing current is much the larger component and for preliminary calculations may be considered to be thes ame as the exciting current.

The magnetizing current is calculated by determining the flux density in each part of the magnetic circuit and the m.m.f. required for the whole circuit. This m.m.f. expressed in ampere-turns is the product of the maximum value of the magnetizing current and the turns of the phases acting at a given instant. This is calculated by means of the usual tabulation.

Stator Core or Yoke.—The main flux per pole divides in the core, one-half going each way. The maximum density at any instant is equal to one-half the total flux divided by the net area

TABLE LV

MAGNETIC DENSITIES AND M.M.F.'S

(For Dimensions Refer to Fig. 146)

Part	Flux per Pole	Area	B_{max}	A.T. per Inch	Length Path	Total A. T.
Stator core	$\frac{\phi}{2}$	$h_1 \times l$	$1 \times B_{avg}$		$\frac{\pi D_1}{2 \times \text{poles}}$	
Stator teeth	ϕ	$t_0 \times l \times \frac{\text{slots}}{\text{poles}}$	$1.57 \times B_{avg}$		n_1	
Air gap	ϕ	See below	$1.57 \times B_{avg}$		g	
Rotor teeth	ϕ	$t_3 \times l \times \frac{\text{slots}}{\text{poles}}$	$1.57 \times B_{avg}$		n_2	
Rotor core	$\frac{\phi}{2}$	$h_2 \times l$	$1 \times B_{avg}$		$\frac{\pi D_3}{2 \times \text{poles}}$	
Total						

of the core, and this density sweeps around the circle of the core, followed by a period of zero density and then by one of reversed maximum density, as in the core of the revolving armature of a d.c. machine. The net cross-section of the iron is the radial depth h_1 multiplied by the effective length of iron, $l = .9L_1$. The material is sheet steel of high permeability. The ampere-turns per inch length of path are obtained from a magnetization curve of the specified material. The length of the path is indefinite as each magnetic line has a different length, but the average length is closely approximated as one-half the pole pitch measured on the outside perimeter of diameter D_1. The product of magnetizing force and length gives the ampere-turns required for this particular part.

Stator Teeth.—The representative or equivalent density is figured at a point in the teeth one-third the distance from the narrowest part towards the widest, as shown at t_0 in the figure. The total cross-section of the path in the teeth is therefore $t_0 \times l \times$ (teeth per pole). The average density is the total flux divided by this area. Owing to the peaked or sinusoidal space distribution of the flux, the maximum density (at the center of

the wave) is 1.5 to 1.57 times the average density. The number of ampere-turns required depends upon the maximum density. The length of path in the stator teeth is obviously n_1, the depth of the slots.

Theoretically this maximum in the stator teeth, gap and rotor teeth is 1.57 times the average density, but saturation in the teeth flattens the wave so that the ratio is sometimes as low as 1.4. This can only be accurately determined by analyzing this circuit as several magnetic circuits in multiple, and making a saturation curve for each tooth of a pole. Roughly, the constant may be varied with the apparent tooth density thus:

Apparent Tooth Density	Factor
80,000	1.5
120,000	1.45
150,000	1.40

But this is not exact, as the relative lengths of the paths in the teeth and gap affect the ratio.

This constant is used by calculating what the maximum density in stator and rotor teeth would be if the constant were 1.57, and calling this the apparent density, then choosing a new constant based on this preliminary work and making the actual calculation with the revised constant of lower value.

Air Gap.—The flux is not uniformly distributed in the air gap, especially if the openings of the slots are fairly large. The flux passes through each tooth and spreads out from the iron in bunches when it leaves the teeth to cross the gap. The peripheries of the stator and rotor present unequal and dissimilar surfaces passing each other. The lesser of the two is the controlling factor and is the one considered in figuring the reluctance. The effective area of the section of the air gap is

$$(t_1 + g) \times \text{slots per pole} \times L_1.$$

The $(t_1 + g)$ allows for the spreading of the flux in one direction and L_1 allows for the spreading along the laminations. For more accurate results the Carter coefficient may be used.

The ampere-turns per inch are 0.313 × maximum density.

Rotor Teeth.—The calculation of the ampere-turns for the rotor teeth is similar to that for the stator teeth.

Rotor Core.—Similar to stator core.

Magnetizing Current.—The sum of the ampere-turns thus found is the amount required per pole to produce the maximum instantaneous value of the useful flux of one pole. Thus, for the whole machine, the total ampere-turns required would be this number of ampere-turns multiplied by p, the number of poles.

This is produced by the resultant action of the various phase windings of the machine. If the machine is wound with two phases, then this m.m.f. must be produced at all times; but to evaluate the current required in the windings it is convenient to consider that instant at which the current in one phase is a maximum and the current in the other phase is zero. Then the required m.m.f. must be equal to the number of turns in series in that phase multiplied by the current at that instant, which is the maximum value of the exciting current required. Assuming a sine-wave shape for this exciting current, the effective value would be the maximum divided by $\sqrt{2}$. It is recognized that the exciting current does not follow a true sine-wave shape. The error introduced by this assumption is no greater than the variations in the permeability of diferent types of iron or variations in the mechanical dimensions of the air gap, so that it is not worth while except in special cases to consider the irregular wave shape. On this basis the effective value of the exciting current per phase in a two-phase machine would be

$$I_m = \frac{\text{ampere-turns per pole} \times \text{poles}}{\sqrt{2}S} = \text{effective amp.}$$

In a three-phase machine the magnetization is produced by the combined effect of the three windings and the maximum exciting current in one-phase, and may be evaluated by the consideration that at the instant in which the current in one phase is a maximum the current in the other two phases is one-half the maximum value. Consequently the maximum instantaneous m.m.f. in ampere-turns is:

$$SI + \tfrac{1}{2}SI + \tfrac{1}{2}SI, \text{ or equal to } 2SI.$$

Placing this magnetomotive force equal to the required ampere-turns for all poles of the machine, and again assuming that the

effective value of the magnetizing current is equal to the maximum value divided by $\sqrt{2}$, we have for the three-phase machine a magnetizing current given by the equation:

$$I_m = \frac{\text{ampere-turns per pole} \times \text{poles}}{2\sqrt{2}S} \text{ effective amp.}$$

The form factor of the magnetizing current may be 1.15 instead of the theoretical 1.11, and if we can predetermine the form factor we may multiply the effective value, determined as in the preceding, by the ratio of 1.11 to 1.15 for a more accurate determination.

Resistance per Phase of Primary Winding.—The direct-current resistance or ohmic resistance at 75° C. is given by the formula used in all resistance calculations previously discussed, namely:

$$r = \frac{.01 \times S \times (\text{m.l.t.})}{12{,}000qm},$$

where S = turns in series per phase;
q = cross-section of one conductor in square inches;
m = number of conductors or circuits in parallel in one phase.

.01 is the resistance at 75° C. of conductor 1000 ft. long and 1 sq. in. cross-section.

The mean length of one turn is given by the equation:

$$\text{m.l.t.} = 2(L + x) + \frac{10Dk}{\text{poles}} \text{ inches.}$$

(See A. C. Generators.)

Because of the eddy currents set up in the primary conductors by the gross flux and the eddy currents set up in the core and mechanical housing by the leakage flux, the effective resistance of the primary winding is about 25 per cent greater than the value calculated by the above formula. This allowance takes care of what is termed the load loss.

Resistance per Phase of Secondary Winding.—*Definite-wound Rotor.*—In a definite-wound rotor the resistance per phase is found in the same manner as the resistance per phase of the

primary, except for the difference in the pitch diameter of the windings. The secondary winding usually has much fewer turns in series than the primary winding, and the resistance is correspondingly smaller in proportion to the primary resistance and hence cannot be used in the conventional method of calculating an induction motor as a transformer. For the purpose of such calculations the secondary resistance is prorated as if the secondary had the same number of turns as the primary but each conductor correspondingly decreased in size. The secondary resistance reduced to primary turns is then equal to

$$(\text{actual effective sec. res.}) \times \left(\frac{\text{number prim. turns per phase}}{\text{number sec. turns per phase}}\right)^2$$

For the determination of the proper external resistance to be connected in the secondary circuit for purposes of starting and speed regulation, the actual number of turns in the secondary and the actual secondary resistance must be considered for suitable proportions.

Squirrel-cage Secondary.—If the secondary is of the squirrel-cage type it is most convenient to calculate its resistance by first calculating the total I^2r loss in the secondary at rated load and then dividing this by the square of the primary current to obtain the effective resistance of the secondary in terms of the primary turns. This is accomplished by assuming the rated current in the primary and equal ampere-turns or ampere conductors in both primary and secondary, by which means the actual current in each secondary bar may be calculated and therefrom the watts lost in I^2r.

With this explanation of the principle the following procedure will be obvious.

$$\text{Amp.-cond. per sec. slot} = \frac{\pi D\sigma}{\text{(number of sec. slots)}}.$$

σ = amp.-cond. per inch for full-load current.

$$\text{Current density in sec. bars} = \frac{\text{(amp.-cond. per slot)}}{\text{(area of one bar)}}.$$

Watts lost in bars =

$$0.825 \times 10^{-6} \text{ (vol. of all bars)} \times \text{(density in bars)}^2$$

Area of bars per pole = (area of one bar) (number of bars per pole).

$$\text{Current density in rings} = \frac{(\text{current density in bars}) \times (\text{area of bars per pole})}{4\,(\text{area of ring})}$$

Watts lost in rings =

$$0.825 \times 10^{-6}\ (\text{vol. of both rings}) \times (\text{density in rings})^2$$

$$\left.\begin{array}{l}\text{Resistance of sec. in terms of} \\ \text{primary for a three-phase motor.}\end{array}\right\} = \frac{\text{total loss}}{3(\text{full-load prim. current})^2}.$$

If the primary is connected two-phase, the resistance per phase of the secondary in terms of this primary is found by substituting the factor 2 in the place of the 3 in the denominator of the equation. Sometimes a three-phase connected secondary is used with a two-phase connected primary, in which case the resistance per phase of the three-phase winding is calculated normally; and this three-phase resistance multiplied by 1.5 will give the equivalent resistance of a three-phase secondary in terms of the two-phase primary, and may be used in the calculation of the characteristics of the machine by the conventional methods.

Leakage Reactance.—When the motor is loaded the currents in the secondary set up a counter m.m.f. which causes part of the flux to pass along the air gap, between the primary and secondary windings, instead of into the secondary core. This flux does not interlink both members and is therefore a leakage or useless flux. It is proportional to the load current and to the permeance of this path.

The leakage flux passes through a path in air and in iron in series, and as the reluctance of the part in the air is very great compared with the reluctance of the part in the iron, the latter may be neglected without appreciably affecting the final result. This leakage flux may be divided into two components, one interlinking the primary winding only and one interlinking the secondary winding only, thus giving primary leakage reactance and secondary leakage reactance respectively.

For convenience the flux interlinking each winding may be divided into three parts and the equivalent permeance, or **flux per ampere conductor,** for each part evaluated as for d.c. and a.c. generators, except for the tooth-tip flux which has a different path in induction motors.

Slot Flux.—The equivalent permeance of the slot leakage flux is, as before:

$$P_s = 3.2L\left(\frac{u_1}{3w_1} + \frac{v_1}{w_1} + \frac{2r_1}{w_1 + q_1} + \frac{p_1}{q_1}\right),$$

in which the symbols refer to the dimensions in Fig. 147.

As before, if the slot has the same width at the opening as elsewhere (open slot) the second and third terms in the parenthesis disappear.

The Tooth-tip Flux.—On account of the very short air-gap used in induction motors this flux crosses the air gap back and forth

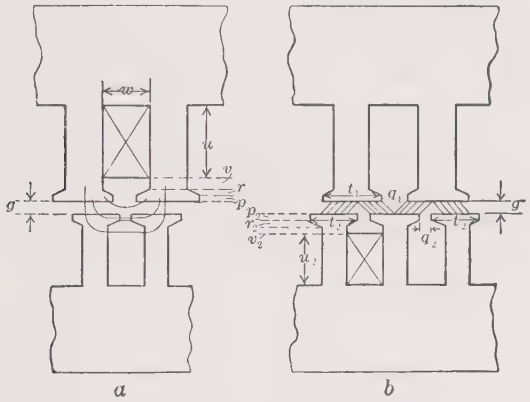


Fig. 147.—Primary and Secondary slots and teeth showing dimensions affecting leakage reactance.

from primary tooth face to secondary tooth face and *vice versa*, giving a more complicated distribution than in a.c. generators. This is known also as the **zig zag** flux. The permeance of this path varies from a minimum, when two slot openings are opposite each other, to a maximum, when a tooth of one member is opposite the opening of a slot of the other. Considering Fig. 147, and concentrating attention on the upper member (assumed the primary and designated by giving its dimensions a subscript 1, as q_1) we have in (*a*) the condition for minimum permeance and in (*b*) the condition for maximum permeance of the path of the leakage flux of the primary.

It is obvious that in (*b*) the flux of the upper slot has to cross

the gap and return by two symmetrical paths so that the reluctance of the two paths in series is:

$$R_{\min} = \frac{g}{\frac{t_2 - q_1}{2}} + \frac{g}{\frac{t_2 - q_1}{2}} = \frac{4g}{t_2 - q_1},$$

and, since minimum reluctance gives maximum permeance, the permeance, being the reciprocal, is:

$$P_{\max} = \frac{t_2 - q_1}{4g}.$$

If we considered the minimum permeance to be zero and the average permeance to be one-half the maximum, we should have for the average permeance, $\frac{t_2 - q_1}{8g}$. But the minimum permeance is not zero and Arnold has shown that the average value is nearer $\frac{t_2 - q_1}{6g}$. Taking this value, the tooth-tip flux per ampere conductor becomes:

$$P_t = 3.2L\left(\frac{t_2 - q_1}{6g}\right).$$

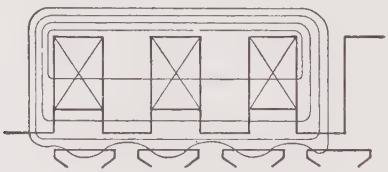


FIG. 148.—Paths of leakage fluxes showing belt leakage.

Belt Leakage.—Some authorities include a belt-leakage flux surrounding the whole group of the slots per pole per phase. This is reasonably approximated by increasing the slot permeance, P_s, and tooth-tip permeance, P_t, by 5 per cent for each slot per pole per phase above one. This factor would be $\left(1 + \frac{s_1 - 1}{20}\right)$.

End Connections.—The flux per ampere conductor around the end connections of the coils is taken the same as for a.c. generators as:

$$P_e = 1.17 s_1 l_c \log \frac{2lc}{Uc} \text{ for single-layer winding,}$$

$P_c = .59 s_1 l_c \log_{10} \frac{2lc}{Uc}$ for double-player winding;

l_c = length of end connections of one coil at one end;
U_c = perimeter of a group of s_1 coil sides, see page 173.

In modern windings, with two coil-sides per slot, the end connections have one-half as many effective conductors as the slot portion. Therefore in a double-layer winding the equivalent flux per ampere conductor would be one-half that of a single-layer winding.

The total leakage flux per ampere conductor of one member, say, the primary, would then be equal to:

$$P_1 = P_s + P_t + P_c,$$

and the leakage reactance would be:

$$x_1 = 2\pi f p s_1 c_1^2 P_1 k_1' k_2' \times 10^{-8} \text{ ohms per phase,}$$

in which p is the number of poles;
c_1 is the effective conductors per slot;
s_1 is the slots per pole per phase;
f is the primary frequency;
k_1' and k_2' are the winding distribution and pitch constants for the primary winding.

Secondary Leakage Reactance.—The three equivalent leakage fluxes per ampere conductor are calculated for the secondary winding in the same manner as for the primary winding, thus:

$$P_s'' = 3.2L\left[\frac{u_2}{3w_2} + \frac{v_2}{w_2} + \frac{2r_2}{w_2 + q_2} + \frac{p_2}{q_2}\right],$$

$$P_t'' = 3.2L\left[\frac{t_1 - q_2}{6g}\right],$$

$$P_c'' = 0.59 s_2 l_2 \log_{10} \frac{2l_2}{U_2},$$

$$P_2 = P_s'' + P_t'' + P_c''.$$

Secondary leakage reactance per phase in terms of secondary winding is:

$$2\pi f p s_2 c_2^2 P_2 k_1'' k_2'' \times 10^{-8} \text{ ohms.}$$

For convenience in calculations of the characteristics of the motor, it is desirable to have the value of the secondary reactance

prorated to be in terms of the primary winding. As in a transformer, this is done by multiplying by the square of the ratio of the number of series turns in each, or, what is the same thing, by $\left[\frac{c_1 s_1}{c_2 s_2}\right]^2$. This gives for the secondary reactance, in terms of the primary winding:

$$x_2 = 2\pi f p s_2 c_2^2 P_2 k_1'' k_2'' \times 10^{-8} \times \left(\frac{c_1 s_1}{c_2 s_2}\right)^2 \text{ohms per phase.}$$

The symbols used are:

f = primary frequency;
p = number of poles;
s_2 = secondary slots per pole per phase;
c_2 = effective conductors per secondary slot;
k_1'' and k_2'' = the distribution and pitch constants for the secondary winding;
l_2 = length of end connections at one end;
U_2 = perimeter of a group of s_2 secondary coils.

Losses in Induction Motor.—The losses in an induction motor are:

Coreloss;
Friction, bearing and windage;
Primary copper loss;
Secondary copper loss.

The first two of these are approximately constant for all loads, and the last two vary as the square of the current per phase.

Coreloss.—The distribution of the magnetic flux in an induction motor is quite irregular, both in the core and in the teeth. The losses are therefore greater and their calculation more involved than in machines having uniform density in each part. In the primary, the frequency of the passage of the secondary teeth introduces pulsations which increase the losses. In the secondary, the frequency, being proportional to the slip, is so low that the coreloss is negligible.

In order to avoid too lengthy and complex calculations, use is made of empirical constants, by which the easily calculated losses are multiplied to derive the practical loss. The total loss consists of hysteresis and eddy loss in the primary core and primary teeth. The loss in watts per cubic inch for one cycle per second at any magnetic density is found by the curves given in Fig. 7.

The losses are then calculated as follows:

Hysteresis loss:

In primary core = $k_h C_1 f V_c$ watts,

In primary teeth = $k_h C_2 f V_t$ watts.

Eddy loss:

In primary core = $k_e C_3 f^2 V_c$ watts,

In primary teeth = $k_e C_4 f^2 V_t$ watts,

where V = volume in cubic inches, C = respective loss per cubic inch from curves, f = frequency, k_h = empirical constant, 1 to 1.5, k_e = empirical constant, 3 to 4.

The higher values of k_h and k_e are to be used where open slots are used and where the frequency of the passage of the secondary teeth past one primary tooth is high. The coreloss is the sum of all the above losses.

Friction and windage loss varies greatly with the style of motor and form of structure, bearings, etc. The loss is made up of bearing friction and wind friction. The latter may be purposely made large, as the motor may be designed with fan blades in order to circulate the air for the purpose of keeping the motor cool by ventilation. While each manufacturer has a formula by which the friction loss can be calculated correctly for machines built according to a particular plan, no general formula can be assumed. The nearest approach to an estimate is obtained from the percentage given in the table below.

TABLE LVI

USUAL VALUES OF LOSSES

Rating, H.p.	$\frac{Im}{I}$	$\frac{XI}{E}$	Friction Loss / Input	Coreloss / Input	$\frac{r_1 I}{E}$	$\frac{r_2 I}{E}$
1	0.45	0.20	0.06	0.05	0.05	0.05
5	0.35	0.14	0.035	0.04	0.04	0.04
20	0.30	0.13	0.025	0.035	0.035	0.035
50	0.27	0.11	0.015	0.03	0.025	0.03
100	0.26	0.11	0.015	0.03	0.025	0.03
200	0.25	0.11	0.015	0.03	0.025	0.03

Primary and Secondary Copper Losses.—The calculation of the effective resistance of the primary and secondary windings is

given above. The primary or secondary copper loss is equal to the product of the number of phases, the effective primary or secondary resistance per phase, and the square of the current per phase. The total copper loss is the sum of the primary and secondary copper losses.

Slip and Speed.—The slip for any given current is approximately:

$$s = \frac{r_2 I}{E},$$

where the symbols are defined in a preceding paragraph.

The synchronous speed is:

$$N = \frac{120f}{p} \text{ r.p.m.}$$

where f is the frequency of the supply and p the number of poles. The actual speed of the motor is then:

$$N_1 = (1 - s)N.$$

Values of Other Characteristics.—The energy component of the "No Load" current is:

$$I_e = \frac{\text{(Total coreloss)} + \text{(Total friction loss)}}{nE},$$

where n is the number of phases and E the volts per phase.

The total "No Load" current per phase is then:

$$I_{00} = \sqrt{I_e^2 + I_m^2},$$

where I_m is the magnetizing current per phase.

The impedance per phase at standstill (**short-circuit** impedance) is:

$$Z = \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2},$$

where r_1 and r_2 are the effective values of the primary and secondary resistances per phase, the latter being reduced to primary turns and x_1 and x_2 the primary and secondary reactances per phase, the latter being reduced to primary turns.

The current per phase at standstill (**short-circuit** current) is:

$$I_s = \frac{E}{Z}.$$

The starting torque in pounds at 1-ft. radius is:

$$T_s = \frac{7.06nr_2E^2}{NZ^2},$$

where n, r_2, E, and Z are as defined above, and N is the synchronous speed in revolutions per minute.

The slip at maximum output is:

$$s_m = \frac{r_2}{r_1 + Z}.$$

The maximum output in watts is:

$$P_m = \frac{0.5nE^2}{1.3(r_1 + r_2) + Z} \text{ watts.}$$

This should be from 1.5 to 3 times the rating of the motor. A 25-cycle motor usually has a greater maximum output or overload capacity than a 60-cycle motor.

Heating.—Since in most motors the primary member is subject to both a core loss and a copper loss and is stationary, the heating of this member is very important. As the secondary contains only a copper loss and is usually revolving, its rise in temperature is usually much less than that of the primary. Moreover, since the secondary is usually of a low voltage, the windings have little insulation and the heating is of no great importance. Therefore it is seldom necessary to calculate the heating of the secondary.

The heating of the primary when, as is usually the case, it is stationary, may be calculated in the same manner as the heating of the stationary armature of a salient-pole alternating-current generator, and the same constants will apply.

There is usually a $\frac{1}{2}$-in. air duct for every 2 ins. of stacked punchings in both stator and rotor to provide a direct path for the air, as the air gap is too narrow to serve effectively for ventilation.

Calculation of Performance Characteristics

The most rational and accurate method of predetermining the characteristics of a polyphase induction motor from the calculated constants of the design is the analytical method advocated by Steinmetz. In this the various circuits of the motor are logically represented as in Fig. 149*a*, by the use of the transformer principle.

This can be simplified by the equivalent single-circuit diagram, Fig. 149*b*, which follows the method of transformer analysis by assuming a one-to-one ratio of transformation of voltages, and gives the same equations.

In this method the value of the slip for a desired load is assumed and substituted in the various formulae. As the slip is the only unknown quantity the output is easily calculated. It is customary to estimate the value of the slip at rated load by the formula given in a preceding paragraph, and to assume values for other loads on the basis that at half load the slip will be approximately one-half that at rated load. By assuming values of slip for $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{1}$, $1\frac{1}{2}$ load and for maximum output (see page 297), a set of curves may be plotted from which the slip, efficiency, power factor, current, and torque for any value of output may be obtained.

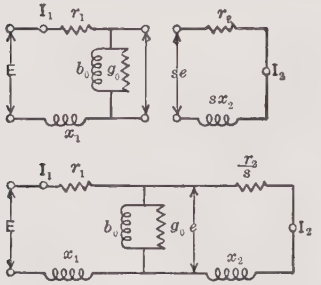


FIG. 149.—Diagrams of circuits of induction motor showing real circuit and the equivalent circuit.

E = impressed voltage per phase;

n = number of phases;

r_1x_1 = primary resistance and reactance per phase;

r_2x_2 = secondary resistance and reactance per phase prorated to terms of primary turns;

g_0 = primary no-load conductance = $\dfrac{\text{coreloss}}{nE^2}$;

b_0 = primary no-load susceptance = $\dfrac{Im}{E}$;

s = an assumed value of slip as .01.

Secondary conductance = $g_2 = \dfrac{sr_2}{s^2x_2^2 + r_2^2}$.

Secondary susceptance = $b_2 = \dfrac{s^2x_2}{s^2x_2^2 + r_2^2}$.

Total conductance $g_1 = g_0 + g_2$.

Total susceptance $b_1 = b_0 + b_2$.

$$c_1 = 1 + r_1g_1 + x_1b_1.$$

$$c_2 = x_1g_1 - r_1b_1.$$

Counter e.m.f. $= e = \dfrac{E}{\sqrt{{c_1}^2 + {c_2}^2}}$.

Current per phase $I = e\sqrt{{g_1}^2 + {b_1}^2}$.

Total volt-amperes input to motor $= P' = nEI$.

Watts output of armature $= P_2 = ne^2g_2(1 - s)$.

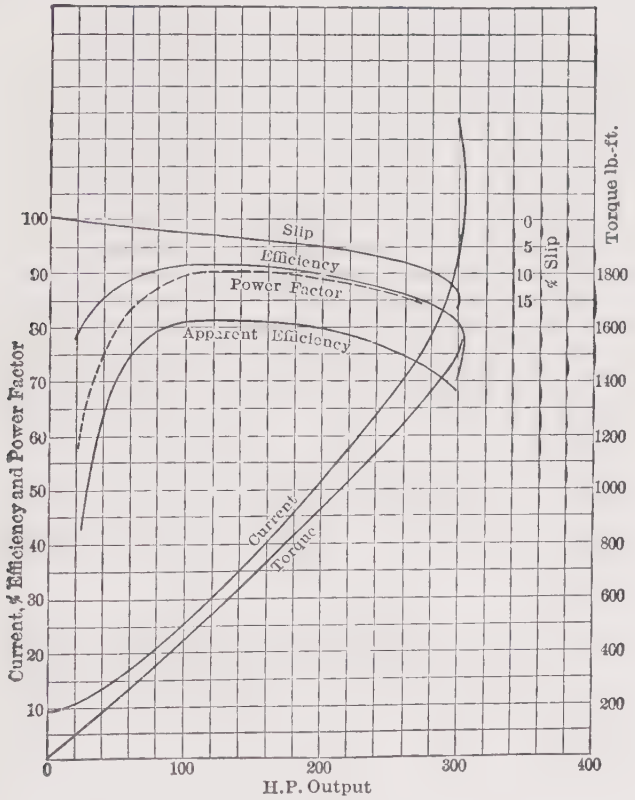


FIG. 150.—Load and Efficiency curves of an induction motor showing usual method of plotting.

Watts output at pulley $P_0 = P_2 -$ (friction in watts).

Watts input to motor $= P_1 = ne^2(g_1c_1 - b_1c_2)$.

Efficiency $= \dfrac{P_0}{P_1}$. Power factor $= \dfrac{P_1}{P'}$.

Design of Starting Rheostat

Let R_2 = actual resistance per phase of the secondary winding;
R_3 = actual resistance per circuit of external starting resistance, both connected Y;
$r_x = (R_2 + R_3)\frac{S_1^2}{S_2^2}$ = resistance of secondary circuit reduced to terms of the primary;
$X = x_1 + x_2$ = total motor reactance per phase in terms of the primary;
E = voltage per phase on primary;
N = synchronous speed in revolutions per minute;
$Z = \sqrt{(r_1 + r_x)^2 + X^2}$.

Then the stationary starting torque is:

$$T_s = \frac{E^2 r_x}{Z^2} \times \frac{7.06n}{N}, \text{ lbs. at 1 ft.}$$

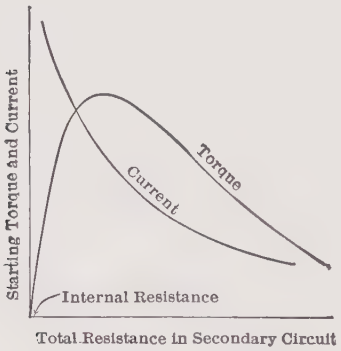


Fig. 151.—Curves showing variation in starting torque and starting current for various values of resistance in the secondary circuit.

and the starting current in the primary:

$$I_s = \frac{E}{Z},$$

starting current in the secondary:

$$I_2 = I_s \times \frac{S_1}{S_2};$$

For a required or assumed starting torque:

$$r_x = \frac{T_s Z^2 N}{E^2 \times 7.06n};$$

$$R_2 + R_3 = r_x \left(\frac{S_2}{S_1}\right)^2;$$

$$R_3 = (R_2 + R_3) - R_2.$$

EXAMPLE OF THE DESIGN OF AN INDUCTION MOTOR

1. Required a 5 h.p. motor for 110 volts, three-phase and 60 cycles at 1200 r.p.m. Must be 6 pole.
2. For a small motor assume a very low value of V, such as 2200.

$$\text{Then } D = \frac{12 \times 2200}{\pi \times 1200} = 7 \text{ ins.}$$

3. Estimated input, kv-a. $= \dfrac{5 \times 746}{.82 \times .83 \times 1000} = 5.5.$
4. Length of core:
   Assume overhanging teeth, $\rho = .8$,

$$\sigma = 450$$

$$B = 30{,}000$$

$$k_1 = .96 \qquad k_2 = .90 \qquad k_1 \times k_2 = .864,$$

$$\text{then } L_1 = \frac{5.5 \times 144 \times 10^9}{.8 \times 450 \times 30{,}000 \times 2200 \times 7 \times .864} = 5.5 \text{ ins.}$$

5. For preliminary purposes try a Y-connection;
   then E per phase $= 64.$ $\quad I = 29.$
6. Approximate turns in series per phase:

$$S = \frac{\pi \times 7 \times 450}{2 \times 3 \times 29} = 57.$$

7. Number of slots $= \dfrac{\pi \times 7 \times 450}{180} = 55.$
   Must be divisible by 6: try 54 and 60.
8. Number of effective conductors on perimeter of armature.

$$c \times s = 57 \times 2 \times 3 = 342.$$

9. Nearest combination 6 conductors in 54 slots $= 324$
   6 conductors in 60 slots $= 360.$
   Try 54 slots with 6 effective conductors in each.

10. Size of effective conductors, $q = \dfrac{29}{1250} = .023$ sq. in.
11. Copper per slot $= .023 \times 6 = .138$ sq. in.
12. Assume a gap $= .02$ in. Inner diameter stator $= 7.04$ ins.
13. Slot pitch at face $= \dfrac{\pi \times 7.04}{54} = 0.41$ in.
14. With overhanging teeth the width of slot may be 60 per cent of pitch.
    Slot width: $w = .41 \times .6 = .246$ say 0.25 in.
15. Assume slot factor $= 0.35$.
    Slot area should be $= .138 \div .35 = .40$ sq. in.
    Approximate depth of slot $= .40 \div .25 = 1.60$ ins.
    Width of copper in slot $= .25 - .05 = .20$ in.
16. Must have a number of wires per slot divisible by 6, aggregating .138 sq. in. and which can be fitted into a space $.20 \times 1.1$.

    (Note: In small motors with overhanging teeth, less than standard insulation may be used and most of it in the form of slot armor, rather than on the coils.)

    After many trials we find that 54 wires of No. 15 B. & S. with d.c.c. will fit.

    diam. bare $= .057$ d.c.c. $= .067$

    area $= .0255$ sq. in.

    Arrangement 3 wide by 18 deep.

    width of slot $= .067 \times 3 + .05 = .25$

    depth of slot $= .067 \times 18 + .60 = 1.60$.

    Make opening 0.10 in. (See Fig. 152.)

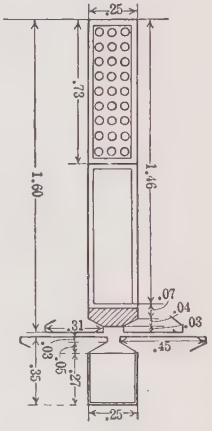


Fig. 152.—Dimensions of slots for the example calculated.

17. Three wires in multiple to form one conductor. Nine turns per coil. Two coil-sides per slot, 18 coils per phase.

    6 coils in series per circuit.

    3 circuits, 3 poles, in multiple.

    Pitch of coils to be seven slots.

18. Secondary slots and winding:

    Choose a squirrel cage of 43 slots. (In a small motor it is difficult and not customary to have a large number of slots in the rotor because of the restricted space.)

    Pitch of rotor slots at face $= 0.51$ in.

    Make slot width $\frac{1}{2}$ of .51 say 0.25 in.

    Use $\frac{1}{2}$ as much copper in secondary because a squirrel cage, having no end connections, has a low resistance and resistance is needed for starting.

$$\text{Copper per slot} = \frac{.0255 \times 54 \times 54}{43} = .06 \text{ sq. in.}$$

$$\text{Width of bar} = .25 - .01 = .24 \text{ in.}$$

$$\text{Depth of bar} = .06 \div .24 = .25 \text{ in.}$$

Depth of slot = .25 + .10 = .35.

Make slot opening $\frac{1}{16}$ in.
Rotor bars = .24 × .25. Rotor slots = .35 × .25 × $\frac{1}{16}$.

(See Fig. 152.)

19. Final turns in series:

$$S = \frac{54 \times 54}{3 \times 3 \times 3 \times 2} = 54.$$

20. Flux per pole: $k_1 = .96$ $k_2 = .90$.

$$\phi = \frac{64 \times 10^8}{4.44 \times 60 \times 54 \times .864} = 500{,}000.$$

21. No leakage factor need be used here because leakage flux is taken care of by the "Leakage Reactance."
22. Design and calculation of magnetic circuit.
Stator Core: diam. at back of stator slots:

$$= 7.04 + 2 \times 1.6 = 10.24 \text{ ins.}$$

Effective length = 5.5 × .9 = 4.95 ins.
Flux in core = 250,000.
Need 4 to 5 sq. ins.
Make radial depth of core = .88 in.
Then outside diam. = 12 ins.
Stator Teeth: diam. at equivalent section of teeth

$$= 7.04 + \tfrac{1}{3} \times 3.2 = 8.10$$

Pitch = .471. Equiv. width tooth = .471 − .25 = .221 in.
Teeth per pole = 9.
Air Gap: width of tooth face = .41 − .10 = .31.

$$t + g = .31 + .02 = .33 \text{ ins.}$$

Rotor Teeth: diam. at equivalent section

$$= 7 - \frac{2 \times .35 \times 2}{3} = 6.53 \text{ ins.}$$

Pitch = .476.
Equivalent width = .476 − .25 = .226.
Rotor Core: diam. at bottom of slots = 6.3.
Radial depth desired = .75.
An inner diam. of 5 ins. gives a radial depth of .65 which will do.

23. Excitation.

		Square inches	ϕ $\times 10^6$	Av. $B10^3$	Max. $B10^3$	H	L	Amp.-turns
S. C.	$4.95\times.88$	4.35	2.5	58	58	7	3.14	22
S. T.	$4.95\times.221\times9$	9.9	5	51	79	19	1.6	30
G.	$5.5\times.33\times9$	16.3	5	30.7	48	15,000	.02	300
R. T.	$4.95\times.226\times7.2$	8	5	63	98	57	.35	20
R. C.	$4.95\times.65$	3.22	2.5	64	64	10	1.4	14
Total amp.-turns per pole								386

Exciting Current.

$$I_m = \frac{386 \times 6}{2\sqrt{2} \times 54} = 15.3 \text{ amp.}$$

24. Primary Resistance.

One effective conductor = 9 × No. 15 wires.

$$L + x = 5.5 + 2 = 7.5.$$

Pitch diameter of winding = 7.04 + 1.6 = 8.64 ins.

$$\text{Mean length turn} = 2 \times 7.5 + \frac{10 \times 8.64}{6} \times \frac{7}{9} = 26.2 \text{ ins.}$$

$$\text{At } 75^\circ \text{ C. } r_1 = \frac{.01 \times 26.2 \times 54}{.00255 \times 9 \times 12{,}000} = .0515 \text{ ohm per phase.}$$

Effective value = .0515 × 1.25 = .064.

$$\frac{29 \times .064}{64} \times 100 = 2.9 \text{ per cent.}$$

Primary copper loss = $(29)^2 \times .064 \times 3 = 162$ watts.

Secondary Resistance:

Use one ring (1 × .125) at each end. Diam. = 6.3 ins. outside.

$$\text{Amp.-conductors per slot} = \frac{29 \times 6 \times 54}{43} = 219.$$

$$\text{Copper density in bars} = \frac{219}{.06} = 3660.$$

Volume of all bars = .06 × 6.5 × 43 = 17 cu. ins.

Watts lost in bars = $.775 \times 10^{-6} \times 17 \times (3660)^2 = 176$ watts.

$$\text{Area of bars per pole} = \frac{.06 \times 43}{6} = .43.$$

$$\text{Copper density in rings} = \frac{3660 \times .43}{4 \times .125} = 3170.$$

Volume of two rings $= .125 \times \pi \times 6.3 \times 2 = 5.$
Watts lost in rings $= .775 \times 10^{-6} \times 5 \times (3170)^2 = 39.$
Total loss in secondary = 215 at 60° c.
Resistance per phase of secondary in terms of primary turns.

$$r_2 = \frac{215}{3 \times (29)^2} = .086 \qquad 3.9 \text{ per cent}$$

(Effective resistance because of low frequency in secondary).

25. Sketch of Slots and Coils, see Fig. 152.
26. Primary Leakage Reactance:

$u_1 = 1.46 \quad v_1 = .07 \quad r_1 = .04 \quad p_1 = .03 \quad w_1 = .25$

$q_1 = .10 \qquad l_c = 7.6 \quad U_c = 2(.73 + 3 \times .25) = 2.96.$

$$P_s = 3.2 \times 5.5\left(\frac{1.46}{3 \times .25} + \frac{.07}{.25} + \frac{2 \times .04}{.25 + .10} + \frac{.03}{.10}\right) = 49$$

$$P_t = 3.2 \times 5.5\left(\frac{.45 - .10}{6 \times .02}\right) = 51$$

$$P_c = .59 \times 3 \times 7.6 \log \frac{15.2}{2.96} = 9.6$$

$$110$$

$x_1 = 2\pi 60 \times 6 \times 3 \times 36 \times 110 \times .864 \times 10^{-8} = .24$ ohm per phase.

Secondary Leakage Reactance:

$u_2 = .27 \qquad v_2 = 0 \qquad r_2 = .05 \qquad p_2 = .03 \qquad w_2 = .25$

$q_2 = .062 \qquad l_c = 0 \qquad U_c = 0.$ No end connections.

$$P_s = 3.2 \times 5.5\left(\frac{.27}{3 \times .25} + \frac{2 \times .05}{.25 + .062} + \frac{.03}{.062}\right) = 21$$

$$P_t = 3.2 \times 5.5 \times \frac{.31 - .06}{6 \times .02} = 21$$

$$P_c = 0 = 0$$

$$42$$

$$x_2 = 2\pi 60 \times 6 \times 2.39 \times 1 \times 42 \times .864 \times 10^{-8} \times \left(\frac{6 \times 3}{1 \times 2.39}\right)^2 = .112$$

per phase.

Total reactance per phase $= x_1 + x_2 = .35$.

$$100\frac{Ix}{E} = \frac{29 \times .35}{64} \times 100 = 16 \text{ per cent.}$$

27. Coreloss: Assume $\eta = .002$ $\epsilon = .0001$.
Volume of stator core = 148 cu. ins. Stator teeth = 111 cu. ins.

Core	$H = 1.25 \times .0072 \times 60 \times 148$	=	80
	$E = 2 \times .000017 \times 3600 \times 148$	=	18
Teeth	$H = 1.25 \times .0117 \times 60 \times 111$	=	96
	$E = 2 \times .00003 \times 3600 \times 111$	=	26
		Gross Coreloss	220

28. Assume friction at 3.5 per cent of input = 160 watts.
29. Efficiency and Losses:

Friction	=	160 watts	3.5 per cent
Coreloss	=	220	4.9
Primary I^2R	=	162	3.6
Secondary I^2R	=	215	4.8
All losses		757	16.8
Output		3750	
Input		4507	

Efficiency per cent 83.

30. Heating of Stator:

$$P_t = 220 + 162 \times \frac{11}{26} = 289 \text{ watts.}$$

$$A_t = \pi \times 12 \times 5.5 + \frac{\pi}{4}(12^2 - 7^2) \times 2 = 356 \text{ sq. ins.}$$

$$T_t = 30 \times \frac{289}{356} = 25° \text{ C.}$$

$$M = \frac{3.22 \times 5.5 \times 54}{230 \times .025} = 166. \qquad N = \frac{1.96 \times 7.6 \times 108}{230 \times .02 + 110} = 14.$$

$$k_3 = \frac{170}{1 + .00025 \times 2200} = 110.$$

$$T_c = \frac{162 + 166 \times 25}{166 + 14} = 24° \text{ C,}$$

TABLE LVII

Induction Motors

Mechanical Data. (Dimensions in Inches)

Type	1	2	3	4	5	6
	3-ph.	3-ph.	3-ph.	3-ph.	3-ph.	3-ph.
Number of poles	4	4	4	6	6	8
Rating in h.p.	5	15	30	5	10	20
R.p.m.	750	750	750	1200	1200	900
Prim. volts bet. lines	440	220	220	220	440	220
Prim. connections	Y	Y	Δ	Δ	Δ	Δ
Frequency	25	25	25	60	60	60
Stator						
Outer diam. punchings	17	21	28	17	18.25	26
Inner diam. punchings	12.06	15.07	19.07	12.094	11	19.07
Total length of iron	6	7.5	8	5	3.75	6
Number of ducts	0	0	0	0	0	0
Width of each duct						
Total number of slots	60	72	72	54	72	96
Depth of slot	1.25	1.25	1.44	1.56	1.5	1.25
Width of slot	0.32	0.34	0.39	0.50	0.3	0.33
Width of slot at face	0.32	0.34	0.39	0.25	0.3	0.33
Wires per slot	36	16	16	64	30	16
Size of wire	No. 14 B. & S.	No. 10 B. & S.	No. 8 B. & S.	No. 14 B. & S.	No. 15 B. & S.	No. 13 B. W. G.
Wires in multiple	1	2	2	2	1	2
Turns in series per ph.	360	96	96	288	360	128
Per cent coil pitch	100	72	67	100	100	75
Rotor						
Outer diam. punchings	12	15	19	12	10.95	19
Inner diam. punchings	8	11	12	8.5	6	14
Total number of slots	37	47	67	72	127	71
Depth of slots	0.56	0.47	0.47	0.94	0.5	0.47
Width of slots	0.56	0.56	0.56	0.34	0.12	0.56
Width of slots at face	0.063	0.063	0.063	0.19	0.02	0.063
Wires per slot	1	1	1	4	1	1
Size of wire or bar	0.5×0.45	0.35×0.5	0.35×0.5	0.34×0.11	0.35×0.09	0.35×0.5
Number in multiple	1	1	1	4	1	1
Cross-section each ring, square inches	1.3	0.94	2	.....	0.198	1.2
Resistance rel. to copper	2	2	2	.....	1	2
Air gap on one side	0.03	0.035	0.035	0.047	0.025	0.035

INDUCTION MOTORS

Electrical Data. (All quantities per phase)

Type	1	2	3	4	5	6
Volts per phase, E	254	127	220	220	440	220
Current per ph. at rating	6.6	38	42	8.1	7.3	31.2
Flux per pole, megalines	0.636	1.2	2.07	0.29	0.48	0.645
Magnetizing current I_m	2.1	9	13	2.64	2.4	12.3
Friction, watts	80	175	530	150	295	390
Coreloss, watts	150	390	1240	180	210	760
Primary res. at 60° C., ohms	2.86	0.193	0.13	1.15	2.78	0.213
Secondary res. at 60° C. ohms	1.76	0.18	0.17	1.11	3.5	0.21
Short-circuit current	36	212	362	29.4	26	205
Y = (Sh. cir. cur.) $\div I_m$	17	23.6	28	11.2	10.8	16.6
Reactance per ph.	5.52	0.55	0.54	8	5.05	1.10
Eff. at rating	0.83	0.853	0.883	0.83	0.86	0.875
Power factor at rating	0.905	0.91	0.917	0.845	0.90	0.83
$I_m \div I$	0.32	0.24	0.31	0.33	0.33	0.39
Fr. $\div P_1$	0.018	0.013	0.02	0.033	0.034	0.023
Coreloss $\div P_1$	0.033	0.029	0.049	0.04	0.024	0.044
$IR_1 \div E$	0.074	0.058	0.025	0.042	0.046	0.03
$IR_2 \div E$	0.046	0.054	0.032	0.041	0.058	0.03
$IX \div E$	0.143	0.165	0.103	0.29	0.084	0.156
Slip, per cent	0.047	0.042	0.032	0.03	0.038	0.03

INDEX

A

Air-blast transformer, 231, 260
Amortisseur winding, 146
Ampere, definition, 1
Ampere-conductors, direct current, 56
  induction motor, 274
  per slot, 278
  salient pole, 150
  transformer, 246
  turbo, 200
Armature diameter, direct current, 55
  induction motor, 276
  salient pole, 148
  turbo, 200
Armature heating, direct current, 63
  salient pole, 171
  turbo, 211
Armature loss, 105, 183, 210
Armature reaction, direct current, 47, 56
  salient pole, 143
  turbo, 200

B

Back ampere-turns, 49, 88
Barrel winding, 133
Brushes, friction, 68
  loss, 67
  movement, 51
  size, 66

C

Calculation, sample, direct current, 112
  induction motor, 301
  salient pole, 186
  transformer, 263
  turbo, 218
Carter coefficient, 81
Chain winding, 132
Commutation, 50
Commutating poles, design, 96
  effect of, 94
Commutator, design, 65
Compensating winding, 98
Compounding, 31
Connection, a.c. generators, 134
  induction motor, 274
Copper, resistance, 16
Coreloss, direct current, 103
  induction motor, 294
  salient pole, 181
  transformer, 252
  turbo, 215
Core-type transformer, 228, 244
Cramp constant, 89
Cross ampere-turns, 49, 88
Cruciform-type core, 244
Current density, direct current, 59
  induction motor, 279
  salient pole, 154
  transformer, 243
  turbo, 202

D

Dielectric strength, 22
Dimensions, main, direct current, 57
  induction motor, 277
  salient pole, 152
  turbo, 200
Direct-current machines, 27
Distributed core, 228
Distribution constant, 139
Dyne, value of, 3

E

Eddy loss, 14
Effectiveness of field, 211
Efficiency, direct current, 106

Efficiency, induction motor, 274
salient pole, 183
transformer, 240
turbo, 215
Examples of design, direct current, 121
induction motor, 307
salient pole, 193
transformer, 267
turbo, 225
Excitation, direct current, 105, 107
induction motor, 287
salient pole, 175
transformer, 251
turbo, 213

F

Field, effectiveness, 211
Field winding, salient pole, 179
series, d.c., 92
shunt, d.c., 90
turbo, 213
Flux per pole, direct current, 75
induction motor, 282
salient pole, 153
transformer, 243
turbo, 204
Forced oil transformer, 231, 262
Form-factor constant, 143
Fractional-pitch constant, 142
Frequency, a.c. generator, 129
induction motor, 273
Friction of brushes, 68

G

Gap, direct current, 57
induction motor, 283
salient pole, 160
turbo, 205
Gauss, definition, 2
Gilbert, definition, 2

H

Heating, armature. (*See* Armature heating.)
commutator, 68
constants, 111
direct current, 108
field, 91, 181, 215
Heating, induction motor, 297
salient pole, 184
transformer, 257
turbo, 216
Henry, definition, 3
Hysteresis, 13

I

Inductance, deduction, 4
unit, 3
Induction generator, 124
Induction motor, connection, 274
currents, 274
efficiencies, 274
examples, 307
general, 268
losses, 295
maximum output, 297
power factor, 274
sample calculation, 301
starting, 300
usual dimensions, 277
Insulation, direct current, 61
induction motor, 280
salient pole, 156
transformer, 248
turbo, 203

K

Kilowatt, definition, 2

L

Lap winding, 37, 133
Leakage coefficient, direct current, 76
salient pole, 163
turbo, 204
Length, effective, 84
Losses, values of, direct current, 107
induction motor, 295
salient pole, 183
turbo, 216

M

Magnetic circuit, direct current, 76
induction motor, 283
salient pole, 160
transformer, 251
turbo, 206

Magnetic density, gap, direct current, 56
induction motor, 277
salient pole, 150
turbo, 201
iron, direct current, 77
induction motor, 283
salient pole, 161
transformer, 243
turbo, 204
Magnetization curve, a.c. generator, 165
various materials, 12
Magnetizing current, induction motor, 287
transformer, 251
Maximum output, induction motor. 297
Maxwell, definition, 2

O

Oersted, definition, 2
Ohm, definition, 1
Oil-cooled transformer, 258

P

Peripheral velocities, direct current, 56
general, 23
induction motor, 276
salient pole, 149
turbo, 200
Permeability, definition, 2
Permeance, definition, 2
Phase voltages, 130
Pitch constant, 142
Pole arc, 24
Pole pitch, 24
Power factor of induction motors, 274

R

Rating, direct current, 32
general, 20
induction motor, 273
salient pole, 127
transformer, 231
Reactance, leakage, direct current, 68, 75
Reactance, leakage, induction motor, 290
salient pole, 171
transformer, 253
turbo, 216
synchronous, 174
Reaction of armature, direct current, 47, 87
salient pole, 143
turbo, 200
Regulation, direct current, 107
salient pole, 175
turbo, 213
Reluctance, definition, 2
Resistance, induction motor, 288
transformer, 252
Resistance of armature, direct current, 63
salient pole, 169
turbo, 210
Resistance of field, salient pole, 180
series, 93
shunt, 92
turbo, 214
Rotor, construction, 197, 209

S

Salient-pole generator, 124
Series-field winding, 92
Shell-type transformer, 228, 248
Shunt-field winding, 90
Slip, 296
Slot factor, direct current, 60
induction motor, 280
salient pole, 155
turbo, 202
Slot insulation, direct current, 63
induction motor, 280
salient pole, 158
turbo, 202
Space factor, 245
Speed, direct current, 54
induction motor, 273
salient pole, 148
turbo, 200
Squirrel-cage rotor, 289
Stability factor, 86
Starting of induction motor, 300

Steel, 11
Steinmetz calculation, 298
Stray-load loss, direct current, 105
  salient pole, 182
  turbo, 215
Stray-power loss, 104
Synchronous generator, 124
Synchronous reactance, 174

T

Thermal resistivity, 112
Tooth density, equivalent, 84
Torque, induction motor, 297
Transformer bushing, 250
Transformer, constant-current, 237
  design constant, 241
  examples, 267
  general, 226
  series, 238
  volts per turn, 242
Turbo-generator, examples, 225
  general, 196
  sample calculation, 218
Turn, mean length, 63, 169, 253, 288

V

Ventilation, transformers, 261
  turbo-generator, 216
Volt, definition, 1
Voltages, standard, a.c. generators, 129
  direct current, 34
  induction motors, 273

W

Water-cooled transformer, 261
Watt, definition, 2
Winding, connection, 135
  pitch, 38
  three-phase, 136
  types, alternating current, 131
    barrel, 133
    chain, 132
    direct current, 36
    lap, 37, 133
    multiple drum, 37, 132
    multiplex, 45
    series, 41
    wave, 41
Wire table, 15

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